4.6 Introduction to Angular Momentum: what is conserved when bodies interact?

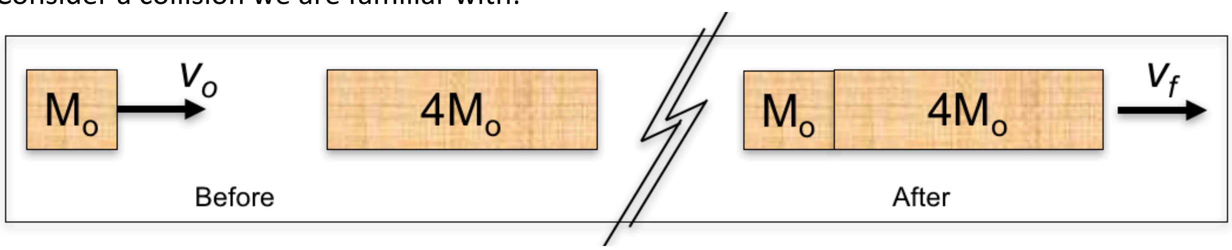
While we recognize Newton's second law: $\sum \vec{F} = m\vec{a}$, we also introduced force as a *single* interaction between two bodies whereby momentum is exchanged: $\sum \vec{F} = \frac{d\vec{p}}{dt}$, or force causes the change in momentum of each object in opposite ways. Thus if $\sum \vec{F}_{system} = 0$, then $\frac{d\vec{p}}{dt} = 0$, and we learned that momentum of a system is conserved *if there is no external force*. Now we take the same steps with rotation dynamics. We know that

$\sum \vec{\tau} = I\vec{\alpha}$, but also

$\sum \vec{\tau} = \frac{d\vec{l}}{dt}$, *torque* causes the rate of change of *angular momentum*.

if $\sum \vec{\tau} = 0$, then $\frac{d\vec{l}}{dt} = 0$, *angular momentum* doesn't change… is conserved when there is no external torque.

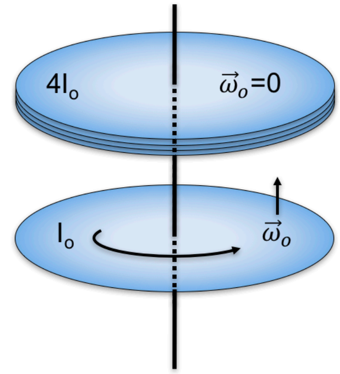
Consider a collision we are familiar with:



A mass runs into and sticks to a stationary mass 4 times its mass. The resulting mass, at *5M_o* moves off at $v_f = \frac{v_o}{5}$ by conserving momentum. The smaller mass receives an impulse to the right, and the larger object receives the equal and opposite impulse to the right. Additionally, we can see that in this process we've lost 80% of the kinetic energy to heat. It's easiest to see if we express kinetic energy as:

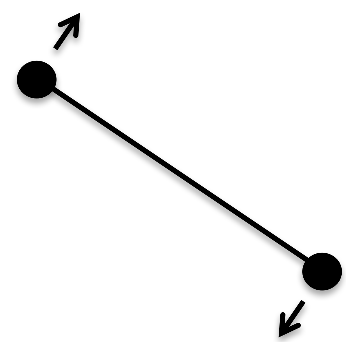
$E_K = \frac{p^2}{2m}$, because we know that momentum is the same before and after, but the moving mass increased by a factor of 5 in the collision. Let's do this for a rotational collision. Imagine long ago, there were turntables, upon which we could drop stationary vinyl records. Imagine that I accidentally dropped many records on a rotating turntable at once such that the records together had a moment of inertia four times that of the rotating turntable. Imagine that the motor driving the turntable is not engaged so the turntable is free to rotate… when I say "free to rotate," what does that tell you about the system?.

Exercise 1: The base of a turntable is rotating freely at rotational velocity ω_o on a very good bearing when four disks are dropped onto it. Assume that the 4 disks are all identical to the original rotating disk. Please find:



a) The final rotational velocity. Please support your answer with the correct concepts and formulae and pay particular attention to which lens you use. Develop this carefully.
b) Is kinetic energy conserved? If not how do you know? If so, how much energy is lost in heat dissipation?
c) What is the angular impulse $\Delta\vec{l}$, received by each body. Please include direction.
d) If you imagine the interaction between the disks as they land whereby the stationary disks speed up and the rotating disk slows down so they can have the same speed. Can you identify a frictional interaction that may generate heat?

Exercise 2: You see two equal masses tied together with a string spinning in space at constant angular speed, ω_O, when a motor at the center pulls them both inward such that the final diameter of their paths is 1/3 the original diameter, or, d => <u>1/3</u> d_O.



a) This happens in outer space…. And we wonder what causes this change. Is there anything acting on this from the outside? What should be conserved? Consider each of the 4 lenses when you answer this question.
b) What happens with the moment of inertia with this change? I => ___ I_O
c) Let's say we conserve angular momentum. What should be the new angular velocity? ω => ___ ω_O,
d) Would this change the kinetic energy? If so, by what factor: KE =>___ KE_O
e) Now, let's say we conserve rotational kinetic energy. What should be the new angular velocity? ω => ___ ω_O,
f) Would this change the angular momentum? If so, by what factor: L => ___ L_O,
g) Hopefully, you have found that you cannot conserve both angular momentum and kinetic energy in this interaction. Which do you think is conserved? … who do you trust and where did the extra energy (or angular momentum) come (or go)? Which lens is the most helpful and why?