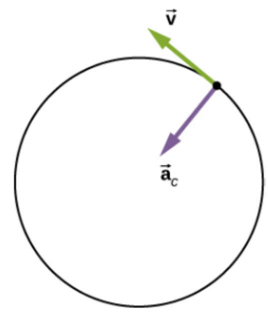
**5.0 Central Attraction:** centripetal acceleration

When we see a body moving in a circular path, we know that there must be force acting on it because without a force, things travel in a straight line. We likely want to look at this through a *dynamics lens.* From our experiences we can also see that the force is radially inward:

a) Gravity pulls the moon into a circular orbit. The force of gravity on the moon pulls the moon directly inward toward the earth.
b) If you attach a rock to the end of a string and spin it around your head in a horizontal circle, the string pulls the rock radially inward.
c) If you roll a marble around the bottom of a cylindrical container, the normal force of the wall on the marble is inward.




We know that $\sum \vec{F} = m\vec{a}$, so if the force pulling the object is inward, we can be sure the resulting acceleration is inward too. At right, you see that the velocity of an object in circular motion is tangential to the circle (and would continue in this direction without some force accelerating it), and the acceleration is radially inward.

We can show that the acceleration of an object in uniform circular motion scales like the <u>square</u> of the velocity and <u>inversely</u> with the radius of the circular path. Consider that at some velocity, *v,* you go around a half turn of radius *R*, and come back with velocity -*v*, remembering that that $\vec{a} = \frac{d\vec{v}}{dt}$:

a) If you double your speed the change in velocity (-2*v*) also doubles. However, at twice the speed, you take half the time to make the turn, so the acceleration should increase by a factor of 4, or the square of the speed.
b) If you double the radius of the turn, the change in velocity is still the same (-2*v*), but you take twice as long to do the turn, so the acceleration should be half as much

Thus, we are not surprised that centripetal acceleration is $a_c = \frac{v^2}{R}$.

<u>Forces cause acceleration (dynamics lens) not the other way around</u>. I often hear that *centripetal acceleration* means there's *centripetal force*. There's no such thing as centripetal force, just like there's no such thing as linear force. A force is an interaction between two bodies whereby they exchange momentum. Force can cause linear acceleration or centripetal acceleration. When we see centripetal acceleration (or any other acceleration), we know that there is some force pushing or pulling on the object to cause this acceleration. So, if we see circular motion, we should consider the dynamics lens and look for the force or *net force* toward the center of the circular path.

Exercise 1: Please first read the previous paragraph.
You see a 10 kg rock in space moving with constant speed of 10 m/s in a circle of radius 20 m. You wonder about the rock, and look at it through different lenses.

- Do you think there's a force acting on it? Why?
- Find the acceleration of the rock, including direction of the acceleration.
- Calculate the force necessary to accelerate this rock.
- What kind of force is this? – if you say, "it is centripetal force!" I will be sad. I will be pleased if you say, "I have no idea what force is acting on it, because I can't see anything that the rock is interacting with, so I have to look around at what object must be applying a force of_____(put answer from d) on the rock to make it accelerate at______(put answer from c)."
- Then you see a string attached to my arm as I spin the rock in a circle. What kind of force is it? Find the tension in the string.
- Then the string breaks – what happens to the rock? Please Draw a Picture
- ….instead of a string, you see a large sphere in the middle of the rock's circular path. What kind of force might be acting on the rock now?
- ….instead, you notice that the 10 kg rock is actually a small 10 kg toy car driving around in a 20 m circle on a flat parking lot at 10 m/s. Now what force is acting on the car? Please find the coefficient of friction necessary to keep the car moving in this circle. Is this Static friction or dynamic friction?

Exercise 2:
You're taking a turn with a car or bike on flat pavement and there is a coefficient of static friction of 1.2 between the rubber and the road. To execute a turn of a radius of 10 m,

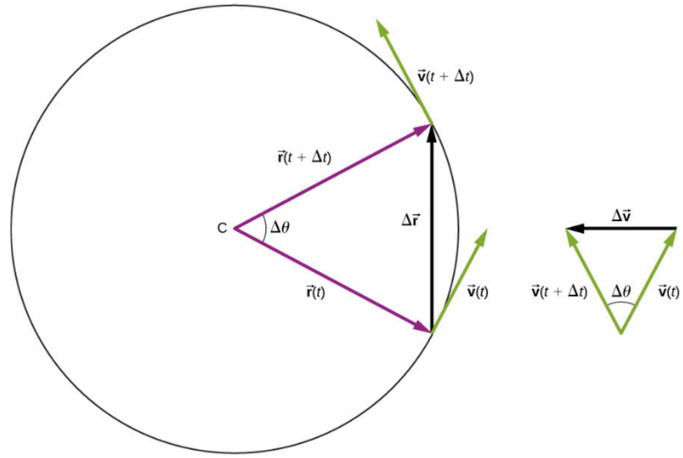
- What force causes the centripetal acceleration allowing you to make this turn?
- How fast can you go without sliding out?
- State your lens; set up the drawings and logic; and reflect on your answers in light of your experiences.
- Why did we use static friction rather than dynamic friction?

Exercise 3:

Prove to yourself that we can also write centripetal acceleration as $a_c = \omega^2 R$, and $a_c = \frac{4\pi^2 R}{T^2}$, where $\omega$ is angular velocity. Astronomers like the second expression because *T* is the period or time it takes for a full revolution, or length of year (of a planet for instance).

Exercise 4:

**PROVE IT!** Proving the formula for centripetal acceleration is a classic proof I expect you to be able to do. Please learn it. The drawing at right shows a picture of something moving in circular trajectory. The diagram to the right shows how to find the change in velocity using the velocities at the two points in the cicular trajectory.




Please prove to yourself that the two isocelese trianges are similar, and thus their parts are in proportion. You can then start with the definition of acceleration as the rate of change of velocity and express $\Delta\vec{v}$ and $\Delta t$ in terms of radius and speed. In this proof it is also important to take the limit of $\Delta t = \Delta\vec{v} = \Delta\theta = 0$. Why? Do we want the average acceleration, or the *instantaneous acceleration,* $\overrightarrow{a} = \frac{d\vec{v}}{dt}$? The average acceleration over one revolution is zero because the change in velocity is zero. The acceleration, like the velocity is constantly changing, so it's the instantaneous accelertion that is of interest. When we take the limit of short time or small $\theta$, we see that $\Delta\vec{r}$, the change in position becomes the same as the distance traveled along the perimeter of the circle and $\frac{\Delta r}{\Delta t} \Rightarrow v$, the tangential speed. Please carry out this proof with a good drawing.