

4.0 Introduction to Rotation

We started this course with four lenses of mechanics.

- Momentum: $\vec{p} = m\vec{v}$; system momentum is always conserved; momentum is a vector.
- Energy: Energy is a scalar, is always conserved, but changes forms.
- Dynamics: $\sum \vec{F} = m\vec{a}$, we have a protocol to solving these. We practice it.
- Kinematics: Explicit time dependence of motion: position, velocity, acceleration.

We start now with the rotational analogues of these four lenses:

- Angular Momentum $\vec{l} = I\vec{\omega}$; is how hard it is to stop a body from spinning. Just like *linear momentum*, angular momentum is a vector, and in an isolated system is conserved. The initial example we had for linear momentum conservation was two carts colliding. The rotational analogue would be if you dropped a stationary mass onto a rotating platform. In an inelastic collision, they would stick together and rotate, but at a lower rotation rate than the platform was previously spinning.
- Rotational Kinetic Energy: We introduce a new form of kinetic energy $E_R = \frac{1}{2}I\omega^2$. If something is spinning, then the different parts have different speeds constituting kinetic energy.
- Rotational Dynamics: Turning forces, or “torques” cause rotation rates to change (rotational acceleration): $\sum \vec{\tau} = I\vec{\alpha}$. rotational inertia (or moment of inertia) I , is the resistance to *angular* acceleration. Large, massive objects have a larger “ I ” and are harder to rotationally accelerate (start and stop spinning). Just like with linear forces, a torque is a single turning interaction between two bodies, affecting each in opposite directions. So, if I put a torque on an object, it puts an opposite torque on me.
- Rotational Kinematics: Explicit time dependence of rotational motion:
 - angle (θ),
 - rotational velocity ($\omega = \frac{d\theta}{dt}$) or how fast something is spinning, and
 - angular acceleration ($\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$) or how fast this rotation is speeding up or slowing down.

These quantities are also vectors, just like the linear analogues because you can rotate an object about any axis in either direction. For instance, if two objects are spinning in opposite directions at the same ω , the angular velocities of each object would have opposite signs, as would their angular momenta.

Example 1:

Two identical disks ("A" and "B") are spinning in opposite directions in space, and $\omega_A = 3\omega_B$.

How would you compare:

- Their initial angular momentum? $l_A = __ l_B$
- The two initial kinetic energies? $E_{RA} = __ E_{RB}$

The two bodies collide and stick together:

- What is the linear analogue to this situation?
- What lens is most important to use here?
- How does the final rotational velocity compare to the initial rotational velocity of A, ω_A ?
- Is kinetic energy lost in this collision? If not how do you know? If it is, please find an expression indicating how much energy is lost.