

## **Part 1**

# **FORCES**

### **The Nature of Forces**

### **Forces as Interactions between Objects**

### **Newton's Law of Gravitation**

### **How Forces Relate to Motion**

*If you push something, it speeds up. If you press against something, you feel it press back against you. The more massive something is, the stronger its gravitational pull.*

Brian Greene, Physicist  
*The Elegant Universe*

# FORCES

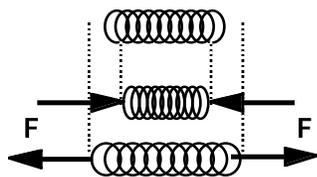
*Newton's laws tell us how matter behaves when it is acted on by forces. The only two things we need to know about the physical world that Newton's laws don't tell us are: What is the nature of matter? What is the nature of forces that act between bits of matter? These two questions are still the central concerns of physics.*

David Goodstein, Caltech

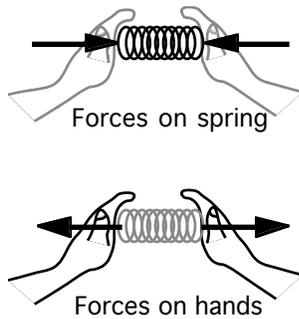
To understand the laws of physics - and in particular, Newton's laws and how they relate to the motions of objects and systems, it is necessary to understand the nature of forces. Indeed, what are often referred to as Newton's three laws of motion are really about the nature and effect of forces. And the essence of Newton's law of gravity is that it so clearly expresses a universal truth about the nature of forces at a fundamental level. So we will begin our discussion of Newton's laws with a discussion of forces. I differ here from most introductory texts on physics - which nearly always begin with a discussion of *motion*, of kinematics or the description of how things move. But while all of that is important - and must also be understood to successfully understand Newton's laws and their significance in describing mechanical systems, *forces* are really the most visceral of the concepts we will encounter. We can *see* the motions of objects (and hence have an intuitive sense of their paths in space, and their speeds or velocities), and we can *define* accelerations (how those velocities change) and energy and momentum and the other concepts we will encounter in the study of mechanics. But we can *feel* forces - both those we apply to other objects and those we are subject to. So that is why I want to start the discussion with forces.

## THE NATURE OF FORCES – Interactions Between Two Objects

When you push against an object or pull it or try to twist it about some pivot point, you say that you exert a force on it. That force may or may not cause the object to move - but the force is exerted in either case. But the force that you exert requires the *existence* of the other object, that is, you cannot exert a force against *nothing*! Moreover, when you are in contact with the object either pushing or pulling or twisting - ie, exerting a force on it - it is also simultaneously in contact with you and resists your force with an exactly equal and opposite force. That is, you can *feel* the contact in such a way that the harder you push or pull or twist, the greater the sensation of the object acting against your hand. That idea is central to understanding forces - and will ultimately be referred to as *Newton's Third Law*. (In fact, as we will see, that characteristic of forces will also lead to the laws of conservation of momentum and of energy that will become very important ideas in problem solving in this course.)



To understand this central idea, it is useful to think of the behavior of a spring. That is, consider a spring that is sufficiently stiff that when relaxed and under no tension, its coils do not touch each other. Such a spring can either be compressed by exerting forces against both ends toward one another or it can be extended by pulling the two ends apart. In either case, the spring resists that distortion and tries to restore its original relaxed configuration. Likewise, the greater the force applied to compress or extend the spring, the greater the resistance offered *by* the spring (and the greater the distortion from its rest-length.). The discussion of springs is important because it relates to all forces associated with tensions and compressions.



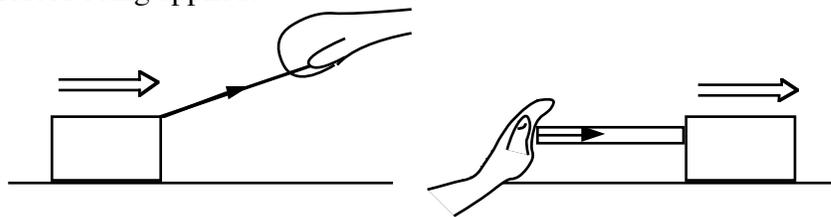
But as important as it will be to know how much springs stretch or compress due to applied forces, the essential idea of this discussion is that the spring exerts exactly equal and opposite forces on the agent (in this case, your hands) that cause the extension or compression. That is, when you compress the spring by pushing on its ends, it tries to restore to its normal length and exerts equal and opposite forces outward on your hands. Similarly, when the spring is extended by pulling at both ends, it will exert equal forces inward on each of your hands. Both tension and compression forces can thus simply be thought of as spring forces.

