Part 5

MOMENTUM

Newton's Laws and Momentum

Impulse and Momentum

Momentum of a System

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Momentum Problems

MOMENTUM

The significance of Newton's laws is difficult to over-emphasize. In addition to giving one the tools to predict the motions of objects which are subject to known forces, they lead to three profound principles of nature that lead to problem solving strategies even when the details of the forces that act are *not* known sufficiently well to be able to simply apply Newton's second law directly to obtain an object's acceleration - and hence be able to deduce its motion. Those principles are called *conservation laws* - and will follow directly from Newton's second and third laws. But before we introduce the conservation laws, we need to examine Newton's laws more closely.

NEWTON'S LAWS AND MOMENTUM

To begin, let's look more carefully at the law of motion - in particular, what we normally abbreviate as $F = ma$. More precisely, the second law states that the net force - or the vector sum of all the forces - acting on an object is numerically equal to the product of its mass and the vector acceleration that results from the net force. And the mathematical expression for the second law is given by

$$\vec{\mathbf{F}}_{net} = \sum \vec{\mathbf{F}} = m\vec{\mathbf{a}} \qquad (1)$$

But Newton did not state the second law in that form. The language of the time used the word "motion" to mean the product of the mass of an object and its velocity. Newton stated his second law as "The rate of change of the motion of an object is equal to the net force imparted upon it." That is, *his* mathematical expression for the second law would have been

$$\frac{d\vec{\mathbf{p}}}{dt} = \vec{\mathbf{F}}_{net} = \sum \vec{\mathbf{F}} \qquad (2)$$

where $\vec{\mathbf{p}}$ is the product of the mass times the velocity of the object. We now use the word *momentum* to represent that quantity. That is, the momentum of an object is defined as

Momentum: $$\vec{\mathbf{p}} = m\vec{\mathbf{v}} \qquad (3)$$

And Newton's second law states that the rate of change of the momentum of an object is equal to the net force that acts on that object. Notice that since velocity is a vector quantity, then so is momentum. So the effect of a net force is to change the vector momentum of an object - either its magnitude, its direction, or both.

There are several important ideas that appear in the discussion of momentum. The concept of momentum, the product of mass and velocity, follows from Newton's second law. We will define the *impulse* delivered by a force acting over some amount of time - and that impulse has the effect of changing the momentum of an object or system of objects if all such impulses are accounted for. But while the chapter appears to be discussing *momentum* it is really about *systems* of particles and how Newton's second and third laws apply to a system of interacting objects.

By defining the momentum of a *system* as the vector sum of all the individual particle momenta, we will see that, as a consequence of Newton's *third* law, the system momentum can only be changed by external forces that act on the system. The forces within a system – *i.e.,* the *internal* forces acting between objects within a system - can have no effect on the total momentum of the system itself. That idea is particularly important in dealing with collisions between objects (or, more generally, interactions between them) because it states that momentum is *conserved* during collisions if there are no external forces acting during the collision. Finally, it is very useful to define the *center of mass* of a system - then describe the motion of the *system* as the motion of its center of mass. That allows all motion of even very complicated systems to be reduced to the motion of the center of mass and the motion of the various parts of the system relative to the center of mass, treated as two separate problems.

But it is the application of Newton's third law to interacting objects that leads to the grand principle of the conservation of linear momentum. And *that* idea is the real importance and significance to introducing the concept of momentum.

Impulse and Momentum Change

Since Newton's second law relates the net force on an object to its mass and acceleration, and the momentum of the object is just the product of its mass and velocity, Newton's second law can be written:

$$\vec{\mathbf{F}}_{net} = m\vec{\mathbf{a}} = m\frac{d\vec{\mathbf{v}}}{dt} = \frac{d\vec{\mathbf{p}}}{dt} \tag{4}$$

The differential relationship between the net force and the rate of change of the momentum leads to an important relationship between the *impulse* imparted by all the forrces that act on an object and the subsequent change in the object's momentum.

Impulse

The impulse, $\vec{\mathbf{J}}$, imparted by a force acting over a time interval is defined as the integral of the force over the time during which the force acts.

Impulse due to a force:

$$\vec{\mathbf{J}} = \int \vec{\mathbf{F}}dt = \vec{\mathbf{F}}_{Avg}\Delta t \tag{5}$$

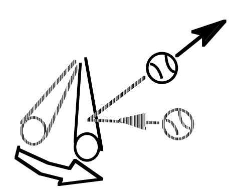
The integral of the force over the time interval during which the force acts is equivalent to product of the *average* force acting during that time interval and the time interval itself. Newton's second law requires that the impulse due to *all* the forces that act on an object is the change in the object's momentum. That is, since

$$\vec{\mathbf{F}}_{net} = \frac{d\vec{\mathbf{p}}}{dt}$$

then the *net* impulse - or impulse due to the net force is equal to the change in momentum

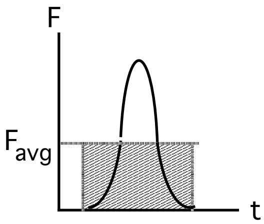
Impulse due to *all* forces:

$$\vec{\mathbf{J}}_{net} = \int \vec{\mathbf{F}}_{net}dt = \int \frac{d\vec{\mathbf{p}}}{dt}dt = \Delta\vec{\mathbf{p}} \tag{6}$$



The effect of the net impulse on an object – *i.e.*, the impulse of all the forces acting during a given time interval - is thus a change in its momentum which is numerically equal to the net impulse. If, for example, a baseball is struck by a bat, a large force acts on the ball for a short time. Although neither the force nor the time may be known, the impulse is necessarily equal to the change in momentum - a result of Newton's second law.

