2.2 Graphical Analysis of Motion

We want to be able to correctly construct and interpret graphs of position-time (*x-t*), velocity-time (*v-t*), and acceleration (*a-t*). Most students understand the concepts involved, but make mistakes. My advice is to practice using these graphs and converting one to the other. We remember:

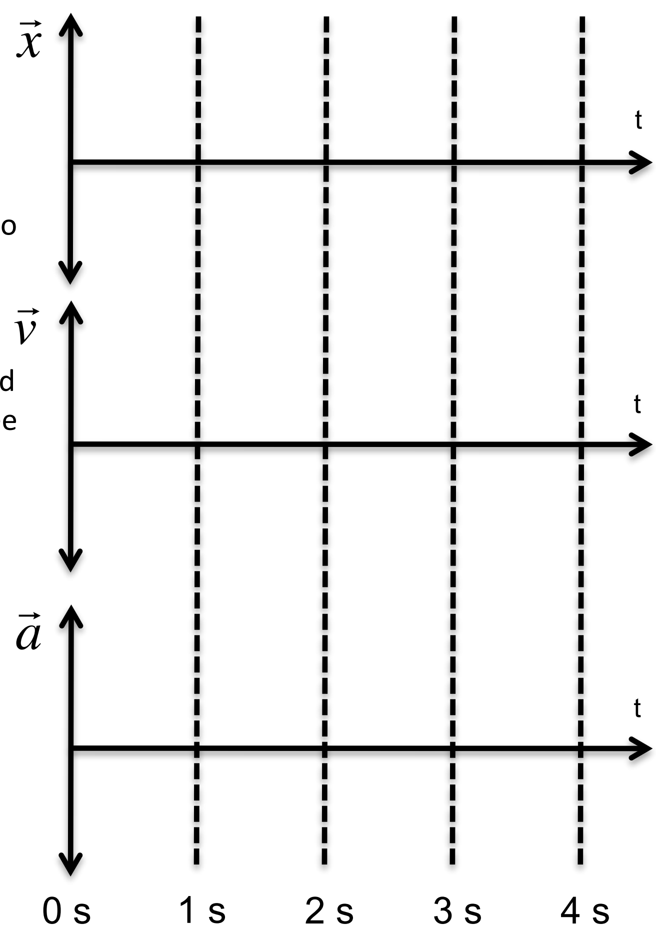
v = dx/dt; velocity is the derivative of position. Consequently:

- velocity is the *slope* of the *x-t* graph.
- Change in position, *dx = v*dt*. So change in position is the area under the *v-t* curve, and is velocity integrated over time.

a = dv/dt; acceleration is the derivative of velocity, Consequently:

- acceleration is the *slope* of the *v-t* graph.
- Change in velocity, *dv = a*dt*. So change in velocity is the area under the *a-t* curve, and is acceleration integrated over time.
- $a = d^2x/d^2t$; the second derivative of position. Thus acceleration is the *curvature* of the *x-t graph.*

So, let's say you start with a *v-t* graph. You would differentiate it to get a graph of acceleration as a function of time, and you would integrate it to get a graph of position over time, although for the latter, you'd have to know what the starting position was. The mistake most students make is to just copy the same graph. Let's give it a try.



<u>Exercise 1</u>: My mass is 70 kg, and the mass of my bike is 10 kg. I'm riding my bike at a constant speed of 10 m/s. At 0s, my displacement is *x = -15 m*, I see a car. At t = 1s, I apply my breaks and smoothly slow to a stop over a period of two seconds.

a) Please graph my acceleration, velocity, and displacement as a function of time. Label the axes correctly. *Then* please also find:
b) the force exerted by my breaks;
c) and the work done by my breaks and the average power.
d) Was energy conserved in this process? How?
e) Was momentum conserved in this process? How?