Part 6

ROTATIONAL MOTION

Rotational Kinematics

Rotational Dynamics

Torque and Angular Acceleration

Rotational Energy and Moment of Inertia

Angular Momentum Conservation

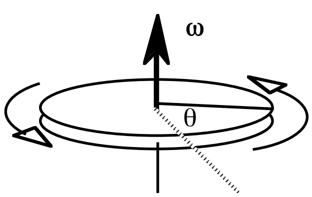
Rotation Problems

ROTATIONAL MOTION

All of the discussion of the motion of objects and the principles that relate to the motion has so far been in terms of *translations* - or the movement of the *center of mass* of the object or system being described. To complete the description of the motion of objects and systems, we need to also consider *rotations* - which we will be able to reduce to two classes of problems: rotations about a fixed axis and problems which involve both rotations and translations. For single objects (for example a ball that rolls) those motions can described in terms of motion of the center of mass and motion about the center of mass. For multiple mass objects (for example masses suspended over pulleys, etc.), that will involve setting up separate equations for the translational an rotational motions of the different objects. We will see that the ideas are very similar to problems involving just translation. And so the material will serve as a review of those ideas. There will also be two new ideas - *torque* and *angular momentum* - introduced. The equations related to rotational motion will be essentially identical to those involving translations – with each quantity having an analogy to quantities you have been dealing with all term. Those new concepts will be very similar to the ideas of force and linear momentum. And a third *conservation law* will be added to the two (energy and linear momentum) that we have already made good use of.

Rotational Motion

Rotations about an axis can be described in terms of the angle swept out by an imaginary line on the rotating object. Consider a disk which is rotating about its axis of symmetry. Let the angle θ represent that angular displacement of the disk with respect to some arbitrarily chosen direction in space (which could be called the *x*-axis). The definitions of the dynamic quantities which characterize its rotational motion based on changes in that angle are given below:



The **angular velocity** - $\vec{\omega}$ - is just the rate at which the rotation angle changes or the *rate of rotation* and is related to the angle itself by $\omega = d\theta/dt$ in exactly the same way that linear velocity is related to the rate of change of a linear coordinate by $v=dx/dt$. Just as linear velocity is a vector quantity, the angular velocity is also a vector quantity. But the direction of the vector is not in the direction of motion of any particle, but rather is assigned a direction along the axis of rotation and defines the sense of the rotation about that axis. The vector direction of $\vec{\omega}$ is determined by the *right-hand rule* where the right thumb points in the direction of the vector if the fingers curl in the direction of the rotation. Angular velocity has units rad/sec or just sec^{-1}.

It is important to note that an object's angular velocity is just the rate of rotation about a point or axis. The *vector* direction associated with the angular velocity just defines the axis of rotation -*and* gives the sense of rotation by using the right-hand rule. This has the effect of letting the vector $\vec{\omega}$ contain all the information about the rotation – *i.e.,* the axis about which the rotation occurs, the sense of rotation, and how rapidly the object rotates. But it should also be clear that nothing actually *moves* in the direction of the angular velocity vector.

The **angular acceleration** - $\vec{\alpha}$ - is the rate of change of the angular velocity velocity vector in the same way that linear acceleration is the rate of change of the linear velocity vector (which can mean either a change in its direction, or both) and is given by the equation $\vec{\alpha} = d\vec{\omega}/dt$. The same RH rule applies to determining the direction of the angular acceleration vector. If the axis of rotation does not change, the angular acceleration is just the rate of change of the magnitude of the angular velocity vector $\alpha = d\omega/dt$. The units of angular acceleration are rad/sec^2 or simply sec^{-2}.

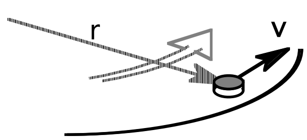
It should be noted that angular acceleration vector can either identify the change in rotation rate (for example, if the "spin rate" increases or decreases) *or* the rate at which the axis of rotation changes direction - for example when a rotating bicycle wheel goes through a turn. In both cases, the angular acceleration is the rate of change of the angular velocity vector. *Most* problems that we deal with will involve rotations about a fixed axis, so the angular acceleration only has to do with how the rate of rotation changes.