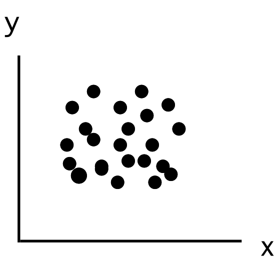
The impulse could be determined <u>if</u> the time dependence of the force were known. Or the impulse could be determined if we knew the <u>effect</u> of the impulse, *ie*, the change in momentum that resulted from the impulse.



The <u>average</u> force could be determined if we knew the impulse and the total time over which the force acted - even if the force were not constant (or even known). The figure shows a graph of a variable force acting on an object over some time interval. The impulse is the area under that curve (*ie,* $\int \mathbf{F}dt$) which is also equal to the *average* force over that time interval multiplied by the time interval itself (from the Mean Value Theorem).

Momentum of a System of Particles and the Center of Mass of a System

Most problems, of course, involve more than just one object - and, generally, the various parts of a system somehow interact with each other. For example, even when we are describing the motion of a baseball flying through the air, we are really talking about the collective motion of, say, 10^{24} atoms that make up the baseball! That, of course, would be an impossible problem to deal with - and, happily, there is no need to describe the motion of every particle. But throughout the earlier discussion of Newton''s laws and how they govern the motions of objects, we ignored how the internal parts of the object interacted with each other. What we will see in this section and those that follow is that we can separate the description of the motion of what is called the *center of mass* of the object (or system) and the motion of all the elements of the object or system with respect to the center of mass. And no *new* laws - or even complications - are necessary to be able to accomplish that. What we will see is that Newton's laws apply directly to the motion of the center of mass of an object or system as if it were just a point particle located at the center of mass. And that is precisely what was done in all of the "F=ma" problems which applied Newton's second law to obtain the acceleration of an object which then led to a complete description of motion. It is why force diagrams are often constructed showing all of the forces acting at the center of mass of the object (as in *free-body diagrams*) - and it is the acceleration of the center of mass that is calculated using Newton's second law.



Consider an "object" which is actually a collection of interacting particles. Assume that we are interested in following the motion of this collection of particles. (If you need a specific example, think of something - a baseball, a galaxy, a swarm of bees ... it really doesn't matter which.) The momentum of a system of particles is just defined as the vector sum of the individual momenta of the particles or objects that make up the system - that is, the total vector momentum of the particles.

$$\vec{\mathbf{P}}_{system} = \Sigma\vec{\mathbf{p}}_k = \Sigma m_k \vec{\mathbf{v}}_k \qquad (7)$$

The idea of the *center of mass* of a system of particles can be understood in terms of the system momentum: The idea is that the center of mass (CM) is the point within the object or system which moves so that the momentum is just the total mass of the system times the velocity of the center of mass.

$$\vec{\mathbf{P}}_{system} = \Sigma m_k \vec{\mathbf{v}}_k = M\vec{\mathbf{V}}_{cm} \qquad \text{where } \vec{\mathbf{V}}_{cm} \text{ is just} \qquad \vec{\mathbf{V}}_{cm} = \frac{\Sigma m_k \vec{\mathbf{v}}_k}{M}$$

That means that the location of the center of mass can be located with respect to the x, y, and z axes by

$$x_{cm} = \frac{\Sigma m_i x_i}{M}\ ,\ \ y_{cm} = \frac{\Sigma m_i y_i}{M}\ ,\ \text{and}\quad z_{cm} = \frac{\Sigma m_i z_i}{M}$$

In general, the center of mass of a solid object is located at its geometric center. The above equations just give a formal definition of the location of the center of mass with respect to any choice of origin.

The motion of the Center of Mass

Defining the momentum of a system as the vector sum of all its parts allows one to examine the change in the momentum in terms of the forces that act on each part of the system. That is, the rate of change of the system momentum is just the vector sum of the net force acting on all of the parts of the system. But the net force on each part of the system includes both whatever forces act on that part due to external influences *and* the sum of all the interactions with other parts of the system. That is, the net force on the system is the sum of both all the external forces and all the internal forces.

$$\frac{d}{dt}\vec{\mathbf{P}}_{system} = \Sigma\frac{d\vec{\mathbf{p}}_k}{dt} = \Sigma(\vec{\mathbf{F}}_{\text{ext}} + \vec{\mathbf{F}}_{\text{int}})$$

where the summation includes every particle of the system. But the forces on each part can be written in terms of the external forces that act and the forces of interaction between the parts of the system - or internal forces. But the internal forces will include *action-reaction pairs* - or terms like $\vec{\mathbf{F}}_{1\to 2} + \vec{\mathbf{F}}_{2\to 1}$ (or just $\vec{\mathbf{F}}_{12} + \vec{\mathbf{F}}_{21}$) where the two forces are equal and opposite, hence cancel. That is, the vector sum of all the internal forces within a system is necessarily zero by Newton's third law.

