Physics 141

SUPPLEMENTAL NOTES AND PROBLEMS

FORCES, MOTION, and NEWTON'S LAWS

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The word physics, translated literally from the Greek, means "natural things." It refers to rainbows and snowflakes, clouds and lightning, waterfalls and whirlpools, rosy dawns and brilliant sunsets, ocean surfs and ripples on puddles, to the multitude of splendid forms and ceaseless transformations that we experience as the material world.

Hans C. von Baeyer, William & Mary

Newton's laws tell us how matter behaves when it is acted on by forces. The only two things we need to know about the physical world that Newton's laws don't tell us are: What is the nature of matter? What is the nature of forces that act between bits of matter? These two questions are still the central concerns of physics.

David Goodstein, Caltech

It is the most persistent and greatest adventure in human history, this search to understand the universe, how it works and where it came from. It is difficult to imagine that a handful of residents of a small planet circling an insignificant star in a small galaxy have as their aim a complete understanding of the entire universe, a small speck of creation truly believing it is capable of comprehending the whole.

Murray Gell-Mann, Caltech

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INTRODUCTION

What the study of physics is about – and how you should go about it.

Part 1 – FORCES

The nature of forces – interactions between objects.

Part 2 – KINEMATICS

The mathematical description of motion.

Part 3 – NEWTON’S LAWS

How forces are related to the motions of objects.

Part 4 – WORK AND ENERGY

Work and energy principles – The conservation of energy

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INTRODUCTION

This introductory calculus-based course in physics is the foundation for many other courses in science and engineering. Topics include the description of motion, Newton's laws, the concepts of forces and accelerations, work and energy, impulse and momentum, torque and angular momentum, and the three conservation laws (energy, momentum, and angular momentum) that characterize the motions of objects and the principles that govern that motion. That is, it is ultimately about *forces* and *motion* - and, more specifically, the principles that apply to determining the motion of objects and small systems of objects. The material covered here falls in that category of topics called *classical mechanics* as distinct from other areas of physics such as thermodynamics, electromagnetism, optics, quantum theory, atomic or nuclear or condensed matter physics, or relativity and cosmology. Here, we will limit our attention to how things move and the forces that govern how they move and the principles that apply to that motion.

This set of supplementary notes and problems has been assembled to give an overview of the main ideas of this course. It is intended to help you in your study by identifying the underlying principles and concepts and how they interrelate. You should not think of this supplement as a stand-alone text or a substitute for the lectures or discussions or your own study of the textbook itself. But rather you should look at each of the sections in this guide prior to beginning the chapters that cover the material in detail - and then again after you have spent time studying the chapters and working on the homework problems. The questions and problem sets at the end of each of the sections are intended to make you think differently than the typical numerical homework exercises. Many of the questions are more suitable as discussion questions than as problems that you simply work out using standard techniques. You should use the questions and problems to periodically test your own understanding of the material. If you cannot explain the principles involved in these questions and problems, then you do not yet have a good grasp of the material. Nearly every problem or question in these sets has at some time been used as a quiz or test question. If you have trouble with questions of this type, go back to the textbook, your lecture notes, and the material presented in this supplement and look not at the equations but the ideas. And make good use of the other resources available to you - including instructor office hours.

Physics is ultimately about forces - the interactions between bits of matter - and how those forces relate to motion. It is interesting not because we can solve so many problems, but because we can reduce the huge variety of things that happen in the universe to a few fundamental concepts and principles and forces!

There is a difference between *studying* science and *doing* science. Doing science is much like assembling a jigsaw puzzle but a puzzle without the boundary "edge" pieces - or even a picture to guide you, although you may be convinced that it will make sense as you assemble it. (One of the *thema* of science is that the universe is ultimately understandable.) To do such a puzzle, you must examine each piece carefully, learning its shape and colors and patterns and symmetries, turning it over and seeing it from different perspectives. Then you must look to see how it connects to other pieces - whether it adds to some fragmented part of the existing picture or whether instead it seems to open new vistas. For no matter how familiar you are with any one piece, no matter how well you know and understand it, it will still make very little real sense until it is seen in the context of the other pieces that surround it. Then as pieces begin fitting together, they will create a larger more understandable pattern or picture.

When you *study* science, you are covering ground that others have already covered. The logical connections are already understood by others - and your study of the material involves making those connections for yourself - to form a coherent picture in your own mind. You must be willing to explore - take an idea or question and examine it, see where it came from, turn it over and look underneath it, pull it apart if necessary and examine its components. Then reassemble it and see how it fits with other ideas. It is also much like doing a jigsaw puzzle. The difference between studying science and *doing* science, however, is that the picture you are trying to assemble has been done by others - and hence there is a guide picture. But just looking at the guide picture is no substitute for assembling the puzzle yourself. The discovery - in fact, the *excitement* of the discovery - is very personal. That others already understand the ideas and how they go together does not detract from the value of your own quest to understand.