Notice that the angular velocity and angular acceleration of a rotating object have the same relationship to the angular displacement that linear velocity and linear acceleration have to a translational displacement along a straight line. Because of that relationship, any equation that was useful in the description of motion in translational motion has a counterpart in rotational motion descriptions:

$$\omega = \frac{d\theta}{dt} \qquad \alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2} \qquad \text{and} \qquad \omega(t) = \int_0^t \alpha\, dt \qquad \theta(t) = \int_0^t \omega(t)\, dt$$

And if the angular acceleration is a constant, we get the same kinematics equations that resulted from constant acceleration in one-dimensional motion:

$$\omega = \omega_o + \alpha t \qquad \theta = \theta_o + \omega_o t + \tfrac{1}{2}\alpha t^2 \qquad \text{and} \qquad \omega^2 - \omega_o^2 = 2\alpha(\theta - \theta_o)$$



It is useful to note that there is a relationship between the magnitude of the tangential velocity of any part of a rotating object and the magnitude of its angular velocity. That is

$$v = r\omega$$

where r is the radius from the axis of rotation.

Similarly, the tangential acceleration is related to the angular acceleration by $a_t = r\alpha$.

You should recall that the <u>radial</u> acceleration of an object moving in a circle is given by $a_r = v^2/r = \omega^2 r$ which is just the centripetal acceleration expression of circular motion. So the total vector acceleration would, in general, then include both components, that is:

$$\vec{\mathbf{a}}_{tot} = \vec{\mathbf{a}}_r + \vec{\mathbf{a}}_{tan} = -\left(\omega^2 r\right)\hat{\mathbf{r}} + \left(r\alpha\right)\hat{\boldsymbol{\theta}}$$

where $\hat{\mathbf{r}}$ and $\hat{\boldsymbol{\theta}}$ are just unit vectors in the radial and tangential directions.

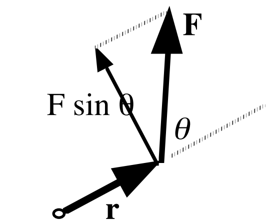
Torque and Newton's Second Law for Rotation:

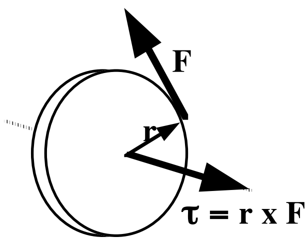
What <u>causes</u> rotational accelerations? The answer, of course, is that forces cause things to accelerate. But if an object is constrained to rotate about an axis, only the component of a force tangential to the rotation can change the rotational motion. How much the angular velocity changes depends on the magnitude of the tangential force, how far its application point is from the axis of rotation, and the rotational inertia of the object. The quantity that contains that information is called the *torque*.

Torque

Torque is defined as the tangential component of a force times the "moment arm" , which can be written mathematically as the vector product (or cross product):

$$\vec{\boldsymbol{\tau}} = \vec{\mathbf{r}} \times \vec{\mathbf{F}}$$





The magnitude of the torque is $rF\sin\theta$. $F\sin\theta$ is the component of the force perpendicular to the radius arm and the direction of the torque vector is *also* given by the right hand rule (consistent with the *cross product* of the vectors $\vec{\mathbf{r}}$ and $\vec{\mathbf{F}}$): Point your RH fingers in the direction of $\vec{\mathbf{r}}$, curl them toward $\vec{\mathbf{F}}$, and your thumb points in the direction of the torque. This torque direction is consistent with the direction of the angular velocity direction that would result from a rotation caused by the torque.

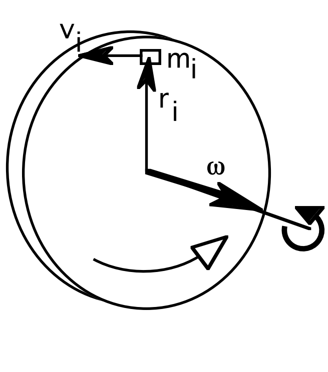
As in translational motion where a force parallel to the velocity of an object increases its speed, a torque vector in the direction of the angular velocity will increase the rate of rotation. The units of torque are Newton-meters or kg-m^2/s^2.

Newton's second law for rotation then becomes: $\vec{\boldsymbol{\tau}} = I\vec{\boldsymbol{\alpha}}$

$\vec{\boldsymbol{\alpha}}$ is the vector angular acceleration that results from the application of the torque and I is called the *rotational moment of inertia* (or simply, *moment of inertia*) about the axis of rotation of the object that is being described. The most difficult idea here is probably what one means by the "moment of inertia". The quantity I plays the same role in rotation problems that mass does in translation problems. It is a measure of the inertia of the object - its reluctance to having its motion changed. It just means that an object with a large amount of inertia takes a larger force or torque to change its linear or angular velocity. It should not be surprising that the moment of inertia depends on the mass of the object. But it also depends on how the mass is distributed about the axis of rotation, as we will see in the following discussion.