$$\frac{d}{dt}\vec{\mathbf{P}}_{system} = \Sigma(\vec{\mathbf{F}}_{\text{ext}} + \vec{\mathbf{F}}_{\text{int}}) = \vec{\mathbf{F}}_{\text{total ext}} + \Sigma\vec{\mathbf{F}}_{\text{int}} = \vec{\mathbf{F}}_{\text{total ext}} + \underbrace{(\vec{\mathbf{F}}_{12} + \vec{\mathbf{F}}_{21})}_{0} + \underbrace{(\vec{\mathbf{F}}_{13} + \vec{\mathbf{F}}_{31})}_{0} + \ldots = \vec{\mathbf{F}}_{\text{total ext}}$$

The rate of change of the momentum of the system is thus equal to the net *external* force acting on the system - and any forces that are internal to the system can have no affect on the momentum of the system. But that means that the change in motion of the center of mass of the system is determined entirely by the external forces that act on the system, since

$$\frac{d}{dt}\vec{\mathbf{P}}_{system} = M\frac{d}{dt}\vec{\mathbf{V}}_{CM} = \vec{\mathbf{F}}_{\text{total ext}} = M\vec{\mathbf{a}}_{CM} \qquad (8)$$

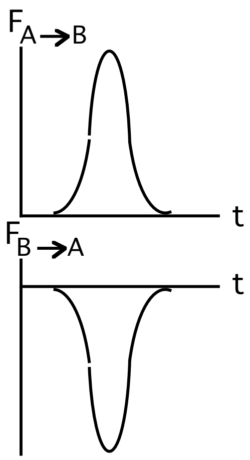
This is the fundamental (but largely unspoken!) idea behind all the previous Newton's law problems we have addressed. That is, for *any* object - a projectile, a block sliding down an incline, a planet in orbit about the sun, the object's acceleration can be determined by considering all of the *external* forces that act on it - ignoring the forces that are internal to the object. In a later section, we will consider rotations about the center of mass, for example, when a ball rolls down an incline and the motion involves both an acceleration of the center of mass and rotational motion *about* the center of mass.

The Law of Conservation of Momentum

A profound idea follows from Eqn (8). Since the rate at which the system momentum changes equals the net external force acting on the entire system (and depends not at all on any internal forces), if the vector sum of the external forces is zero, then the momentum of the system does not change - hence momentum is said to be *conserved*.

$$\frac{d}{dt}\vec{\mathbf{P}}_{system} = \vec{\mathbf{F}}_{\text{total ext}} = 0 \quad \text{or} \quad \vec{\mathbf{P}}_{system} \text{ is constant}$$

That is, combining Newton's second and third laws leads to the important result that the momentum of a system can only be changed by the action of external forces. Or, in the absence of external forces, the momentum of a system does not change. This idea is called the ***conservation of momentum.***



This principle can be thought of as a direct application of Newton's third law - the statement that all forces are interactions between two objects and that each object is subject to equal forces of interaction which are necessarily opposite in direction. When two objects collide, each object simultaneously exerts equal and opposite forces on the other.

The top graph represents the force some object A exerts on object B as a function of time when they collide. The bottom graph is the force B exerts on A as a function of time. According to Newton's Third Law, at any given moment, the forces must be equal and opposite. So the two graphs are identical - except for sign. But that means that the *impulse* of A on B is numerically equal, but opposite in sign, to the impulse of B on A - since the impulse is just the integral of the force over the time interval in which the two objects are in contact.

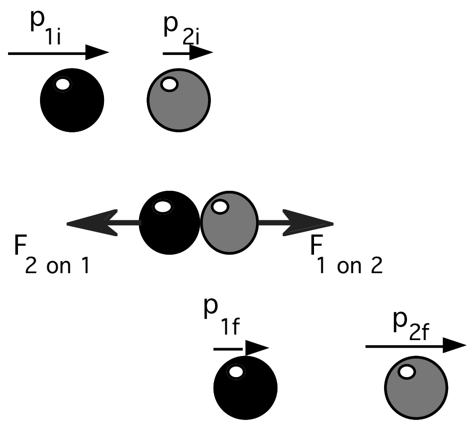
But if the only forces that act on the two objects are the forces they exert on each other, then the corresponding momentum changes of the two objects must also be equal and opposite, since the impulse of a force on an object equals its momentum change, if that is the only force acting. The consequence, of course, is that the two momentum changes add to zero and there was no change in momentum as a result of the collision *as long as no other forces were acting on either object during the collision*.

If there are no external forces acting on a system, its momentum is conserved.

The Significance of Momentum Conservation

The value of a conservation law is this: If a problem presents itself and some quantity can be defined such that the value of the quantity remains unchanged and does not depend on the details of the problem, then a solution to the problem can often be obtained without even requiring knowledge of those details. That means that if one knows the value of the quantity before some event takes place, and that it does not change during the event, then the results of some interaction may be obtained without having to solve the problem in detail. The are three conservation laws that follow from Newton's laws - the conservation of momentum, energy, and angular momentum. We will ultimately examine each of these principles.