## Tensions and Compressions

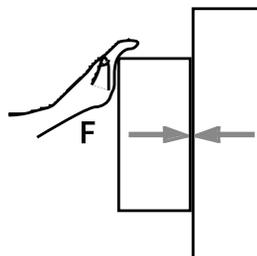
All tensions and compressions are thus similar to spring forces. When you pull on a rope, it does not stretch very much, but it does stretch slightly and hence exerts a reaction force on you equal to the force you apply to it. And when a solid is compressed, it distorts slightly and responds with an equal and opposite reaction force against the object that compressed it.

When you pull something with a rope or string or when a solid rod is compressed between two objects, the tension in the rope or string or the compression forces within the rod are essentially like the spring we have discussed. That is, although the details are different, the rope or string is pulled taut and *stretched* ever so slightly so that it is pulling back with the same force with which it is being stretched. And that is what we call the tension in the string. Or the rod is being compressed ever so slightly (like an extremely stiff spring) and is simultaneously exerting equal and opposite forces on whatever it is that is compressing it. So both tensions and compressions can be modelled by a spring.

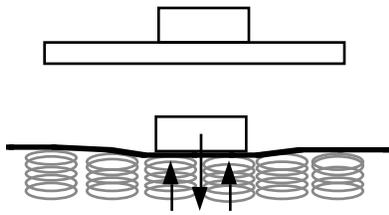
And those tensions or compression forces act at *both* ends of the rope or rod. That is, pulling an object with a rope means you have to apply a force at one end of the rope (which in turn pulls back on your hand) and the rope applies a force on the object being pulled (which in turn pulls back on the rope). This series of interactions at each end of the rope in effect allows you to exert a force on the object being pulled with the rope as an intermediary. The tension in the rope is equal to the force you apply and it is that tension that acts on the object. The effect is similar when a solid rod is being used to push an object. You push against the rod (which in turn pushes against your hand) and the rod pushes against the block (which resists the rod by pushing back). All of these are examples of interactions with equal and opposite forces being applied.



## Normal Forces



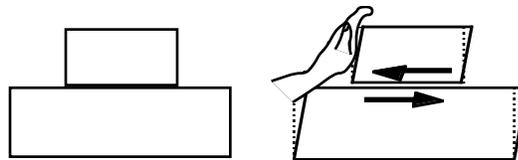
When two solid surfaces are in contact, each surface exerts a force on the other equal to the amount of force pushing the two surfaces together. Those forces are perpendicular to the surfaces themselves, and are therefore called *normal forces*. Each of the two surfaces distort some - just as the spring distorts - and it is the resilience of each of the solid surfaces that exerts the forces on the other. This concept of a *normal force* is real obscure for many students - and it needn't be. The normal force that acts between two objects in contact is just the compression force between the two surfaces. The word "normal" here means "perpendicular". That is, the normal force between two objects in contact is the force perpendicular to the surfaces in contact and is just the force that each object exerts on the other due to the compression force between them. And exactly the same normal force acts on each object but in opposite directions.



When an object sits on a table, the force compressing them together is generally just the weight of the object. So the surface of the table distorts just enough so that the resilience of the surface exerts an equal and opposite force upward on the object - hence supporting it. (It is as if the surface were supported by springs!) If an additional force were applied downward, more distortion would occur and the compressional normal force would increase until the forces between the two surfaces were again equal and opposite.

### Shear Forces

Shear forces are forces that act parallel to two interacting surfaces in contact. Again, the surfaces distort some and are attempting to restore their equilibrium configuration - and it is the resilience of the surfaces - the attempt to restore the equilibrium shape - that represents the shear force. The frictional force that exists between two surfaces either in sliding contact or in resisting sliding can be thought of as shear forces.



Both shear forces and the compressional normal forces act between two surfaces in contact. The difference between the two is the direction the forces act. Normal forces are always perpendicular to the surfaces and shear forces are always parallel to the surfaces.

### FORCES AS INTERACTIONS - The Essence of Newton's Third Law

What is common to all of the above described situations is the essence of what has become known as *Newton's Third Law*. Forces are *always* the result of two different objects interacting with each other. Your hand pushes against some object and that object simultaneously exerts an equal and opposite force against your hand. That fundamental truth about forces is often forgotten - or at least left unsaid - in the course of setting up problems, and yet is very important in understanding the very nature of forces. No force can exist by itself. Forces always exist in pairs - and those pairs are equal in magnitude and opposite in direction and act on two different objects, ie, the objects that are interacting. Understand that point, and you will understand the nature of forces and the fundamental reasoning behind the laws of conservation of momentum and conservation of energy that we will encounter later in the course.