GREAT IDEAS THAT HAVE CHANGED OUR WORLDVIEW

Although you can think of "Physics" as just a course that must be taken - or a body of knowledge that must be mastered at some level in order to proceed in some science or engineering or architecture curriculum, it is really about *ideas*. Its origins are in philosophy - or more specifically, *natural philosophy* - the quest to understand how the universe works. In that light, it is useful to see what some of those grand ideas are about and how they have changed the way humans view their universe. And even though some of those ideas are now familiar, they were not always so - and they certainly are not intuitive. These are indeed revolutions of human thought - triumphs of the human intellect.

The Copernican Revolution

We are all comfortable with the idea that the earth is one of nine planets orbiting our sun which is itself just one of the literally billions of similar stars in our Galaxy which is just one of billions of galaxies. But those ideas were slow in coming - and it was only with the publication of *On the Revolutions of the Heavenly Spheres* by Nicholas Copernicus in 1543 (on the day he died) that a model of the solar system with Earth as just one of the planets was formally presented. Nearly a century later, Galileo suffered greatly at the hands of the Roman Inquisition for espousing that idea - and spent the last eight years of his life under house arrest unable to publish his works.

Newton's Synthesis and the Universality of Physical Laws

Isaac Newton, who was born the year Galileo died one hundred years after the publication of Copernicus' *Revolutions*, developed the laws that govern the orbits of the planets. In the process, he made an enormous intellectual leap in stating that those laws apply equally well to *all objects in the universe!* That idea - that physical laws govern the universe and how it works - is a central idea in all of science. Newton articulated the relationship between forces and the changing motions of objects. But more than that, he expressed that forces are interactions between objects and determine how those objects behave. In that sense, Newton's laws are intimately related to the idea of *causality*. A consequence of Newton's synthesis is that the entire universe is a system of interacting objects - a mechanism that is governed by physical laws. The broad subject which includes those principles is called *classical mechanics*.

Energy and Entropy

These two ideas were developed over a period of about one hundred and fifty years - and can be thought of as describing the organizing principles that explain the macroscopic universe, from describing chemical processes - and hence life-processes - to describing the large scale behavior of galaxies and clusters of galaxies. The concept of *energy* is a central idea in all of science and engineering. And if the universe can be thought of as a mechanism, it can be said that energy is what drives that mechanism. And while energy can be both transferred between objects and transformed, it can always be accounted for. But even though energy is always conserved, it becomes less available for doing work - a principle expressed in the concept of *entropy*. That principle gives to natural processes a "direction" - that is, it assigns an order in time to natural processes. In that sense, the concepts of energy and entropy put limits on what can happen - and the order in which those events can occur.

The Conservation Laws

Expressing that energy can be either transferred or transformed - but can always be accounted for, is the equivalent of stating that it is a conserved quantity. The principle of energy conservation is just one of the conservation laws that are particularly useful in summarizing the laws of physics. When a quantity is invariant or unchanging, it becomes a very useful quantity to evaluate in the analysis of the behavior of objects or systems of objects. Newton himself first expressed the idea that certain quantities are conserved. He postulated, for example that mass is conserved in all processes in closed systems - ultimately, as we now know, because all matter is made up of constituent atoms - and they can always be accounted for. He also expressed the conservation of momentum in describing interacting objects or systems of objects. And energy conservation follows directly from his laws of motion. The conservation laws thus take on great importance in understanding the behavior of systems.

The Atomic Hypothesis

For well over two millenia, since the time of Democritus and Lucretius, the idea that everything is made of distinct atoms has been a part of natural philosophy, although neither understood nor even accepted even by many natural philosophers and scientists over much of the intervening time. Even as recently as the beginning of the twentieth century - within the lifetime of some humans - the existence of atoms as individual constituents of matter could not be verified without some doubt, and it had been assumed that any direct observation would be impossible. But the *atomic hypothesis* has become the *atomic fact* - and we now know that all materials are constructed of individual atoms that are distinguishable and distinct. Furthermore, the study of atoms in the first half of the last century has led to an understanding of the very structure of the atoms themselves and to the remarkable conclusion that we now know the characteristics of every type of atom that is even possible *in the entire universe* up to atomic number 110, or so, and the organizing principles for any that are heavier which are either discovered or manufactured - even in the core of some supernova in some distant unobservable galaxy. Even if the "modern physics" concepts of quantum mechanics and relativity - which changed our worldview from the classical physics of the previous centuries - are replaced by a "new" physics, it will not change what we know about the possible atoms that can exist. All that can be changed are the underlying descriptions - the mathematical formalism or interpretation that describes nature at the atomic level - but not the knowledge of the nature of the atoms themselves.