A force applied to an object in such a way to change its rotation rate would also do work on the object and subsequently change its kinetic energy. By looking at the kinetic energy of an object in rotation, we will be able to identify how the distribution of an objects mass affects its rotational dynamics and will see a relatively simple way to determine the energy associated with its rotation.

Rotational Kinetic Energy and Moment of Inertia



Any object which has rotational motion has kinetic energy associated with the rotation. This is just the total kinetic energy of all the particles that comprise the object. Since each particle has a kinetic energy that depends on its speed v and the speed is given by $v = r\omega$, and every part of the entire object rotates with the same angular velocity ω, the total kinetic energy of all the particles is given by:

$$K = \sum \tfrac{1}{2} m_i v_i^2 = \sum \tfrac{1}{2} m_i r_i^2 \omega^2 = \tfrac{1}{2} \sum (m_i r_i^2)\, \omega^2 = \tfrac{1}{2} I \omega^2$$

That is, the kinetic energy of rotation of an object which has angular velocity ω about an axis is

$$K_{rotation} = \tfrac{1}{2} I \omega^2$$

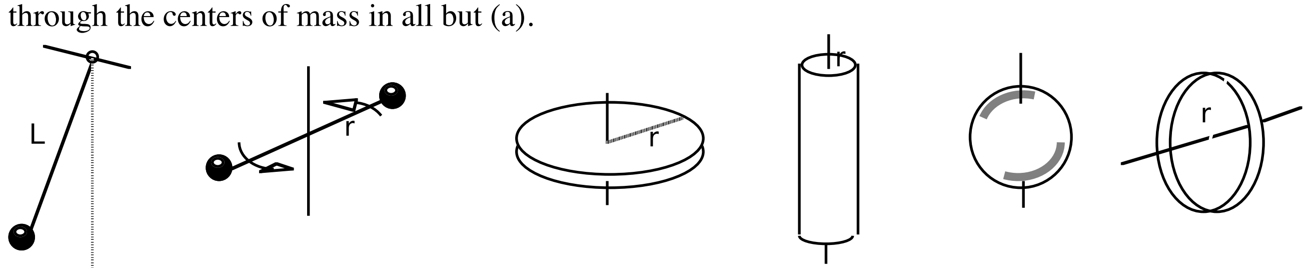
The quantity $I = \sum (m_i r_i^2)$ is the *moment of inertia* and in all problems involving rotation plays the same role as the mass in translational motion problems.

Notice that the form for the kinetic energy is just that of the kinetic energy of an object that is in translational motion - one-half a measure of its inertia times the square of its rate of motion. The moment of inertia depends both on the mass and how the mass is distributed - and so does the kinetic energy. For a given rate of rotation, if more of the mass is located far from the axis of rotation, the object has a larger kinetic energy (because of the higher speeds of the particles farther from the axis.) If the mass is distributed close to the axis, it would have a smaller kinetic energy.

It should be noticed that the expression for the kinetic energy of rotation is not a definition (as was $I = \sum (m_i r_i^2)$), but rather a result of adding the kinetic energies of all the particles that comprise the object to obtain the total kinetic energy of the object rotating about an axis. What is *defined* in this expression is the rotational moment of inertia I.

Moments of Inertia

Different objects have different values for their moments of inertia in the same way that different objects have different masses. But even objects with the same mass will have different values for I depending on how the mass is distributed with respect to the axis of rotation. And notice that also means identical objects will have different values for I depending on the location of the rotation axis. Below are the moments of inertia for a few common objects about the axes shown. The axes are through the centers of mass in all but (a).



$I = mL^2$	$I = 2mr^2$	$I = \frac{1}{2}mr^2$	$I = \frac{1}{2}mr^2$	$I = \frac{2}{5}mr^2$	$I = mr^2$
(a)	(b)	(c)	(d)	(e)	(f)

Notice that the farther the mass is distributed from the axis, the larger the value of the moment of inertia (compare the disk, sphere, and hoop, for example). You should also notice that the moments of inertia are always of the form of a mass times the square of a dimension (and have units kg-m^2).

If an object is rotated about an axis that does not go through its center of mass (as in (a) above), the moment of inertia about that axis is given by $I = I_{cm} + mL^2$, where L is the distance of the CM from the axis of rotation and I_{cm} is the object's moment of inertia about an axis parallel to the rotation axis which through its CM. This is called the *parallel axis theorem*. This is sometimes useful when an object is suspended from a point removed from the center of the object - for example if the hoop is suspended from a nail and rotates, or if the pendulum shown has a large ball rather than a tiny one (in (a), I_{cm} of the ball is much smaller than mL^2).