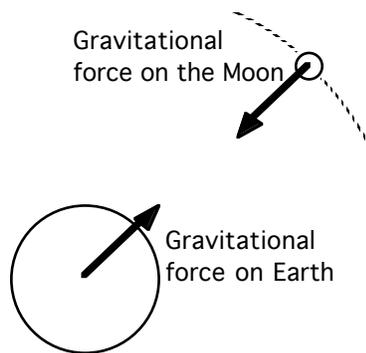
**Forces are *always* the result of two different objects interacting with each other.**

### Newton's Law of Gravity - Forces between masses

Newton knew that objects that fall to the earth accelerate at a uniform rate - and that the rate they fall is independent of the mass of the object, or at least that would be true in a vacuum. And the force that we attribute to that phenomenon is called the *gravitational* force. So he pondered the question of how far from the earth gravity acts. Does it, for example, extend out as far as the moon's orbit? And that, of course, raised the question of why the moon orbits at all. He concluded that earth's gravity must be what holds the moon in its orbit - that is, the moon must be in a constant state of freefall about the earth subject to the pull of earth's gravity just as an apple falls under earth's gravitational influence near its surface.

But if earth's gravity extends to the moon, then his third law requires that the earth and moon must feel equal forces - each due to the other. Furthermore, Newton reasoned that the force that acted on

each object must be proportional to its own mass. That is, the gravitational *force* could not be the same on all objects since more massive objects would then fall *slower* than less massive objects (according to Newton's second law relating forces and accelerations). Combining those ideas mathematically requires that the mutual interaction force must depend on the *product* of the masses of the two objects that gravitationally interact. Since the moon's orbit is nearly circular, and he knew the distance to the moon and the period of its orbit, Newton was able to calculate that the moon's acceleration in its orbit was much smaller than the acceleration of an object near earth's surface - consequently, he could conclude that the gravitational force of attraction between the earth and the moon had to depend on the separation of the two, and that led to the correct mathematical expression for the relationship between the gravitational force and the masses and separation of the two interacting objects. [Since he assumed that the gravitational attraction diminished proportional to  $1/r$  raised to some power  $n$ , he could determine  $n$  by comparing the calculated acceleration of the moon in its orbit to the acceleration of objects which fall to the earth. The power  $n$ , he found was equal to 2. That is, the gravitational force diminishes by  $1/r^2$ , where  $r$  is the distance between the centers of the earth and whatever it interacts with.]



Strictly speaking, Newton derived the relationship for the earth-moon system. But since the same equation should apply equally well to, say, falling apples as to the moon in its orbit, Newton made a grand intellectual leap to declare that the same mathematical relationship must apply to *every pair of objects in the universe!* He deduced that the "force law" would have to be universal and have the form

$$F_g = G \frac{m_1 m_2}{r^2}$$

where the  $m$ 's are the masses of the two objects that are gravitationally attracted to each other,  $r$  is the separation between their centers, and  $G$  is some *universal* constant. He then assumed that the same law applies to the planets in their orbits about the sun and derived Kepler's laws of planetary motion - showing that the planetary orbits are elliptical, that reappearing comets (like Halley's) are simply in very elongated elliptical orbits about the sun, and that the planets themselves can have orbiting moons - or even rings of particles (both of which had been observed by Galileo a half century earlier). The *Universal Gravitational Constant*,  $G$ , was determined experimentally by Henry Cavendish in the mid-eighteenth century, about a hundred years after Newton articulated the law of gravity. [The value of the constant  $G$  is now known to be approximately  $6.67 \times 10^{-11}$  N-m<sup>2</sup>/kg<sup>2</sup>.]

**Any two objects have a mutual gravitational attraction that depends only on their masses and the separation between their centers.**

**The magnitude of that gravitational force is given by :**  $F_g = G \frac{m_1 m_2}{r^2}$

The significance of this deduction is hard to overemphasize. Not only did this intellectual triumph explain the workings of the Solar system, its logic depended on the crucial point that all forces are interactions between two objects - and that forces of equal magnitude (and opposite directions) act on each of those objects that interact. That is, the Universal Law of Gravity and Newton's Third Law are intimately connected.