Relativity and Space-Time

Although it will not be a part of this course, the grand ideas surrounding Einstein's relativity have brought profound changes in the way the basic concepts of space and time are understood - and, in fact, have shown that Newton's laws are not strictly valid under all circumstances. (The conservation of mass, for example, is only approximately true since mass and energy are related through Einstein's famous equation $E=mc^2$ - but the conservation of mass and energy *together* is always true.) And as abstract (and even obscure) as the theory of relativity is, it is verifiable to a great degree of accuracy - and even plays a very important role in everyday life. The global positioning satellite technology that allows such precise determinations of positions of objects anywhere on the surface of the earth would not be possible in the absence of calculational corrections based on Einstein's general theory of relativity. And without that technology, even your cell phone - the operation of which depends on communication satellites - would not work properly.

Uncertainty and the Quantum Theory

The fundamental structure of the atom cannot be understood in terms of the classical physics of Isaac Newton. At the very core of our understanding of interactions between the constituent particles of all objects is the quantum theory - and that represents a fundamental change in our view of nature at the microscopic level. And inherent to that change in view is that nature behaves in probabilistic rather than deterministic ways. That idea - and the complex mathematics that goes with it when dealing with atomic and sub-atomic systems - calls into question the very ideas of causality. And although the quantum theory, when used to solve problems that can be experimentally tested, has never failed to achieve a high degree of accuracy, the interpretation of the theory is still developing - and could continue to evolve over a long period of time and could even be replaced by a more fundamental view of nature at the microscopic level.

Information Theory and Genetics

A fundamental change in the way we view, transfer, and store information occurred with the digital revolution. Information became data. Shannon's Law (articulated the same year the transistor was invented - a half-century ago) that information can be accurately transferred through noisy channels only if it is both digitized and redundant (thus allowing accuracy in encoding and decoding information) has led to the astonishing progress we have seen in both information technology and in molecular biology. Both computers and the replication of genes works on the same principle: Information can be faithfully reproduced if that information is in digital form and is sufficiently redundant, and mechanisms exist to decode the information - and that is the essence of molecular biology and the genetic code.

PHYSICS AND PROBLEM SOLVING

Studying Physics

There are some common misconceptions about the study of physics - especially by those in their first year of that study. It is often thought to be about the *equations* - and finding the correct equations which can then be used to solve some given (and often seemingly artificial) problem. Because of that misconception, many students - myself included in my first year - try to memorize the equations that they perceive will be needed to solve the huge variety of possible problems and to learn the procedures and "tricks" for many special case problems. It seems that such an approach ought to work just fine - especially since tests are often just a set of problems that need to be solved. But there are two very serious flaws with that approach: First, even when one *is* successful at it, only certain special case problems are learned and "solved" without really understanding them or the physics that relates to them. And second, it is nearly impossible to *be* successful with that approach, except perhaps in the very short run. Even if it were possible, being "successful" in a physics class without understanding the physics is not being successful. Physics is ultimately about ideas - principles that can be stated and interpreted and tested and justified. And the goal of studying physics should be to learn about those ideas and principles - and how they can be applied to problem solving.

Studying physics does involve a lot of problem solving - in fact, that can be considered one of the primary goals of the study. But learning problem *solutions* and learning problem *solving* are two very different things. And what you are trying to learn in this course is how to solve problems - not the solutions to special case problems. Probably no one - ever - will ask you to reproduce some specific problem solution (with the possible exception of during some test while taking some class). But the approaches to problem solving are an integral part of the study of physics. Problem solving is ultimately about making decisions. And when the principles and ideas of physics are *understood*, those decisions are often easy to make. And even very complex physical world problems can be set up and solved without relying on (often incorrectly) memorized equations or solutions. It is not that you won't end up knowing some equations and the values of some quantities - but they will have been learned, or *internalized*, as a result of having used those equations and quantities repeatedly in problem solving.

