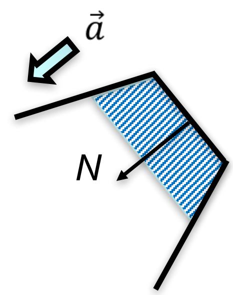
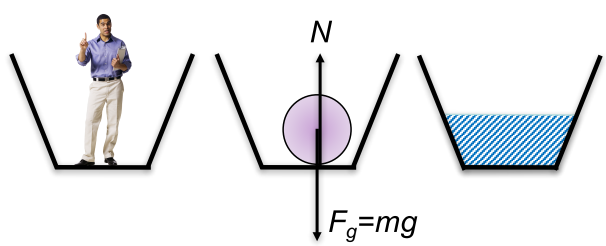
5.3 Loop-the-Loop: *Circular motion in the vertical plane*

When you see something moving in a circle, it's a good bet that dynamics is involved. Make a free body diagram and note acceleration is towards the center of the circle.



A review from section 2.5 (just this page):
If we were in outer space, or in free fall inside the space station as it orbits the earth, there is no apparent force of gravity. How would we keep water inside a bucket, so that the water is touching the bottom of the bucket? "Touching" the bottom of the bucket means that there is a nonzero normal force; that is $N > 0$. Given that this is the only force, it would mean that the bucket would be accelerating in the same direction. So, you could imagine moving the bucket around keeping the normal force positive, which would accelerate it back and forth, or in a circle.

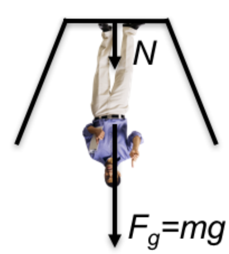
Please recall the elevator problem remembering $\sum \vec{F} = m\vec{a}$. Consider keeping a man, a ball, or water in a bucket as shown at right.



Exercise 1:
If the object is kept in the bucket, what do we know? Identify all correct statements. Answers at chapter end.

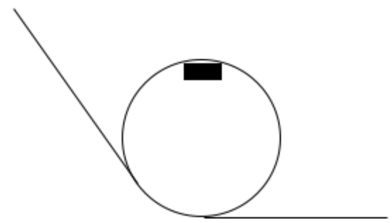
a) $N > F_g$.
b) $F_g > N > 0$.
c) $N > 0$.
d) acceleration must be upward or zero only.
e) acceleration can be zero or any value upward or downward.
f) acceleration can cannot be downward more than gravity.

Exercise 2:
How about if we see someone in a bucket upside down, what do we know? Identify all correct statements. Answers at chapter end.



a) $N > F_g$.
b) $F_g > N > 0$.
c) $N > 0$.
d) acceleration must be downward more than gravity.
e) acceleration can be any value, but must be downward
f) acceleration can be zero or any value upward or downward.
g) acceleration can cannot be downward more than gravity.

When we see something moving in a circle, we only know that it is accelerating radially inward. We look for the forces that could provide that acceleration according to $\sum \vec{F} = m\vec{a}$. So, for instance with a "Loop-the-Loop" carnival ride, the forces include gravity and the normal force that cause centripetal acceleration inwards. So, at the top the centripetal acceleration is downward. If we want the cart to stay in contact with the track, we know that the normal force is downward. What does this say about the centripetal acceleration necessary to keep the cart on the track?

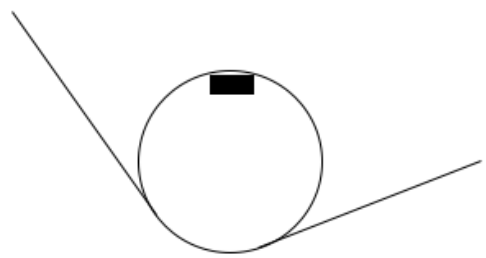


Exercise 3:
Please make a good free body diagram of the cart at the top of the loop and the bottom of the loop. Please identify what you know for both the *acceleration,* and the *normal force*:

- at the top of the loop
- at the bottom of the loop

be sure to support your answer with a diagram and lens identification.

Exercise 4:
You go on a R = 10 m, loop-the-loop ride where the cart is let go on a low friction track and is pulled downhill by gravity. You have to choose how high to start the cart. Say you have a mass of 70 kg, like your instructor and you are sitting on a scale that reads in kg. Don't use this drawing… please make your own.