But in addition to giving one another problem solving strategy, the conservation laws - and in particular the conservation of momentum - offer considerable insight into the behavior of systems of interacting objects. And the insights that come from the conservation laws often set limits on what is possible when describing mechanical systems.



$$\vec{\mathbf{p}}_{1i} + \vec{\mathbf{p}}_{2i} = \vec{\mathbf{p}}_{1f} + \vec{\mathbf{p}}_{2f}$$

The principle of momentum conservation can very often be applied to objects which collide - or interact as a result of their gravitational, electric, or magnetic fields - as long as any other forces that act on the system can be ignored (or eliminated from consideration) during the time the objects collide. For example, if two objects collide, their interaction while in contact is such that the momentum of the one ball is reduced while the other increases by exactly the same amount. That is, regardless of the mechanism of the interaction or any detail of the forces that act between them during the collision, some of the momentum of one of the objects is transferred to the other - and the total momentum of the pair remains unchanged. As we will see, in many cases, that will allow for the final motions of the two objects to be solved for without knowing the forces that acted while they were in contact.

Types of Impulse, Momentum, and Momentum Conservation Problems

The ideas presented in this section allow a variety of problem types to be addressed. Although the problems may appear to be quite different from each other (and sometimes seem to not include enough information), they often have in common that the only forces that act on the system of objects are those that are internal to the system –*i.e.*, forces of interaction between parts of the system.

(1) Impulse - Momentum Problems

Knowing the forces that act and the time interval over which the forces act allows one to solve for the change in momentum of an object by equating the momentum change to the impulse which can be calculated from the forces and the time interval.

Or conversely, knowing the change in momentum allows one to determine the impulse. If the time interval over which the force acts is also known (or can be estimated), the average force can be found.

(2) Collision Problems

During a collision between two objects, we typically assume that the two objects are in contact for a very short time and no external forces are acting. That means that the momentum will be conserved unless some force constrains the motion. Although not strictly so in real collisions, since there are often additional forces acting, very often the *external* forces that act on a system of two colliding objects are negligible compared to the relatively enormous force of interaction (which acts for a very short time) between the colliding objects while they are in contact.

(a) Elastic Collisions:

Some idealized collision problems have an additional condition that will ultimately allow a complete solution to be found. Objects are said to be *elastic* if they can completely restore their shape after being distorted. In a collision between two elastic objects (like perfect billiard balls) - or between two magnets which repel each other (even without physically making contact) - there is an additional relationship that can be included in the analysis. It was observed by Christian Huygens (in the early 1700s) that when nearly elastic objects collide, the combined kinetic energies (defined as $K = \frac{1}{2}mv^2$ for each object) of the two objects are unchanged by the collision. That is, in addition to the vector momentum being conserved, the kinetic energy of the system is also conserved in *elastic* collisions. In such collisions there is no permanent deformation of the objects that collide. The work done on each colliding object by the other is recovered in the "rebound" following the collision - that is, the colliding objects are *perfectly elastic*. [Note: The definition of kinetic energy, $K = \frac{1}{2}mv^2$, does not require the inclusion of the 1/2 to be conserved - but that coefficient is not just an arbitrarily choice. It is a part of the definition of kinetic energy due to the work-energy theorem which we will see in the next chapter.]

(b) Inelastic Collisions:

If permanent deformations <u>do</u> occur during the collision, the collision does not conserve energy - but momentum is <u>still</u> conserved. In inelastic collisions, kinetic energy is not conserved. Some of the original kinetic energy of the objects before the collision goes into deformation of the objects, or is lost to frictional forces as the objects move against each other while in contact. So the kinetic energy is not conserved in those collisions. Examples include when cars collide – energy was "lost" as the cars deform and scrape against each other. But momentum can still be essentially conserved during the collision itself if the friction between the tires and pavement can be considered negligible compared to the collision forces.

If the objects which collide stick together - that is, some trapping mechanism on one of the objects captures the other, or a bullet becomes imbedded in a block, or the colliding cars lock their bumpers together - the collision is called *perfectly inelastic* and the total momentum of the system is *still* conserved as long as external forces acting during the collision itself are negligible.

A Problem Solving Strategy

Defining momentum as the product of mass and velocity, then defining the *system* momentum as the vector sum of all the individual momenta of the parts of the system is only useful if it leads to some important principle or problem solving method that ultimately makes sense. The <u>reason</u> momentum is defined the way it is is so that the total momentum of any system is conserved in the absence of external forces. Having a quantity be conserved is then very useful in problem solving. It means that you can know what to hold constant in setting up a problem. Notice in the examples below that what allowed the problem to be set up was the application of the principle of momentum conservation. Also note that whether momentum is conserved or not does not depend in any way on whether mechanical energy is also conserved. Also notice that the important idea that leads to momentum conservation is Newton's third law. That is, the total momentum of two objects remains unchanged when they collide <u>because</u> the force each exerts on the other is equal and opposite - hence they have equal and opposite *impulses* and equal and opposite *changes* of momenta as a result. So total momentum remains unchanged during the collision.

In setting up problems involving collisions between objects, determine the following:

- Is the problem a one-dimensional or two-dimensional collision? If it is one-dimensional, you can just work with the magnitudes of the momenta. If it is two-dimensional, you must keep track of the components in both the x- and y- directions independently, since momentum is a vector quantity.