But the essence of problem solving is neither the equations nor some fixed set of procedures, but rather the ability to understand what the problem is *about* and what decisions have to be made based on physical principles that would lead to a solution. That is, you want to be able to read the problem, pull it apart and determine what you have to know and what general principles apply. Problem solving is ultimately about being intensely *logical* - and trusting in that logic because it is based in the knowledge of the principles that apply. When faced with some new problem, you will need to think out the problem and form a solution *pathway* which involves a series of logical steps. It is only after you have clearly understood the problem and found a solution path that you can be confident of the solution when you obtain it. It is in that way that physics problems are no different than any other problems - even *life* problems. You first must have a clear picture what the problem is - and then see what principles and ideas apply to it, then formulate a clear logical pathway through it. And you must be patient - not expecting a solution to always come immediately.

"*The formulation of a problem is often more essential than its solution.*" - Albert Einstein

Estimates, Units, Dimensions, Significant Figures and Problem Solving

Most real problems - at least most *interesting* real problems - do not have a simple equation that you can just plug into to solve. Nor do they have a set procedure that you have to follow - an algorithm, if you will - for solving the problem. They will, however, have a lot of common ingredients and approaches - and those are the things you should be learning from the many problems you will be asked to solve. Often, just knowing what kind of quantities are involved in a problem - either in the problem statement or in the kind of answer that is sought - will itself be a clue to how the problem should be approached. For example, if the problem is asking about the time it takes for something to occur, any numerical solution to the problem will need to carry time units (seconds or hours or days or years) as part of the answer. If you want to know how long it takes to drive to Los Angeles, you only need to know the distance (~200 miles) and the average speed at which the trip will be made (~50 mph). The solution is then easy, since the only way to obtain an answer in units of hours is to divide the distance by the speed. If you really needed the time in seconds, a unit conversion from hours to seconds is necessary, but you do not need to redo the problem. Of course, that is a trivial example - but it illustrates the approach that can work for many complex problems as well. But notice that carrying the units along with the calculation will then help you understand the result of that calculation. (The answer to the above question is not just "four", since the number 4 is not a time, but rather "4 hours" or "14,400 sec." or "0.0004566 years", depending on the units you need the answer in.

Very often, an estimate of the answer or an approximate answer is all that is needed - and that always makes the calculations easier. In the above problem, the distance depends very much on where you start in San Luis Obispo to where you intend to go in Los Angeles - so the 200 miles is itself an estimate. The average speed also depends on lots of variables (including the driver or the time of day or whether there was construction on the 101!). So the final answer could be expressed with a lot of uncertainty implied by not including more significant figures than are known. "4 hrs" or "1.5×10^4 sec" or "5×10^{-4} yrs" all convey the essence of the correct answer. The use of scientific notation can also help convey how well the answer is known and the approximate "size" by focusing the reader's attention on the powers of ten. And if that result is used in a subsequent calculation, then using scientific notation is always easier - and significantly reduces the number of calculator errors that often creep into calculation problems.

It is a useful thinking or logic skill - which means a useful approach to problem solving - to develop skill in estimating answers. It is not about doing complicated numerical problems in your head, but rather making reasonable "order of magnitude" estimates or "rounding" values in problems to just one significant figure in order to quickly carry out a calculation in order to get an idea of what a careful solution will yield as an answer. Sometimes it is not important to have an exact answer - just an estimate - and this "quick and dirty" method is invaluable. Sometimes not all the information is even given in a problem - and so making a reasonable estimate for the value of some variable may be the only way to arrive at a numerical result. For example, I don't know how much gasoline is used in the US each year, but if there are 100 million cars, and each is driven 10 thousand miles per year and averages of 20 mpg, the average car burns 500 gallons of gasoline per year (at a cost of about two thousand dollars at current prices). So the total fuel consumption is about 50 billion gallons! Is that right? I have no idea. But the method of approach is reasonable, and the result is probably within a factor of two since each value used in the calculation is a reasonable estimate.