### **The Force Due to Gravity Near Earth's Surface.**

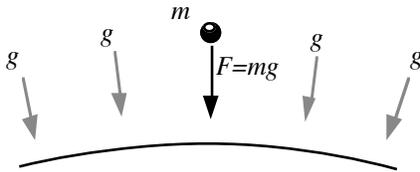
An essential part of Newton's synthesis was that an apple falling from a tree is subject to the very same gravitational force law as is the moon in its orbit. The difference is just that the force equation can then be written in terms of the mass of the earth  $M_E$ , the radius of the earth  $R_E$ , and the mass of the object that is subject to earth's pull, and the height that object is above the surface of the earth:

$$F_g = G \frac{M_E m}{r^2} = G \frac{M_E m}{(R_E + h)^2} \cong G \frac{M_E m}{R_E^2} = mg$$

That is,  $r$  represents the distance between the centers of the earth and the mass  $m$ , and  $r=R_E+h$ . But since  $h$  will be insignificantly small compared to the earth's radius for any object near the surface, the equation can be reduced to a much simpler form where the constant  $g$  just represents the combination of  $G$ ,  $M_E$ , and  $R_E$ . The constant  $g$  is called the *gravitational field constant* and is given by the equation

$$g = G \frac{M_E}{R_E^2}$$

When the values of the constants are substituted, the value of  $g$  is  $9.81 \text{ m/s}^2$ . [We will later interpret this constant as the *acceleration due to gravity* at the surface of the earth.]



The significance of the gravitational field constant  $g$  is that it has the same value at all points near the surface of the earth. (Or rather, it would be constant if the earth were exactly spherical. In fact, the value differs by a fraction of a percent over the earth's surface.) When any mass  $m$  is near the earth's surface, it feels a force of gravitational attraction toward the center of the earth (ie, *downward*) equal to its own mass times the constant  $g$ . Because the direction of earth's gravitational force is always toward the center of the earth, it should be referred to as a *vector field*.

It is hard to single out the most important contribution of Isaac Newton to our understanding of the universe, but one good candidate for it might be the synthesis associated with recognizing that forces had to be the consequences of interactions between two different objects and then the extension of that idea to discovering the fundamental nature of gravitational force - and that the *same* gravitational force law acts between every pair of masses in the universe! Although his "discovery of the law of gravity" was not complete for years, the essential thought process occurred during the year he was away from the university at the time the Black Plague was sweeping through London in 1665 - when he was 22.

### Contact Forces and Field Forces

It is useful to notice that the types of forces we have been describing fall into two very different categories. The tensions, compressions, and shear forces with which we began the discussion are all *contact* forces. That is, the interaction between your hand and some object that you exert a force on requires that the two objects be in contact. However the gravitational force law does not require contact. We refer to that type of force as a *field force*. The reason for that will become clear as time goes on, but the essential point is that all objects have mass, and that mass creates a gravitational field. When other objects are subject to that gravitational field, they are attracted with a force given by Newton's Law of Gravitation - the force equation derived for all masses.

There are other field forces, of course. Magnets also interact with each other without touching. When two magnets are placed near one another, they can either be attracted (and maybe crash into each other) or repel or simply reorient (as with a compass needle) - but in any event, the two magnets can exert forces on each other without ever touching. And Newton's third law applies to that case as well - ie, whatever forces act, they are equal and opposite on the two magnets.

The fact is, the difference between contact and field forces is really a moot distinction on a microscopic scale. That is, *all* forces can ultimately be considered field forces. Even what this discussion calls contact forces really involves the interactions of the atoms of one object with the atoms of the other object through the electric fields associated with the atoms of each object. And fundamentally, physicists think there are only *four* distinct types of forces - gravity, electric and magnetic forces (which can be shown to be intimately related), and two distinct forces that act only within the nuclei of atoms. All other forces that you can imagine are, at a microscopic level, manifestations of those four. At the level of our everyday experiences, all forces are fundamentally either gravitational, electric, or magnetic - and that includes all contact forces between objects which are electrical at a molecular level.

## The Electric Force between Charges – Another Interaction Example

Although this topic will not be a part of *this* course, it does represent another example of an interaction force between objects – one of the four fundamental forces of nature. And, it is very similar to the gravitational force when described mathematically – *ie*, it has the same mathematical form as the Universal Law of Gravitation described earlier. Among the properties of the elementary particles that make up all things in the universe is the property of electric charge. The electron, for example, has one unit of electric charge (1 esu) which has a value of  $1.6 \times 10^{-19}$  Coulombs. (And the electron charge is *negative* – historically an arbitrary choice, but a universally accepted one.) That is, the electric charge of an electron is given by  $q_e = -e = -1.6 \times 10^{-19}$  Coulombs. A proton has the same numerical value, but the charge is positive. *All* elementary particles either have zero electric charge, or  $\pm e$ . All isolated *atoms* are electrically neutral because they have exactly the same number of electrons as they have protons in the nucleus. *Ions* are atoms which have either given off one or more of their electrons – or have captured one or more electrons, so ions can be either positively or negatively charged in units of  $e$ .