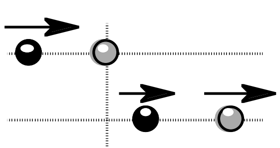
- Do any external forces act on the objects *during* the collision? If there are external forces acting during the collision, decide whether there are components of those forces along the line of motion involved in the collision. If there are no external forces - or if any external forces do not have components in the directions of the momenta you need to consider - then you can conclude momentum is conserved. That is, the total momentum of all the objects involved before the collision is necessarily the same as the total momentum immediately after the collision. (Again, if the collision is two-dimensional, both the x- and y- components of the momentum must be independently conserved.)

- Determine if the kinetic energy of the objects involved in the collision is *also* conserved. If there is no loss of energy *during* the collision, the collision is said to be *elastic*. That implies that you can also set the total kinetic energy before the collision equal to the total kinetic energy after. (That has the effect of giving one more equation, hence allowing you to solve for one more quantity.) Notice that since kinetic energy is a scalar quantity, the kinetic energy is not resolved into components.

- Determine if the masses stick together in some way (velcro, mechanical deformation, etc.) so that the final velocities of both objects are equal (same magnitude and direction). That allows you to determine the final speed of the objects - and correspondingly the final kinetic energies. If you can determine both the initial KE and final KE, you can also determine the energy *lost* in the collision.

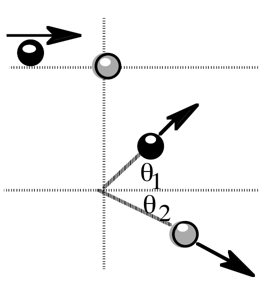
Elastic Collsions:

In elastic collisions, the conservation laws (energy and momentum) ultimately determine how nature decides what the final velocities should be! That is, given the two masses and the initial velocity of the incoming mass, the two final velocities are determined by the simultaneous solution of the two conservation law equations. Whatever happened *during* the collision is not important to the result. It doesn't matter if the balls physically collide, or whether a spring or some invisible force (like magnets) keeps them from actually touching. If energy and momentum are both conserved, the final velocities are determined by the masses and v_o.



Consider a one-dimensional elastic collision of a moving ball with one that is initially stationary. In this case, not only is momentum conserved, but so is kinetic energy. The equations in one dimension thus allow you to solve for both final velocities given the initial velocities and the masses.

$$m_1 v_o = m_1 v_1 + m_2 v_2 \qquad \text{and} \qquad \tfrac{1}{2} m_1 v_o^2 = \tfrac{1}{2} m_1 v_1^2 + \tfrac{1}{2} m_2 v_2^2$$



In a two dimensional collision, the same ideas are there, but now one has to keep track of the components of both masses.

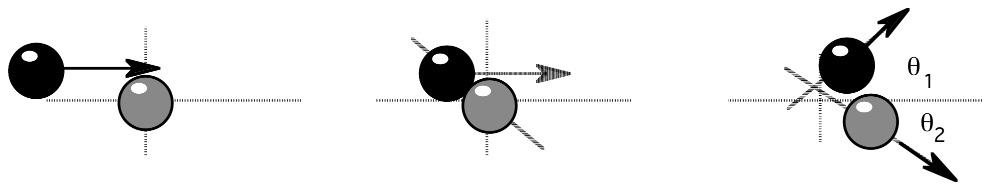
Since momentum is a vector, and if the initial momentum is only in the x-direction, then the final momentum is also only in the x-direction. Hence the sum of the x-components of momentum will equal the initial momentum - and the y-momenta must be equal and opposite (so that $P_y=p_{1y}+p_{2y}=0$).

$$m_1 v_o = m_1 v_{1x} + m_2 v_{2x} = m_1 v_1 \cos\theta_1 + m_2 v_2 \cos\theta_2 \quad \text{and} \quad m_1 v_1 \sin\theta_1 = m_2 v_2 \sin\theta_2$$

If the collision is perfectly elastic as well, then the total kinetic energy is still conserved. Notice that since energy is a scalar quantity – *i.e.*, does not have a direction associated with it - we can just write a single equation. There is no x- or y- component to the kinetic energy.

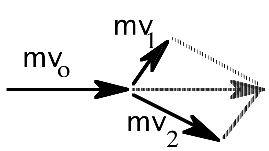
$$\tfrac{1}{2} m_1 v_o^2 = \tfrac{1}{2} m_1 v_1^2 + \tfrac{1}{2} m_2 v_2^2$$

It should now be noticed that some additional information would be needed to completely solve for the final velocities - since there are four unknowns (the velocities and the two angles), but only three equations. That information comes from the details of the collision - exactly how the two balls strike.



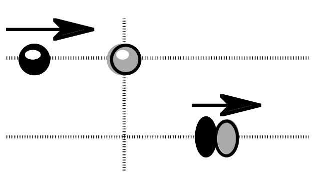
Often the additional information given is one of the scattering angles.

If the two balls have the same mass - and the collision is perfectly elastic - an interesting result occurs. The sum of the two angles is 90° in that case. Ie, the balls leave the collision travelling at right angles with respect to each other. (A result billiard players already know!)



Notice that in that case, the momentum vector equation becomes $\vec{\mathbf{v}}_o = \vec{\mathbf{v}}_1 + \vec{\mathbf{v}}_2$ and the kinetic energy equation becomes $v_o^2 = v_1^2 + v_2^2$. The law of Pythagoras thus requires the angle between $\mathbf{v}_1$ and $\mathbf{v}_2$ to be 90°

Perfectly Inelastic Collsions:



In this case, *i.e.*, perfectly inelastic collisions, you can set the two final velocities equal to each other since the masses stick together. The kinetic energy of the system can then be found after the collision - and hence the energy lost during the collision can be determined. That loss of energy depends only on the initial energy and the two masses involved in the collision - and not on the mechanism that causes them to stick together!

