1.7 Forces, Momentum, and Work:

- A force is a single interaction between two bodies affecting each in opposite directions.
- Momentum (like velocity) is a *vector*, so two bodies moving in opposite directions can have a total momentum of zero.
- Kinetic energy (like mass) is a *scalar*, so two bodies moving in opposite directions both have positive kinetic energy.

In Chapter 1.3, we defined force as a push or pull; an interaction between two bodies where momentum is *exchanged*. Each body receives an *impulse* (Δp) that is equal in magnitude, but opposite in direction, thus conserving total momentum. We further stated that when a force acts on a body, that the body's momentum changes:
$F = dp/dt$;

that the body accelerates:
$F = ma$;

and that work is done on the body, changing the body's energy:
$F = \Delta E/\Delta x$, or $\Delta E = W$, and we define work as $W = F\Delta x$.

Many of us have heard about Newton's Laws. How do these relate to the above?

- 1st Law: If the force on a body is zero, its velocity will not change.
- 2nd Law: If there is a net force on a body, it will accelerate according to $F = ma$.
- 3rd Law: When one body exerts a force on a second body, the second body is exerting a force on the first body of equal magnitude in the opposite direction.

The first and second laws are both consistent with $F = ma$; the first law being a subset of the second law: if the force is zero, then the acceleration is zero and the velocity doesn't change. The third law is consistent with the conservation of momentum:

A force is a **single interaction between two bodies**, so each body must gain an equal and opposite impulse, Δp, over the same time in order to conserve momentum.

It follows that we can turn $F = dp/dt$ around to show that the impulse a body gains is $dp = Fdt$, or $\Delta p=F\Delta t$.

Deeper thoughts: If a force is a single interaction between two bodies whereby they exchange momentum, please consider a book resting on a table. There are forces (F_g and F_N) acting on the book, but the book's momentum doesn't change. If we took away the table, the attractive force of gravity would cause the momentum of the book and earth to change by equal magnitudes in opposite directions. However, because the book is at rest, we can conclude that the repulsive normal force between the table top and book result in a zero net force and zero change in momentum for both the earth and the book.

Important Example

What happens when two carts moving in opposite directions with the same momentum crash and stick together motionless? What happened to the momentum? What happened to the energy?

- Momentum is a vector. We see at the end that the total momentum is zero because the two carts are not moving. There was no outside force, so $\Delta p = 0$, and momentum is conserved. So, the initial momentum must have been zero too. The momenta of the two carts were in opposite directions, so the two momenta are negatives of each other and add to zero.
- Energy is a Scalar. The two objects had initial kinetic energy, and there is no kinetic energy at the end, so we might infer that like momentum, the energies were also opposites of each other. However, we find that after the collision, the carts are hotter where they hit, and careful measurements show that this thermal energy produced is equal to the total kinetic energy that "disappeared" in the collision. So, *kinetic* energy is not conserved in this inelastic collision. However, *total energy* ($E_k + E_{th}$) is conserved as the kinetic energy is converted to thermal energy. This is what we experience, for instance when we drive a nail into some wood with a hammer. The nail is heated up. If instead the two carts compressed a spring in the collision and latched, the kinetic energy would be changed to potential energy in the spring. There is no such thing as negative kinetic energy: if you are moving, then you have positive kinetic energy. Notice the formula for kinetic energy. What happens if the velocity is negative?

Exercise 1: Two carts approach each other from opposite directions, on a low friction track, both at speeds of 10 m/s and have an inelastic collision, sticking together. Cart A, moving to the right, has a mass of 1 kg. Cart B has a mass of 2 kg.

- What is the total initial kinetic energy of the system?
- What is the total momentum of the system?
- What is the final speed of the system after they collide?
- What is the final kinetic energy of the system?
- How much thermal energy was liberated as heat?
- If the collision lasted 0.05 seconds, what was the magnitude of the average force between the carts? Was it the same on both carts? How do you know?