An object can be said to carry an electric charge of some amount  $q$  if it has either an excess of electrons (for some reason that is not necessary to discuss here) or if it has had some electrons stripped away from it. In the former case  $q$  would be negative and in the latter it would be positive. Some object which has been electrically charged with, say, one million excess electrons would hence carry an electric charge of  $q = 1.6 \times 10^{-13}$  Coulombs.

If *two* objects each has some amount of electric charge, say  $q_1$  and  $q_2$ , respectively, and the two objects are separated by some distance  $r$ , then there is a force between the two objects which is given by the following mathematical relationship:

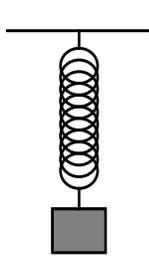
$$F_e = k \frac{q_1 q_2}{r^2}$$

where the constant  $k$  has the value  $9.0 \times 10^9$  N-m<sup>2</sup>/C<sup>2</sup>. That is, each object is subject to the same force of interaction and the two forces are in opposite direction. If the two electric charges have the same sign, then the force acting on each object is away from the other – *ie*, the electric force is said to be positive and it tends to push the two objects apart. If the charges are of opposite sign – one positive and one negative, then the force is said to be negative and it draws the two objects closer together.

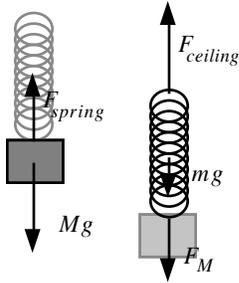
Notice that the form of this electrical interaction between charged particles is identical to the form of the gravitational interaction between two masses. They are both proportional to the product of either the charges or the masses and diminish inversely with the square of the distance between them. (That is, they obey what is called *the inverse square law*.) That has raised questions among theoretical physicists as to whether the two forces – gravitational and electrical – are somehow manifestations of the same thing. One of the grand quests in theoretical physics is the unification of the fundamental forces.

## Representing Forces in Diagrams

It is important to notice in all of these cases, that forces are *vector* quantities - that is, they have both magnitude (or strength) and direction. As such, forces can be represented in figures as vectors - with the length of the vector being proportional to the magnitude of the force and the direction being consistent with the direction of the force. In addition, it is often helpful for the force to be represented in a diagram acting either at the point of contact or at the center of mass (usually the geometrical center) of the object. And finally, in constructing force diagrams, it is important for the diagram to show the forces that are acting on *each object* separately. That is, rather than construct a diagram showing all the forces involved - you pushing on the object and it pushing back on you, for example - it is useful to isolate the object that is being described and *only* show the forces that are acting on *it*. Although that point may not be obvious just yet, it will be an important tool for setting up problems when the goal is to solve for the motions of objects subject to the forces that act. It is probably useful to look at several simple examples.



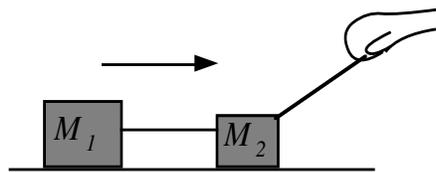
Consider, for example, a mass that is suspended from a spring which is attached to the ceiling. There are many forces involved in this relatively simple problem. The ceiling pulls upward on the spring (which in turn pulls downward on the ceiling). The mass pulls downward on the spring (which is pulling upward on the mass). There is a gravitational pull of the earth on the mass (and the mutual interaction of the mass pulling upward on the earth) as well as the pull of the earth on just the spring's own mass (and the gravitational third law pair force pulling upward on the earth just due to the spring's mass). Of course, in all of this, we have left out the forces exerted by the walls that hold up the ceiling, and on and on and on. But for this discussion, consider all of the forces that act on both the spring and the suspended mass - and consider the diagrams separately.



The only forces that act on the mass  $M$  are the gravitational force ( $Mg$ ) pulling downward and the spring pulling upward on  $M$ . The only forces that act on the spring are the gravitational force ( $mg$ ) acting on the spring's mass pulling downward, the mass  $M$  pulling downward on the spring (which is the third law pair to the spring pulling upward), and the ceiling pulling upward on the spring to keep the whole system from just falling. These forces can best be represented by drawing the spring and the mass separately. Of the five forces shown, the spring force  $F_{spring}$  acting on the mass and the force of the mass  $M$  on the spring ( $F_M$ ) are equal in magnitude and in opposite directions because they are just the interactions of the mass and the spring on each other (Newton's Third Law).