For any object with cylindrical or spherical symmetry (ball, hoop, disk, wheel, pulley, spindle, etc.) the moment of inertia about the axis of rotation is given by $I = bmr^2$ where r is the outside radius of the object and the constant b is some number between 0 and 1. This result will be important in the discussion of the motion of rolling objects.

Newton's Laws for Rotation

It is particularly important to recognize the relationship between the translational quantities and those involved in the description of rotational motion. Any quantity that is used in the discussion of objects which move along an axis has its counterpart in describing the rotation of objects about an axis. For that reason, the equations that are particularly useful in the description of translational motion also have their analogous forms for the description of rotations.

Newton's Second Law: Translation: $\vec{\mathbf{F}}_{net} = m\vec{\mathbf{a}}$ Rotation: $\vec{\boldsymbol{\tau}} = I\vec{\boldsymbol{\alpha}}$

Net forces result in linear accelerations and net torques cause angular accelerations.

Newton's Third Law: Translation: $\vec{\mathbf{F}}_{1\to2} = -\vec{\mathbf{F}}_{2\to1}$ Rotation: $\vec{\boldsymbol{\tau}}_{1\to2} = -\vec{\boldsymbol{\tau}}_{2\to1}$

All forces and torques are interactions between two different objects and results in equal and opposite forces or torques acting on the two objects involved in the interaction. This idea was central to the conservation of momentum ideas in collision problems, and will also be central to the principle of the conservation of angular momentum that follows.

Work and Energy

Just as the work done by a force is defined as the integral of the force over a displacement, so the work done by a torque is the integral over an angular displacement. Although we are not going to use the idea to calculate the work done by torques on objects, it leads to the idea of energy conservation through the work- energy theorem just as it did in the previous discussion.

Work-Energy Theorem: $W_{net} = \int F_{net}\, dS = \Delta K$ or for rotations $W_{net} = \int \tau_{net}\, d\theta = \Delta K_{rotation}$

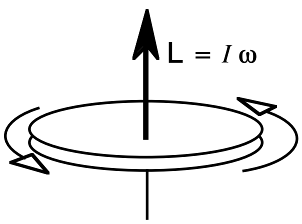
The work done by the net forces causes a change in the translational kinetic energy. Likewise, the work done by the net torque on a system causes a change in the rotational kinetic energy. The net work done by all forces and torques on an object or a system is thus equal to the change in the total kinetic energy of the system - including both translational and rotational energy.

Energy Conservation: $E_{tot} = KE + PE = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + U$

The total mechanical energy of a system can always be accounted for. The kinetic energy of a system is the total of all translational and rotational contributions of all parts of the system. The total potential energy will be the energy stored in the system - and is a way of accounting for the energy associated with work done by conservative forces and torques.

Angular Momentum and the Conservation Laws

Just as it was useful to define the momentum of an object as $\vec{\mathbf{p}} = m\vec{\mathbf{v}}$, and the momentum of a system as the vector sum of all the momenta of the individual parts of the system, it is also useful to define the *angular momentum* in a similar way. And just as Newton's second law requires that the net force on an object changes its *linear* momentum, Newton's second law requires that the net torque on an object changes its angular momentum. That is, angular momentum is defined in such a way that the relationships with which you are already familiar with for translational motion apply to rotational motion ass well.



The angular momentum of an object which rotates about a fixed axis is given by its rotational moment of inertia and its angular velocity. The symbol for angular momentum is $\vec{\mathbf{L}}$ (with units of kg-m^2/sec). It is a vector which points in the same direction as the angular velocity vector.

$$\vec{\mathbf{L}} = I\vec{\omega} \qquad \text{(analogous to } \vec{\mathbf{p}} = m\vec{\mathbf{v}}\text{)}$$

Since Newton's second law states that a net force causes a change in the linear momentum, then it follows that a net torque causes a change in its angular momentum.

$$\vec{\tau}_{net} = I\vec{\alpha} = I\frac{d\vec{\omega}}{dt} = \frac{d\vec{\mathbf{L}}}{dt} \qquad \text{(analogous to } \vec{\mathbf{F}}_{net} = \frac{d\vec{\mathbf{p}}}{dt}\text{)}$$

That is, the angular momentum of an object or a system changes as a result of a net torque on the object or system. If the system that is being described has more than one rotating part, and there are no net torques acting, then the total angular momentum of the system is necessarily constant. As in the case of linear momentum, the *internal* forces and the corresponding torques can have no effect on the total angular momentum of a *system*. Consequently, only *external* torques can change the angular momentum of a system.