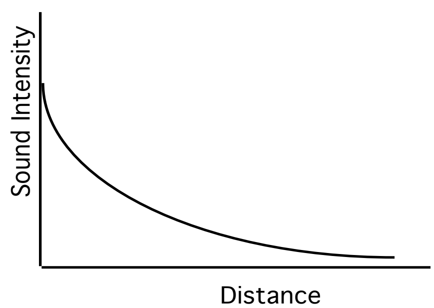
Numerical and Algebraic Problems

There are real and valuable reasons for numerical problems in physics. They allow you to put a scale on problems. They allow you to connect some seemingly artificial problem to life experiences - or conversely, when a problem is about something that is not a part of your experience it allows you put the problem in context. Many times numerical answers can be used to help you decide whether an answer is reasonable. Had you miscalculated and obtained 40 hours for the time to drive to Los Angeles, you might have suspected that you've made a mistake (unless the trip were being made on your skateboard). But it is almost always valuable to set up a problem completely in symbols - i.e., set up a problem algebraically - before substituting in the numerical variables needed to obtain a numerical result. The reason will become clear through solving many problems, but let me continue with the above example: If the time it takes to drive to LA is given by $\Delta t = D/v$, then the time it takes light to get here from the sun is given by the same equation even though the variables have different values. Or if the question is, "By what percentage can you reduce your time by increasing your speed by 10%?", the algebraic expression can often let you answer without doing two different numerical problems to find the time for two different speeds. As we proceed through the course, we will see many problems in which some quantities which might not be known that one might think are important to the problem actually cancel out of the solution - i.e., turn out not to be important at all because they do not affect the answer. Those problems are sometimes solvable only by doing the problem algebraically - and if a numerical answer is needed, making the appropriate substitutions at the end of the problem.

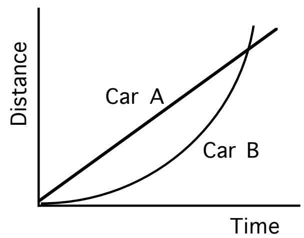
But there is a much more important reason to do problems algebraically. When a problem is done algebraically, the logic of the problem solution becomes very clear and very traceable. If a mistake is made in the problem, where the misstep occurred can usually be found - and that makes it easier to correct any misconception that occurred in the approach to the problem. Furthermore, when a problem is solved algebraically - that is, without ever substituting numerical values into it - what is actually solved for is the relationship between the physical quantities that relate to the problem - and it represents the solution to all identical or equivalent problems regardless of the numerical values of the quantities. And very often real insights can be obtained by, say, graphing the results - perhaps plotting the distance travelled as a function of time for some accelerating car rather than just calculating where it will be a some specific later time.

Functional Relationships and Graphing

The idea of a functional relationship has been a part of your studying mathematics for as long as you have been doing so. And yet, many students are still not altogether comfortable with it. Most students *are* quite comfortable at using a mathematical equation to substitute values into to obtain another value. And in that sense, the equations of physics just become prescriptions for calculating values of things. And although that might be one of the valid uses of the mathematical descriptions that will be be a part of this study, it is hardly the only use - and it is certainly not the most important. Functional relationships between quantities have much more value than just a recipe for doing calculations. For example, if the intensity of a sound diminishes the farther you are from the sound source, there must be a mathematical relationship that expresses that idea. The separation between the source and the listener can be expressed as a variable - and that variable will probably be in the denominator of the expression for intensity as a function of distance from the source. The relationship would probably be something like $I(r)=k/r$ or $I(r)=k/r^2$, or something, where k would be a constant that does not depend on that distance of separation. Such an expression states that as the distance increases, the sound intensity would decrease.



Functional relationships can be represented many different ways. The most common, of course, is in a mathematical expression - an equation which relates the quantities involved. Another way of representing the relationship between two quantities is to create a table of numbers - like a computer printout - that shows a one-to-one correspondence between specific values of one of the quantities and the values of the other. But one method of representing a functional relationship between quantities that is often very revealing is to show a graph of the relationship. Such a graph of the intensity of sound as a function of separation from the source (as in the above example) would show an ever decreasing value the larger the separation. (The curve would be something like a hyperbola if the relationship were $I(r)=k/r$ or would diminish even more rapidly if it were of the form $I(r)=k/r^2$.)



Often actually sketching the function that represents some problem can often give great insight into how a problem "works" - and even represents a tool for solving problems in many cases. For example, suppose car A is traveling at constant speed and passes car B which is initially at rest. Car B then accelerates to catch and pass A. Car A's graph would be a straight line and it's slope would represent the car's speed. If B accelerated from a standstill, its function curves *up* since its slope (which represents car B's speed) continuously increases. The motion of both cars can be graphed as functions of time in order to find when B catches (and passes) A. That is, when the graphs of the two cars intersect, the horizontal location of the intersection along the time axis represents *when* car B passed car A. The vertical location of the intersection along the distance axis represents *how far* they both travelled to the point that B passed A. And furthermore, the slope of B's graph is that car's speed *as* it passed A. There is a lot of information in these graphs.