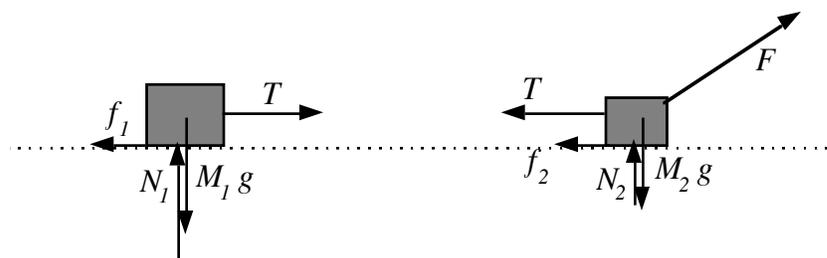
Furthermore, it is not possible from what has been said in the problem how the other forces compare in magnitude, since we do not know if the mass is sitting at rest (at its equilibrium position) in which case the spring force would equal  $Mg$  or whether it is accelerating upward or downward which would require a net force to act on the mass (Newton's second law).

So, what is *likely* to be important in this "simple" problem? Most likely, the problem is to describe the forces that act on the suspended mass so that its motion could be solved for (after examining how to apply Newton's second law to such problems). So of all of the forces that act in this situation, only the two forces acting on the mass are likely to be useful.



Or consider the example of two blocks which are connected by a string being dragged along a horizontal surface. Again, there are many forces involved. Gravity, of course, acts on both blocks pulling the blocks against the table. The table, in turn, pushes upward on the blocks - the reaction force to the blocks pushing downward on the table. The string between the two blocks is pulled taut - and hence is under tension, so pulls to the right on the one block and to the left on the other. And, of course, if the "system" is being dragged to the right (and "lifted") by another string, it exerts a force on the block it is attached to (as well as equally and oppositely on the hand that is pulling). If there is friction between the blocks and the horizontal surface, there are horizontal forces acting both on the blocks (to the left) as well as to the right on the horizontal surface - ie, the friction force is just the horizontal interaction or shear force that acts between each block and the table. But in trying to solve for the motion of the blocks, only the forces acting *on* the blocks need be considered. So the figures below show what forces would need to be considered in solving for the motion of the blocks subject to those forces.

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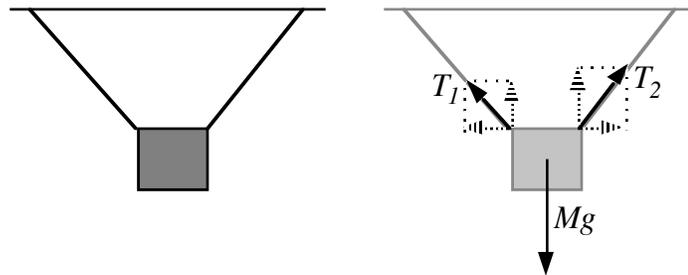


Even then, it seems there are a lot of forces to contend with - but as we shall see in a later section, they can easily be represented in understandable ways. The point here is this: Each object can be

analysed in terms of the forces that act on *it* - forces that are a result of an interaction with some other object. If the force cannot be identified (as well as the object that exerts the force), then a force should not be shown in the diagram.

All forces, of course, are vectors - and that is why they can be represented in diagrams with both magnitude and direction. That also implies that to deal with them correctly, we will have to consider the properties of vectors - in particular, how vectors can be added using their components. So let's consider just one more example of how forces can be represented in a diagram - and show the components that would need to be considered for a complete description.

Consider a single mass that is suspended by two cables, as shown. For the purposes of this discussion, consider only the forces that act on the suspended mass. There are only three forces acting on the mass itself. There is the usual gravitational pull of the earth acting on  $M$ . And there are the two cables, one pulling upward and to the left and the other upward and to the right. For this mass to be in equilibrium, there must be no *net* force acting on it. That implies that the vector sum of all three forces must add to zero - or said differently, the vertical components of the forces must add to zero and the horizontal components of the forces must also be equal and opposite. The second figure shows both the forces and the components of the forces. The two horizontal components of the cable tensions must be equal in magnitude and the two vertical components of the tensions must *add* to equal the gravitational pull on the mass. Only if those conditions are satisfied can the mass be in equilibrium.



Of course, this brief discussion is not intended to make you expert on forces - but rather is intended to give you an idea of the way you will need to *think* about forces in order to set up and solve problems involving forces and motion.

## FORCES AND NEWTON'S LAWS - The Effects of Forces

The discussion of Newton's laws have crept into the discussion of forces, of course, because they cannot really be separated. We have already referred to Newton's *third* law - and that seems odd that we would start with that one. But there is a reason for doing that. Of what are called Newton's three laws of motion, it is the third that is generally the least understood. And yet, it is at the very heart of the nature of forces. Newton's third law states what forces *are* - they are the interactions between objects. Newton's first and second laws of motion are about the *effects* of forces. And it is the second law that you will use most in problem solving.

