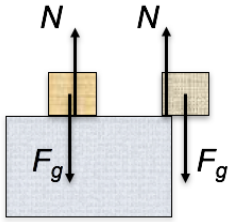
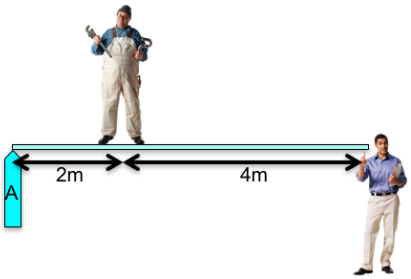
4.5 Introduction to Statics: what is required for something to be immobile?

Statics is all about making sure that buildings and machines are stable. Most engineers start with a full course dedicated this this subject that we will spend a few days on. Consider a bridge touching the ground in only two places. If we're on this bridge, we'd like it to be in *static equilibrium*. We don't want it to move. We originally learned that equilibrium is when $\sum \vec{F} = m\vec{a} = 0$, or the net force on an object is zero, so it's not accelerating. However, this isn't enough. What if there is an upward force on one side of a bridge and an equal downward force on the other side of the bridge? Then it would have *angular acceleration*. This wouldn't be good for us either. Consider the two blocks resting on the larger block at right. The one near the middle is stable because the normal force and the force of gravity are equal and opposite. How about the one hanging over the edge? The force of gravity provides a torque about the point of contact, rotating that block… *into the page*. In a momentum, the block will roll off the edge, the normal force will be zero, and the acceleration will be gravity.



Thus for equilibrium, we have to add $\sum \vec{\tau} = I\vec{\alpha} = 0$, or the net torque on an object is zero, so there's no angular acceleration. For a dynamics problem before, we would use the force equation will all the forces and solve for the unknown. Now we have two *simultaneous equations* and can solve for two unknowns such as the two unknown forces acting on each end of a bridge. Let's try it.



<u>Exercise 1:</u> Please consider that I am helping a *90 kg* friend do some construction work by supporting one side of a plank for him to stand on. In order for him to be in static equilibrium, the forces and the torques must be zero. We would like to know what is the force provided by pylon A and by my finger. The first question is, "what is the body that we are considering is in equilibrium?" It's the plank that has all the forces acting on it that we need to be stable. Please make a good free body diagram of all the forces acting on the plank. We have two unknown forces, but we have two equations. Can you identify these four things? Please set up the equations so that the forces add to zero and the torques add to zero. Remember to pick a positive direction for both the forces (either up or down) *and* the torques (either in or out of the page). Torque requires that you know the radius from the center of rotation. Where is the center of rotation of a static body? The neat thing here is that you can pick any place on the object you like: If the body is static, then it's not rotationally accelerating about any point, so the torques about *any point* is zero. So which point should you pick if you are allowed to pick any point? If you picked the point under my friend, then there would be two torques on the plank: one from my finger, and one from pylon A. The normal force of my friend pushing down on the board (equal to the force of gravity on him because he's not accelerating) is at the center of rotation, so the radius is zero, and the torque is zero. Thus if you pick the center of rotation to be under my friend, your torque equation will have two terms in it, each with an unknown force. If you pick a point right over pylon A, then the normal force of pylon A provides

no torque because it is now at the center, and you have an equation with two terms in it and only one unknown (the normal force of my finger). If you pick the middle of the board, then there will be three torques about the center with two unknowns. Which point do you want to choose as the center of rotation? Please work this out carefully, and show that my finger provides 300 N, and pylon A provides 600 N of force.