Quizzing and Testing

The purpose of quizzing and testing, of course, is to give you a chance to show what you understand and what you can do with your understanding. Quizzes and tests will often involve problems to solve. But they are never just "plug-in" problems where simply knowing an equation and some values will let you "solve" for some unknown quantity by just substituting the variables with numbers and doing the calculation. Any good quiz or test problem should give you the chance to explain why you are doing the problem the way you are - i.e., what the ideas are and how those ideas suggest that your method of solution is correct. My tests will only occasionally involve numerical problems - and when they do, the numbers can be worked out easily without a calculator. That means, of course, that you need to know how to make use of scientific notation, and can round values up or down to make the numerical processing more palatable - that is, easy to do without carrying out calculations in the margins of the test. The point is not to make the tests harder - but rather to make them easier. If you can set up a problem correctly and can explain why it is correct based on principles and can show how to reduce the algebraic formulism to a result which can then be calculated, you have accomplished most of what is needed in a problem. But there is still merit to carrying out the calculations, or simply estimating what the calculation would yield if you actually did the calculations since sometimes an answer that is numerically unreasonable will illuminate a mistake in the problem that can be fixed.

Often, the essence of a problem comes from the set up - and the explanation of the approach or the principles that apply to the problem. Often the underlying concepts can be addressed with questions that require choices - either selecting from among multiple choices or identifying a statement as either true or false. But those decisions should always be based on sound reasons - and those reasons should always be stated. In my view, multiple choice or true/false questions are valuable only if they require that you express a reason for your choice. And in problems of that type, developing the reason or justification before you commit to an answer ("True" or "False" or choice "(b)", for example) will nearly always be more successful than answering the question and then trying to think of the reason. And it will happen from time to time that in formulating an explanation, you will discover that your initial thoughts about the question were incorrect - that what you thought would be a correct response is not consistent with the reasoning that you applied to the problem. All problems are ultimately about making decisions. And each of those decisions should be based on the principles involved in the problem.

-- Ron Brown, Physics Dept.

What follows are a variety of questions and problems that do not require specific "physics" knowledge so much as general knowledge and a curiosity about how problems should be solved. To do the problems, you may need to look up some information in the tables in the textbook - the masses of certain objects (electrons, the earth, ...) - or you may need to just estimate some number if it is not readily available. The answers themselves are not important to your further study, but *obtaining* the answers - that is, the logic behind figuring out the answers - *is* important because it requires you to read the problems carefully, think about them, pull them apart, and try to formulate a solution path. The solutions themselves are not difficult - that is, they do not require a lot of calculations or algebraic manipulations - once you see your way through them. But they do require that you think about them and draw on what you already know. And when you see the logic of the problem, the solution itself will often be easy. In that way, they are not unlike many problems in physics and in life.

There are also some mathematics review problems. They are intended to be relatively simple examples of some of the kinds of problems and techniques that will be used in solving problems. These math problems should be thought of as a diagnostic, in the sense that they represent the types of mathematical skills that you will need in this course. They really should be done alone. And if you have any difficulty with any of them - or if you are just unsure of yourself, you should ask about them. You are expected to have skill with algebra, some geometry, some trigonometry, and differential calculus - as well as the ability to do simple integration problems. Some, but not all of you, have seen vector problems, but even if you have not "covered" vectors, you should try to figure out those problems as well, as they are just an application of what you *do* know about simple trigonometry. Most or all of these mathematical problem types will be surface during this course.

QUESTIONS AND PROBLEMS
Estimates, Calculations, and Mathematical Review

1. What percentage of the mass of a typical atom is represented by the mass of the electrons? [You might think of how an atom is constructed - with roughly equal numbers of protons and neutrons and a number of electrons equal to the number of protons. You should look up the masses - and then *estimate* (rather than calculate) the answer.]

   The mass of the sun is approximately 2×10^{30} kg. Assuming the sun is made entirely of hydrogen, how many atoms is that? [One hydrogen atom has a mass of approximately that of one proton.]

   The mass of the Milky Way galaxy is about 3×10^{41} kg. About how many stars are in our galaxy? [Since you don't know the masses of all the stars, just assume the sun is typical.]

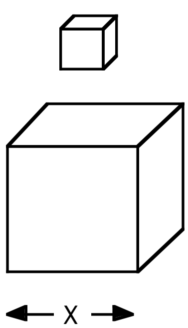
2. The moon is about 1/4 the diameter of earth and is about 4×10^5 km away. The sun is 400 times farther away than the moon. If earth's radius is 6.4×10^3 km, what is the diameter of the sun? [The "missing" piece of information is that the moon and the sun "appear" to be the same size in the sky when viewed from earth. A sketch showing the moon blocking the sun during an eclipse might be helpful.]

   What is the mass density of the sun? [Note: The mass density of an object is its mass per volume.]

   What is the *particle* density of the sun - *ie*, how many hydrogen atoms are there per cm^3?

3. What is the size of the cube that contains one mole of air at standard temperature and pressure? What is the volume of water that contains one mole of water molecules?