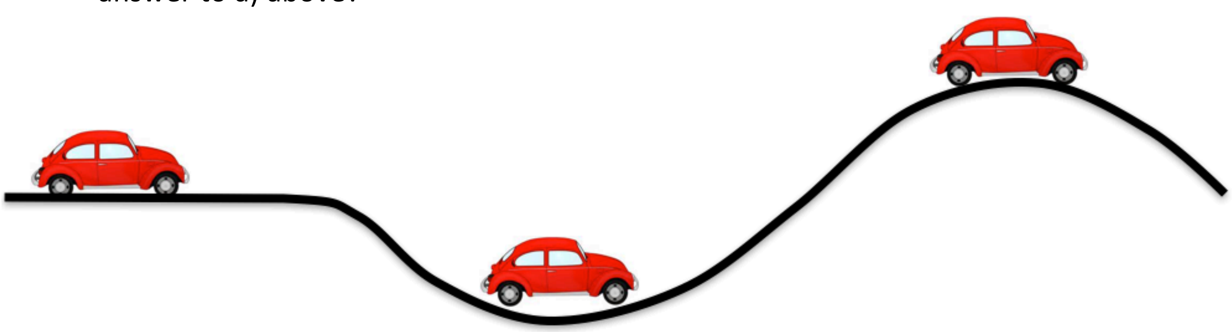
a) If you start from a vertical height of 40 m, what does the scale under you read as you are at the top of the loop? What does it read at the bottom of the loop? Is this a good ride for pregnant women? How does it feel as you round the bottom of the loop?
b) What would happen if you decide to start the cart at the same height as the top of the loop? Why would this happen?
c) Please find the minimum vertical height, above the ground that you must start the ride to stay on the track at the top.

The low friction, circular loop-the-loop ride above has a higher speed at the bottom than at the top. Thus, the centripetal acceleration to keep a circular path is also greater at the bottom. Additionally, please show that because of the way the track faces, in order to keep the cart on the track at the top, there must be a very great normal force at the bottom. This could make for a dangerous carnival ride! Is there some way we could increase the centripetal acceleration at the top (to keep the cart on the track), and lower the centripetal acceleration at the bottom (in order to not smoosh the people)?

Exercise 5:
Look at the carnival ride at right. Notice that the radius of curvature at the top is not the same as the radius of curvature at the bottom. What effect does this have on the centripetal acceleration at the top and at the bottom? Why would they build it this way? Please support your answers with a good free-body diagram, and identify the lenses you use.

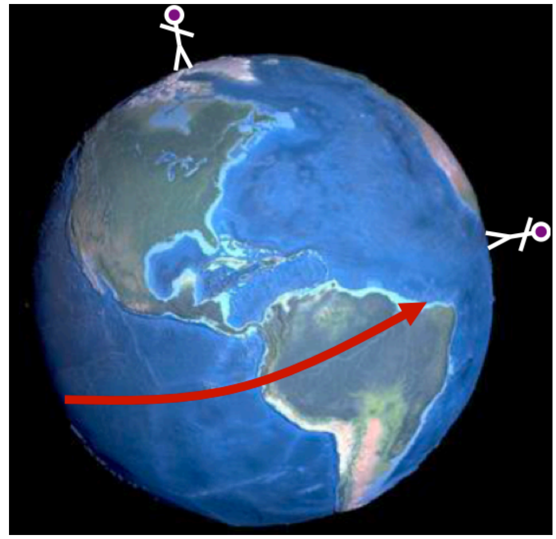
Exercise 6:
Below, you see 3 pictures of cars on a road driving to the right. There is a scale under each person reading the normal force the car exerts on them.

a) For each scenario, please draw a good free body diagram, label the direction of the acceleration, and state what the scale under the person in the car will read: less than zero, zero, greater than zero but less than F_g, equal to F_g, greater than F_g?
b) How do you feel when you are in these places? How does that correlate with your answer to a) above?



Exercise 7:

Do you weigh more at the North Pole?



You stand on a scale at the north pole and then on a scale at the equator to see where you weigh more. Assume that the earth is a perfect rotating sphere. You find that at the equator, you weigh:

a) More because the normal force compensates for both gravity *and* centripetal acceleration.
b) Less because the normal force at the equator is not enough to keep you in equilibrium.
c) The same because the normal force is always the same as gravity.
d) The same because you are in equilibrium in both places
e) None of these.
f) Not enough information is given

If you get confused… as with any mechanics problem, please do a lens analysis.

There is a story that gold was mined in Alaska, weighed, and sent to Fort Knox in Washington DC where they weighed it again. What did they think in DC when they weighed the gold they'd just purchased?

Please don't read these answers until you have made a free body diagram and discussed the answers yourself. Exercise 1 (c,f); Exercise 2 (c,d)