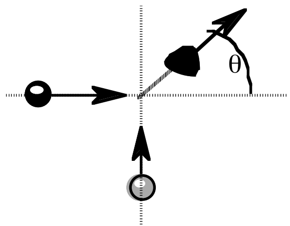
$$m_1 v_o = (m_1 + m_2)V \qquad \text{and} \qquad \Delta\text{K} = \tfrac{1}{2} m_1 v_o^2 - \tfrac{1}{2}(m_1 + m_2)V^2$$

Solving for the final velocity V of the two objects together fom the momentum equation and then substituting into the energy equation gives an interesting result:

$$K_f = \tfrac{1}{2}(m_1 + m_2)V^2 = \left(\frac{m_1}{m_1 + m_2}\right)\left(\tfrac{1}{2} m_1 v_o^2\right) = \left(\frac{m_1}{m_1 + m_2}\right) K_i$$

That is, the final kinetic energy of the combined masses only depends on the initial energy and the ratio of the masses - and NOT on what mechanism made the two stick together!

In two dimensions, the same principles still hold - but again, the momentum must be dealt with in component form. If the initial velocities of the two objects are in the x- and y- directions initially, then in this case, the final momentum will have components equal to the x- and y- components of the objects before they collide.

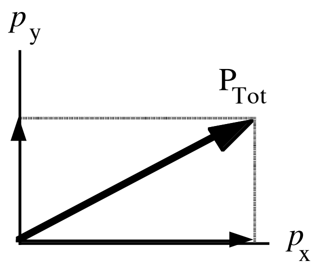


$$m_1 v_1 = (m_1 + m_2)V_x = (m_1 + m_2)V\cos\theta$$

and

$$m_2 v_2 = (m_1 + m_2)V_y = (m_1 + m_2)V\sin\theta \qquad \tan\theta = \frac{V_y}{V_x} = \frac{m_2 v_2}{m_1 v_1}$$

$$K_f = \frac{1}{2}(m_1 + m_2)V^2$$



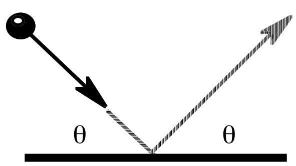
It is useful while setting up two-dimension momentum problems to construct a vector diagram showing the vector momenta. For example, suppose in the above example, that the momenta prior to the collision are different for the two objects. Then the vector momentum prior to the collision can be displayed on a momentum vector diagram. If there are no external forces acting on the two objects during the collision itself, then momentum of the system cannot be changed due to the collision - and hence the final momentum vector is identical to the initial momentum vector in both magnitude and direction. The final velocity of the center of mass is then just the magnitude of the momentum vector divided by the combined masses of the objects which collided.

In the above example, the momentum *change* of each object can also be determined, since each mass has the same final velocity (that of the attached masses), the momentum change of each object is just the difference in their final and initial vector momenta. You should be able to show from their vector diagrams that the momentum *changes* of the two colliding objects are equal and opposite – as required by noting that the impulses must be equal and opposite during the collision itself.

Of course, any interaction with the surroundings after the collision will ultimately result in a change in the momentum of the system – as in the case when two cars collide and the wreckage comes to a stop some distance where the original collision took place.

The above results are much more general than seem to be implied by the examples given. For example, a problem can be considered a collision problem even if the objects never "touch". That is, two objects which interact over large distances, two magnets or two galaxies, for example, would obey the same principles. Or an object which separates into two or more pieces (for example, as a result of an explosion, or something) could also be treated applying the same principles. An explosion, for example, is *internal* to the system and, consequently, cannot change the motion of the center of mass of the object that exploded. Suppose a projectile exploded mid-flight. The center of mass of the fragments when they finally hit the ground would necessarily reside at the same point that the projectile would have landed had it not exploded. That idea follows directly from the statement that the momentum of the system can only be affected by external forces.

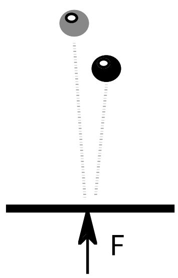
MOMENTUM – Questions and Problems

1. 

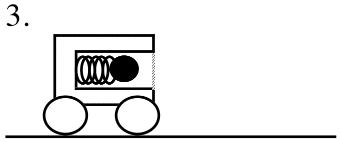
Consider that a marble rolls on a horizontal surface and strikes a wall at some angle of θ . Assume the collision is elastic (which requires the two angles to be the same). [Assume the figure shows the collision of the ball with the wall as you are looking down from above.]

Write an expression for the magnitude of the momentum change and show the direction of the momentum change on the diagram.

If the marble is in contact with the wall for a time Δt, write an expression for the average force exerted on the wall by the marble.

2. 