4. If a game die is a cube one centimeter on a side, what is the approximate length of the edge of a cubic array that contains Avogadro's number of dice?



   A one carat diamond has a mass of 200 milligrams. Diamond, of course, is single crystal carbon. How many carbon atoms are required to make a one carat diamond? How many carats is the diamond crystal that contains one mole of carbon atoms?

   If the atomic crystal structure of diamond can be thought of as an array of cubes - with each cube being 0.36 nm on a side and containing eight carbon atoms - determine the size of a "one-mole" diamond. (Just determine the length of a cube edge that contains one mole of carbon atoms.)

5. If you increase the diameter of a balloon by 10%, by what percentage will you increase the volume? If the increase in diameter is only 1%, show that the volume increase is 3%.

   Note, you might show that if some variable (y, for example) varies according to $y=ax^n$, then the fractional change in y, or $\Delta y/y$, is given by n times the fractional change in x, that is, $\Delta y/y = n(\Delta x/x)$ – a useful approximation that appears in problem solving from time to time. [Hint: A small change in y (*ie,* Δy) is given by dy/dx times the small change in x (*ie,* Δx) that produces the change in y.]

6. What conversion factor allows you to convert your speed in mph to meters/sec?

   How long does it take light to reach earth from the sun? How long does it take a radio signal to be bounced off the moon and returned to earth - ie, make one round-trip?

   What is the length of one light-second (ie, the distance light travels in one second)? Determine how far (in meters) light travels in one year.

7. In Nov., 1999, it was observed that the light intensity from a particular star 157 light-years distant was diminished by 1.7% as it was being eclipsed by an orbiting planet. What is the ratio of the diameter of the planet to that of the star?

   The largest planet in the solar system, Jupiter, has a diameter that is 10% that of the sun. If observed from that distant planet, by what percentage would the sun's light be diminished when eclipsed by Jupiter? Would astronomers on that distant planet be likely to detect planet earth by the same method? Explain.

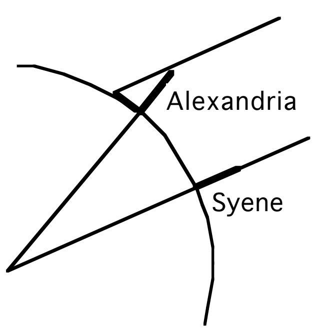
8. Estimate the number of heartbeats you might expect in your lifetime.

   Estimate the number of breaths you expect to take. And what would be the approximate total volume of all the air you inhale? Estimate how many molecules of air you can expect to inhale (and thus exhale).

9. One of Kepler's laws for planetary motion is that the square of the period of a planet's orbit is proportional to the cube of its average distance from the sun. The earth is about 1.5×10^8 km from the sun - a distance called 1 astronomical unit (1 AU). Use proportions to answer the following:

   The planet Neptune was discovered in 1846 and is 30 AU from the sun. What is its orbit period? When will it complete its first orbit since being discovered?

10.



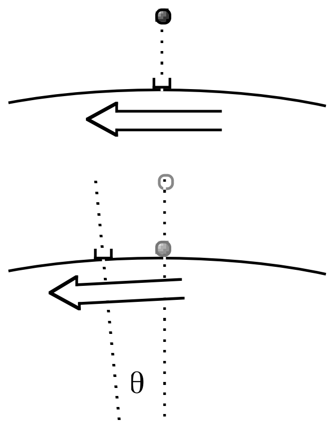
In one of the great scientific deductions in history, Eratosthenes (in about 250 BC) noticed that at exactly noon on the day of the summer Solstice, a vertical pole would cast a shadow 12% of its length in Alexandria whereas it had been reported that a vertical pole cast <u>no</u> shadow at noon on that day in the city of Syene which was due south of Alexandria. He hired a servant to step off the distance between the two cities - a distance of 800 km. From the information given, determine the radius of the earth. (Compare your result to the accepted 6400 km.)

11. Newton's Universal Law of Gravitation states that the force between any two objects with masses m1 and m^2 which are separated by a distance r is given by: $F_g=Gm_1m_2/r^2$ where G is a constant with the value $G=6.67 \times 10^{-11}$ Nm^2/kg^2.

   On earth, the force of gravity on an object of mass m is given by $F_g=mg$, where $g=9.8$ m/s^2. But according to the law of gravity, it is also given by $Fg=GmM/R^2$ where $R=6.4 \times 10^3$ km is the radius of the earth, and M is its mass.

   From the information given, determine the mass of the earth.

12 Suppose you attempt to drop an object from a height of one meter into a cup directly below. Estimate the time the object takes to fall.



