1.3 Dynamics

- Dynamics is the relationship between forces and motion
- Force *causes* acceleration, or change in momentum
- Force can do work to change energy

Dynamics is the study of the forces that can change momentum of an object or do work on an object. To understand this, we need a working definition of force. We can start with "a force is a push or a pull." For example, a cannon exerts a strong force on a cannonball that is launched upward into the air. The cannon ball gains momentum upward from this large sudden force. In contrast, the force of gravity provided by the Earth is a smaller downward force, but acts for a long time on the cannon ball. Over a long period of time, the constant downward force of gravity will reduce the upward momentum of the cannon ball, bring the cannon ball to rest for an instant, and then increase the downward momentum of the cannon ball.

Relationship #1: The amount of force exerted on an object is proportional to the resulting rate of change of the momentum.

$$F = \frac{\Delta p}{\Delta t}$$

In this text, the upper case Greek letter Δ (delta) always means change in whatever quantity follows it; thus, Δp means change in momentum.

It's important to note that the rate of change of the momentum isn't the force, or doesn't make force. The force is the push or pull on the body. However, the force acting on the body produces a rate of change of momentum as indicated above. This is an *empirical* law, meaning it is universally observed to be true.

It is also informative to say that forces make objects accelerate. Isaac Newton originally put it this way: What happens when you push something? It accelerates (speeds up) in the direction you pushed it. The more you push it, the faster its velocity changes. And, the more massive the object is, the more force you will need to produce the same acceleration. This leads us to our second definition of force.

Relationship #2: The force on a body produces acceleration such that the mass times the acceleration is equal to the applied force. Acceleration is how fast the velocity is changing. Or,

$$F = ma = m\frac{\Delta v}{\Delta t}$$

In the cannon ball example, the force of the expanding hot gas accelerates the cannon ball, increasing its speed. After the cannon ball leaves the cannon, the only force on it is gravity, accelerating it downward. As a result, Δv is downward, even if the ball's velocity, v is upward. Consequently, the speed of the ball decreases over time, until it comes to rest high in the air, and switches direction. Note: even when the ball's speed is zero at the top, the force of gravity, the acceleration, and Δv are all downward.

Relationship #3: A force also results in work being done on an object as it moves over a distance. So, the amount of energy gained by an object per distance traveled is equal to the force exerted on it,

$$F = \frac{\Delta E}{\Delta x}$$

or the force on the body results in an energy gradient (change in energy over space). In the cannon ball example, the force on the cannon ball did work on the cannon ball transferring some of the thermal energy of the expanding gas to kinetic energy of the cannon ball. The force of gravity reduced the kinetic energy of the cannon ball as it rose, changing its kinetic energy to gravitational potential energy.

Relationship #2 is the one most commonly quoted. However, relationship #1 (and sometimes #3) is often more helpful. When you put a force on something, its momentum changes, it accelerates, and it can gain or lose energy. The SI (standard) unit of force is the Newton (N) after Isaac Newton.

Again, I stress that none of these relationships *define* force: Force is NOT the rate of change of momentum, mass times acceleration, nor the gradient of the energy. Force is how hard you are pushing or pulling on the object that results in these three things.

<u>Exercise 1</u>:
Now we have a problem: a force changes momentum, but we just learned that momentum must be conserved. How can this be? It might be best to imagine yourself jumping off the side of a small boat by providing a repulsive force with your legs. What happens to your momentum? What happens to the boat's momentum? My mass is 70 kg, and I push off a 140 kg boat so that my final speed is 5 m/s North. The push lasts about 0.5 s.

a) What is my final momentum?
b) What was the average force from my legs on me?
c) What was the final speed and direction of travel of the boat?
d) What was the average force on the boat?

We know that momentum is always conserved, so if one body gets some momentum, another body must get the opposite momentum. Hence, a force is an interaction between two bodies, whereby each gains equal momentum in opposite directions. We see this all the time: if I push you away from me, I get pushed away from you. The cannonball ball changes momentum by exchanging momentum with the cannon (connected to the earth). When the cannon ball gains momentum upward, the force of the expanding gas acts equally on the cannon/earth (downward) and the ball (upward), so each body gains an equal change in momentum in opposite directions. The mass of the earth is much greater than that of a cannon ball, so the earth's change in speed is unnoticed.