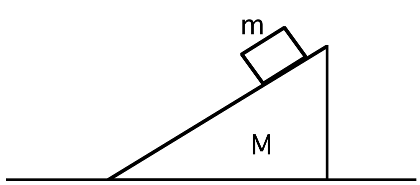
Consider that you drop a ball from a height of one meter and it rebounds to a height of 3/4 meter. If it is in contact with the ground for 10 msec, determine the ball's average acceleration <u>while it is in contact with the ground</u>. Compare the maximum force on the ball to the ball's own weight assuming the maximum force the ground exerts is twice the average force. Explain your reasoning carefully.

3. 

Consider a cart of mass *M* which is initially at rest. Compressed against a spring is a ball of mass m. The spring has spring constant *k* and is originally compressed an amount *Δx*. Suddenly, the spring is released to eject the ball.

a. Write the two independent equations which allow you to determine the speeds of the cart and ball after the ball is ejected. Assume there is no loss of energy when the spring is released. Explain your reasoning.

b. Solve the equations for the two final speeds. Assume you know the values of *m, M, k,* and *Δx*.

4. 

Consider a wedge shaped block of mass *M* and a small block of mass *m*. The small block is released on the incline a height *h* above the table and slides down the incline while the wedge slides to the right.

Assume all surfaces are frictionless.

a. Which of the following quantities are conserved <u>as the small block slides down the incline</u>? Justify your choices.

Kinetic energy   Total energy of the small block   Total energy of both masses

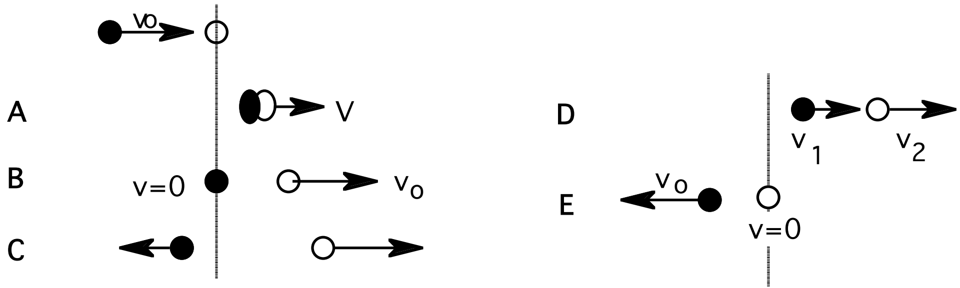
Total momentum   Horizontal component of total momentum

b. Obtain an expression for the speed v of the small block after it reaches the table in terms of known quantities: *m, M, h,* and *g*.

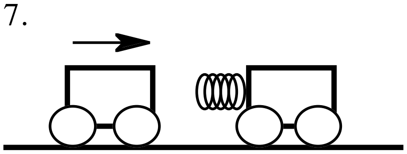
5. Carefully explain why Newton's third law requires that momentum be conserved in a collision of two objects if there are no external forces acting on the objects during the collision.

   Carefully explain why only the external forces on an object can govern the motion of the center of mass of the object.

6. Consider a ball of mass m_1, which strikes ball m_2 (which is initially at rest). The figure shows five outcomes (A-E) of the collision to consider.



   a. Identify each case for which momentum could be conserved. Justify your choices.
   b. Identify each case for which kinetic energy could be conserved. Justify your choices.
   c. If the collision is elastic, which of the above corresponds to the case $m_1 < m_2$?

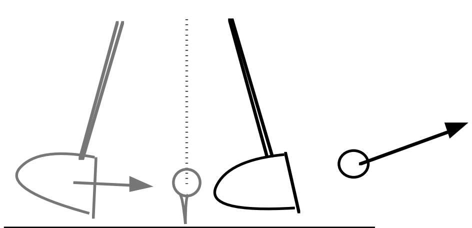
7. 

Suppose a cart crashes into another cart (which has a spring attached as shown). Let the masses be M_1 and M_2 , and the speed of M_1 initially be v_o .

   a. Write two independent expressions which would allow you to solve for the final velocities of the two carts. Briefly justify how you obtained the expressions.

   b. Write the two expressions that would allow you to determine the maximum spring compression during the collision.

   [Note: The two above cases are essentially identical to the (a) perfectly inelastic collision; and (b) perfectly elastic collision.]

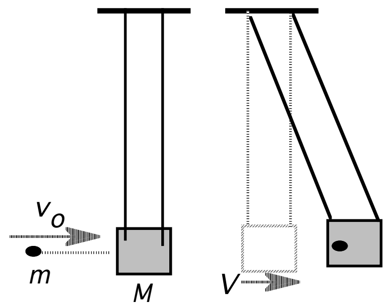
8. 

Suppose a golf ball is struck by a golf club. Assume that the collision is elastic. If the ball has a mass of 50 g and the golf club head has a mass of 200 g, determine the ratio of the launch speed of the ball to the clubhead speed just before contact. [Make the simplifying assumption that it is a one-dimensional collision in which momentum is conserved.]

   If increasing the mass of the clubhead by 10% had the effect of making the swing slower by 10%, would the ball have a higher or a lower launch speed?

   [Note: This is not exactly a pure momentum-conserving collision since there are external forces involved. The clubhead is attached to the shaft which the golfer still is holding (and applying a force to) – and momentum is strictly conserved only if there are no external forces. But it is a pretty good approximation of the relationship between club speed and ball speed when a golf ball is struck by the club.]

9.