During that time, how far has the surface of the earth rotated? (Or, what is the arc length swept out by the earth's rotation during that time?)

Will the object fall "straight down"? If so, by how far will it miss the cup? Or will it land <u>in</u> the cup? If so, what path will it take while it falls?

Try to justify your answers. Sketch the path of the ball from the point of view of an observer who is not also rotating with the earth. When you drop an object into a cup, do you somehow "correct" for the earth's motion? Why or why not?

Mathematical Review Problems

The following mathematical exercises are to remind you of some of the mathematics that you will need to be facile with in this course - and to check your skill level at this time. Work on these independently, then ask about things you don't know or are unsure of.

1. Write the mathematical expression for each of the following statements:

   F is proportional to the square of X and inversely proportional to both Y and the cube of Z.

   At any given time, the rate at which the population changes is proportional to the population at that moment. Let N(t) represent the population at time t.

   If the acceleration is a constant (a), the distance (S) an object will have travelled in some amount of time (t) is equal to the sum of its initial speed (vo) times the time travelled and half the acceleration times the square of the time.

2. Solve the following two algebraic equations for the unknown quantity:

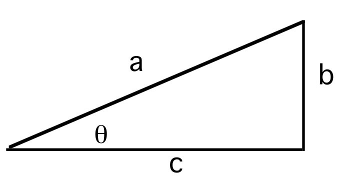
   $2x^2+4x = 6$ $\qquad$ $3y(y-4) + 6 = 0$

3. Solve (showing your work) the following two pairs of simultaneous equations for x and y:

   (a) $x + y = 3$ and $3x + 2y = 7$

   (b) $x\,y=100$ and $x/y=4$

4. Consider the triangle shown and the dimensions *a, b*, and *c* as indicated. Write the sine, cosine, and tangent of θ in terms of the lengths *a, b*, and *c*.



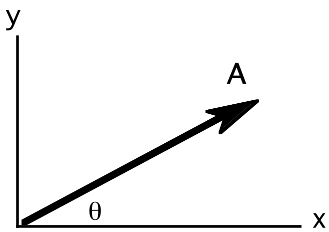
$\sin\theta =$ $\qquad$ $\cos\theta =$ $\qquad$ $\tan\theta =$

5. Give values for the following (based on what you know - do not use a calculator):

$\sin 30° =$ $\cos 60° =$ $\tan 45° =$

$\cos \pi/6 =$ $\sin \pi/3 =$ $\cos \pi/4 =$

6. On the figure below, show the components of the vector A on the x- and y- axes.



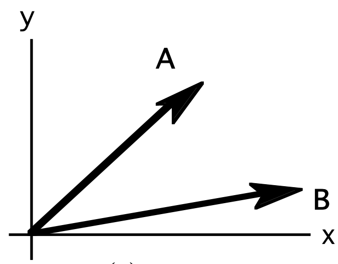
Write expressions for the components A_x and A_y in terms of the magnitude of the vector (A=|**A**|) and the angle (θ) it makes with the x-axis.

$A_x=$ $A_y=$

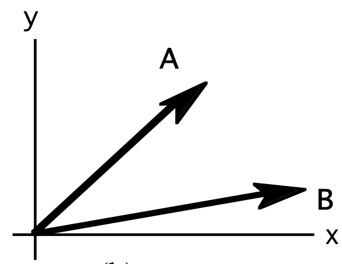
Write expressions for the magnitude A and the direction θ in terms of the components A_x and A_y.

A= θ=

7. On the figures below, sketch (a) the vector sum of the two vectors shown (**C=A+B**) and (b) the difference of the two vectors (**D=B-A**).



(a)



(b)

Write expressions for the components of the vector **C** in terms of the components (A_x, A_y, B_x, and B_y) of the vectors **A** and **B**.

8. Differentiate the following functions with respect to their variables:

$f(x) = 3x^3 - 4x2 + 5x - 3$

$g(y) = (2y^2 - 1)^2$

$(z)=Ae^{bz} + k/z$ (where A, b, k are constants)

9. Find the minimum value of the function f(x) if $f(x) = x^2 - 4x + 3$.

10. Integrate the following: $y = \int (5+2x+3x^2+8x^3)dx =$

11. Suppose: $x(t) = \int_0^t v(t)dt$ where $v(t) = 3 + \int_0^t 4\,dt$

Obtain an expression for x(t):

12. Sketch graphs of $y(\theta) = 5 \sin \theta$ and $y(x) = a + bx - cx^2$ (where a, b, and c are positive constants) [You do not need to put in specific values but should show the basic shapes and label the graphs carefully.] :