### Newton's 1st Law: The Law of Inertia

The first law simply states that the principle effects of forces are to change the motion of objects. That statement is more profound than is obvious at first. The reason is that our intuition tends to tell us that forces *cause* motion - that is, objects cannot be moving unless they have a force acting on them. But a series of experiments by Galileo (in the early 1600s) led him to conclude that if an object had no forces acting on it, it would simply continue doing whatever it was already doing. If, for example, a bowling ball were sitting at rest on a level floor, we can expect that it will continue to do so until some force acts on it to disturb its state of rest. But if that bowling ball were already rolling (never mind how it got *started* moving), then it would continue to do so until it hit something - ie, until some other object

exerted a force on it. That property that causes it to continue in its current state - either at rest or in motion - is called its *inertia*. (And as we will see, its inertia is directly related to its *mass*. The more massive an object, the greater its inertia and the more difficult it is to change its current state of motion (or of rest). Isaac Newton articulated this idea - which had been previously expressed by both Galileo and mathematician and philosopher Rene Descartes - and called it the first law of motion.

**In the absence of a net force on an object, the object either remains at rest or moves in a straight line at constant speed.**

The first law just identifies that changes of motion occur because of forces - and so the state of motion remains unchanged either if no forces act or if all the forces that do act are balanced in such a way to add to zero. This is the basis of all equilibrium problems. The *inertia* of an object is its tendency to remain in its current state of motion. The object's mass is a measure of its inertia.

### **Newton's 2nd Law: The Law of Motion**

The second law is the most famous. It states in precise terms what happens if there *are* unbalanced forces acting on an object. That is, Newton's second law states that the consequence of a net force on an object is that the object accelerates - ie, changes its motion in a specific way. It is this law that will be central to the problem solving that you will do in this course.

**The result of a net force (or unbalanced forces) on an object is an acceleration in the direction of the net force. The acceleration will be directly proportional to the net force and inversely proportional to the mass of the object.**

$$\mathbf{F}_{\text{net}} = \Sigma \mathbf{F} = m\mathbf{a} \quad \text{or} \quad \mathbf{a} = \mathbf{F}_{\text{net}}/m$$

This is the central idea of classical mechanics. If you can determine the forces that act on an object, you can also determine the resulting acceleration, from which the velocity and position as functions of time can be determined. That is,  $F=ma$  lets you solve for the motion of the mass. But to deal with any precision with the second law, we need to spend some time carefully developing the description of motion - and dealing with precise definitions of *velocity* and *acceleration*. And we will look at both one dimensional motion and motion in two and three dimensions. That study will be called *kinematics* - the study of motion.

### **Newton's 3rd Law: The Law of Interaction**

It is the Newton's third law that has been the subject of this section. This may be the most important of the three laws in some ways. It says that *all* forces are caused by some other object. Understanding the third law is essential to being able to solve problems using the second. This important idea will lead to the conservation of linear momentum and the conservation of energy.

**All forces are the result of an interaction between two objects. If one object exerts a force *on* a second object, there is an equal and opposite force being exerted on the first object *by* the second.**

Finally, all of these ideas will be revisited in more detail in the section on Newton's laws. The purpose of introducing them at this point - before a more formal discussion of motion in one, two, and three dimensions - is to take advantage of your own experiences with forces - even if you had not thought about them in this way. But before we try to proceed further in our discussion of Newton's Laws, it is necessary to present a formal description of motion - and that is the topic called *kinematics*.

## FORCES - QUESTIONS AND PROBLEMS

The following problems are intended to help you think some about the section you have just read. It is not essential that you be expert on these at this point. But the ideas will come up again in the course - and getting these ideas down early will help you later on.

1. Explain in your own words what is meant by the statement: "Forces always occur in pairs."
2. When you lift an object, the force your hand exerts on the object is (greater than, less than, equal to) the force the object exerts against your hand. Select from the choices and justify your answer. Does your choice depend on whether you lift the object at constant speed, increasing speed, or decreasing speed? Explain your answer carefully.
3. When an object moves along at constant speed, there must be a constant force acting on it to keep it in motion. Explain whether that statement is true or false. If the answer depends on the circumstances, explain those circumstances.
4. When Newton's second law speaks of a *net* force, what does that mean? Give an example of a problem in which the net force on an object is zero. Give an example of a problem in which the net force on the object is *not* zero. Explain how the object's *motion* is different depending on whether the net force is zero or non-zero.
5. Consider a baseball (or some other object) which is in mid-flight after having been thrown. Describe all of the forces that act on the ball. Explain why there does *not* have to be a force in the direction of the ball's motion after the ball is thrown.
6. Consider that you are holding a ball in the palm of your hand - and are not moving. Describe the forces that act on the ball. What object exerts each of the forces that acts on the ball. What is the corresponding Newton's third law pair (or the "reaction force") to each of those forces?