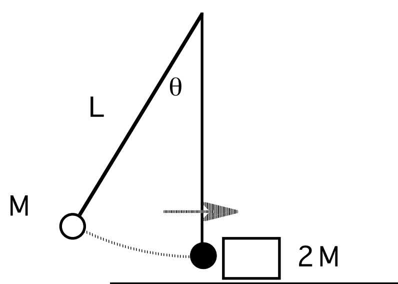
Suppose a bullet (m) is fired horizontally (initial speed v_o) into a block (M) which is suspended from strings of length L, as shown.

a. Obtain an expression for the speed (V) of the block and bullet combination just after the bullet strikes the block and is imbedded in it. What principle is used to determine the expression?

b. Obtain an expression for the maximum height of the block. What principle applies?

c. Show that the fraction of the original kinetic energy of the bullet that is lost in the collision is $M/(M+m)$ – that is, the fraction of the energy lost depends only on the two masses and not on how the bullet is "captured" by the block.

10. Suppose a ball of mass M is attached to a string of length L and displaced through an angle θ. When released, it swings down and strikes a block of mass $2M$.

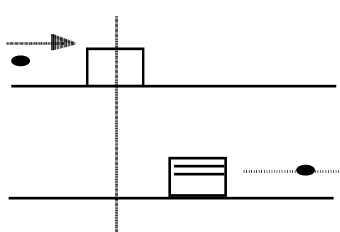


a. Determine the speed of the ball just before impact with the block. (Assume you know M, g, L, θ.)

b. Assuming the collision is elastic, determine the speed of the block just after the collision (in terms of v, the speed just before impact found in (a).)

c. If the collision is elastic, determine the speed and direction of the ball just after impact.

11.



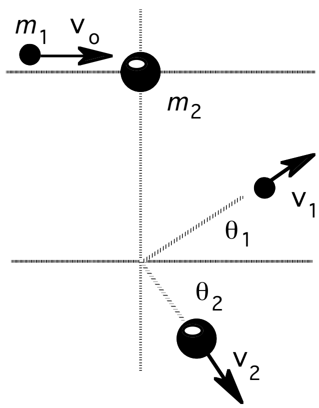
Suppose a bullet is fired at a block on a horizontal frictionless surface. The mass of the bullet is m and the mass of the block is M=10m. If the bullet has an initial speed of v_o, passes <u>through</u> the block and emerges with speed v_f=v_o/10, determine the final speed of the block.

Determine what fraction of the original kinetic energy of the bullet is lost in the collision.

12.

Show that if an object collides with a second object which is originally at rest and they stick together, the final kinetic energy of the combined masses after the collision depends only on the masses and the initial kinetic energy of the first object. That is, the energy lost does not depend on how the two masses become attached – *ie*, whether there is some latching mechanism, glue, magnetic forces, Velcro, or whatever, that ultimately keeps them from separating.

13.



Consider that ball 1 collides elastically with ball 2 in such a way that the two balls scatter at angles θ_1 and θ_2 as shown.

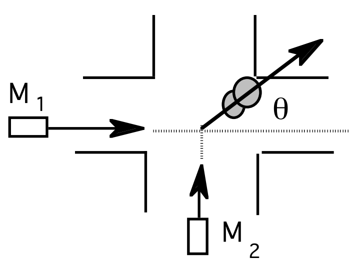
a. Write the equations that would allow you to determine three of the four quantities v_1, v_2, θ_1 and θ_2.

Show that if the balls have equal mass, then $\theta_1+\theta_2$ must be $\pi/2$. [Hint: Write the equation for energy conservation. Notice similarity to law of Pythagoras.]

b. Now assume that m_2 has twice the mass of m_1, *i.e*. the masses are *m* and *2m* and that θ_2 is 30°. Solve for the final speeds and the direction ball 1 travels after the collision (*ie,* θ_1)

c. Suppose the two balls stick together. Determine the speed and direction they will be traveling after the collision. Explain your reasoning. Write an expression for the <u>final</u> kinetic energy in terms of m and the initial speed v_o in that case.

14.



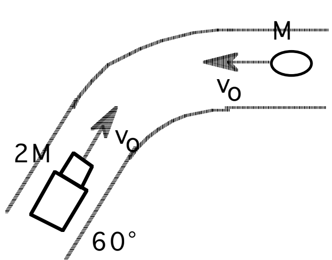
Two cars (masses M_1 and M_2) traveling at initial speeds v_1 and v_2 collide at an intersection as shown. Upon colliding, they stick together and slide along the pavement until they come to rest.

Assume that friction with the pavement does not affect the collision itself and write the equations that would let you solve for the speed of the wreckage immediately after impact and the direction that it travels. Briefly justify your reasoning.

Sketch a momentum vector diagram showing the momenta of the two cars before the collision and the vector sum – ie, the momentum of the system before the collision.

Determine the fraction (or percentage) of the initial total energy of the cars that is lost in the collision itself (*ie,* not including that which is lost as the wreckage slides to a stop).

15.



Consider a that car traveling west and a truck traveling 60° north of east collide, stick together, and slide to a stop. Assume the friction with the pavement has no effect on the collision itself.

Determine the speed and direction of the wreckage immediately after impact if the truck mass is twice the car mass and they have the same initial speeds.

Carefully sketch the momentum vectors of each vehicle before the collision and the combined momentum after.

Determine the energy lost in the collision itself based on the speed of the wreckage immediately following the collision.