If you were to let go of the ball and let it drop, what force acts on the ball and what is the corresponding third law reaction force?

If you were to throw the ball straight up, identify all of the forces that act on the ball after it leaves your hand - and the corresponding reaction forces.

7. A block slides down an incline. Assuming there is friction between the block and the incline, draw a diagram which shows all of the forces that act on the block. [In your diagram, be careful to have each of the forces be shown with an approximately correct magnitude - and in the right direction.] In your diagram, does it matter whether the block is sliding at constant speed, increasing speed, or decreasing speed?

If the block were sliding *up* the incline, how would your diagram change?

8. Consider that you are pressing a book directly against a wall so that it does not slide. (Assume the force *you* apply is perpendicular to the surface of the book.) Draw a diagram to show all of the forces that act on the book. Identify those forces and the *object* that acts on the book to cause each force. What is the Newton third law pair force to each of the identified forces?
9. Suppose you compress a spring against a wall, are the forces you exert at one end and the force the wall exerts at the other necessarily the same? Explain your thinking carefully. If the magnitude of the force you exert is  $F$ , what is the magnitude of the net force on the spring?

If you compress the spring between you hands by applying a force  $F$  at each end, is the tension in the spring  $F$  or  $2F$ ? Explain your thinking carefully.

**For the problems 11-18, assume the following information:**

The Universal Gravitational Constant  $G=6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2$ , Earth's mass is  $6 \times 10^{24} \text{ kg}$ , Earth's radius is  $6.4 \times 10^3 \text{ km}$ , the Sun's mass is  $2 \times 10^{30} \text{ kg}$ , the Moon's mass is  $7 \times 10^{22} \text{ kg}$ , Earth's orbit radius is  $150 \times 10^6 \text{ km}$ , the Moon's orbit radius is  $4 \times 10^5 \text{ km}$ . The mass of an electron is  $9 \times 10^{-31} \text{ kg}$  and the mass of a proton is 2000 times greater.

10. State whether the following statement is true or false: The gravitational force on the moon due to the earth is greater than the gravitational force on the earth due to the moon. Explain your thinking.
  
11. Calculate, using the Universal Law of Gravitation, the force between the earth and a 1 kg mass at the surface of the earth.  
  
Justify that the constant  $g = 9.8 \text{ m/s}^2$ .
  
12. Calculate the mutual gravitational force between two 1 kg masses assuming their centers are separated by 10 cm. What mass object has that same gravitational force due to Earth's pull?
  
13. Consider that some material of unknown mass density  $\rho$  is used to make two identical spheres of radii  $r$ . [The mass density of an object is just its mass divided by its volume.] Express the gravitational force between the two identical spheres when they are placed as close together as possible. Would the gravitational force between them be greater using larger spheres or smaller spheres? Explain your answer carefully.
  
14. Compare the gravitational pull of the Moon on Earth to that of the Sun on Earth. Which is greater and by what approximate factor? [i.e., state that "The sun's pull is 12 times the moon's pull." or what ever it is.]
  
15. Consider that you double the distance between two masses, by what factor has the gravitational force between them changed? Suppose you *both* double the distance and double their masses, by what factor has the gravitational force changed?
  
16. The Earth's radius at the equator is  $6.4 \times 10^3 \text{ km}$ , its radius at the poles is  $6.2 \times 10^3 \text{ km}$ . What is the percentage difference between the gravitational field at the poles compared to the equator?
  
17. The gravitational field constant  $g$  is  $9.8 \text{ m/s}^2$  at the surface of the Earth. What is the value of  $g$  in a satellite's orbit when it is 200 km above the surface of the Earth? What is the value of  $g$  (due to Earth's gravity) in the Moon's orbit? What is the value of  $g$  due to the Moon's gravitational pull?
  
18. Consider that a hydrogen atom is a proton with an electron orbiting in a circle at a radius of 0.1 nm. The electric charges on the two particles are equal and opposite (with a value of  $1.6 \times 10^{-19}$  Coulombs). Use Coulomb's law to calculate the force of interaction between the electron and the proton. Use the law of Gravitation to calculate the gravitational interaction between them. Which force is greater and by what factor? Show that the ratio of the electrical and gravitational forces of interaction between the electron and the proton does not depend on the separation of the two particles.