Conservation of Angular Momentum: If $\boldsymbol{\tau}_{\text{ext}} = 0$, then $\mathbf{L}_{\text{system}} = \text{constant}$

If there are no external torques acting, then the angular momentum is a constant. This conservation law is as important for rotation problems as the conservation of linear momentum is for translation problems. It is a particularly important idea when rotating parts of a system interact. And the principle explains such things as why the orbits of the planets about the sun are in planes (and most are in the *same* plane), why Saturn has rings, and why galaxies tend to form in large rotating discs of stars.

ROTATION PROBLEMS

The kinds of problems involving the ideas of rotational dynamics fall into three broad categories (much like those involving translational motion). Problems can involve either objects which rotate about a fixed axis or can involve objects which have both rotational and translational motions.

<u>Torque and Angular Acceleration</u>

These problems require that you determine the net torque on some object about a given axis of rotation, then determine the resulting angular acceleration. Once you know the angular acceleration, you could also determine the angular velocity at any time or the total angular displacement, etc., using the rotational kinematics relationships.

Many problems of this type also involve two or more objects that are tied together - much like the force problems involving multiple masses - and may involve both rotation and translation. It is crucial that you be able to draw the correct force diagrams for each object, write the correct equations relating forces and accelerations and torques and angular accelerations, then relate the motions of the various parts of the system.

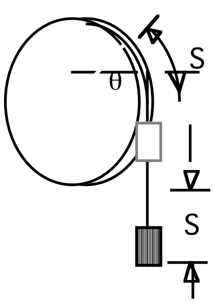
Energy Problems

These problems make use of the energy conservation law - either to convert potential energy to kinetic energy, including both rotational and translational kinetic energy, or to determine the energy losses involved in some system in which there is friction. The principles are identical to those developed earlier in the course: You can always account for the energy of a system. If you only need to know a final velocity, or a change in kinetic energy, or the work done by all the forces, etc., you do not need to calculate angular accelerations. You can just determine velocities or angular velocities directly using energy concepts. The only thing new here is the need to include the rotational kinetic energy of those objects which rotate.

Angular Momentum Conservation

Like the momentum conservation idea that we earlier applied to problems involving two objects which interacted with each other, but had no net external forces acting (as in collisions), the angular momentum concept can be similarly applied. It there are no external torques acting, the total system angular momentum must be constant.

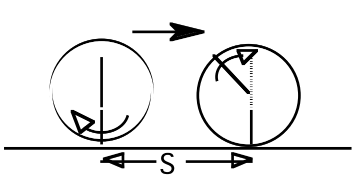
Wheels and Pulleys



Many problems involving rotations also include objects which move in a straight line - and require making a connection between the rotational motion and the corresponding translations. For example, if a mass is lowered by a rotating pulley or if a wheel or ball rolls without slipping, the rotational motion is related to the translation through a simple relationship. The relationship is the same as that between an angle and its arc length.

$$S = R\theta \quad \text{So} \quad v = R\frac{d\theta}{dt} = R\omega \quad \text{and} \quad a = R\alpha$$

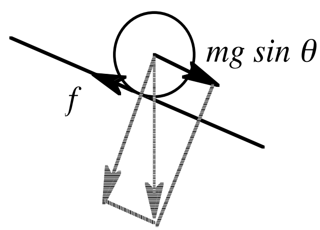
The same relationships apply for an object which rolls without slipping since the arc length that is swept out due to a rotation through some angle is the same as the distance the center of mass travels. So for a wheel or ball that rolls without slipping, the above relationships connect the distance travelled and the speed and acceleration of the center of mass to the total swept angle, the angular velocity and angular acceleration of the wheel or ball about the center of mass where R is the outside radius.



Any object which rolls without slipping must have a frictional force acting to prevent slipping. In that case, the frictional force creates a torque about the axis. But if there is no slipping, there is no loss of energy due to the friction. The kinetic energy of an object which rolls without slipping can be written solely in terms of its speed by making use of $v=R\omega$ and $I_{cm}=bmR^2$:

$$K_{total} = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I_{cm}\omega^2 = \tfrac{1}{2}mv^2 + \tfrac{1}{2}(bmR^2)(v/R)^2 = \tfrac{1}{2}mv^2 + \tfrac{1}{2}bmv^2 = \tfrac{1}{2}mv^2(1+b)$$

Notice that the total kinetic energy of an object that rolls without slipping is divided between its translational energy and its rotational energy. And the fraction of its total kinetic energy that is rotational just depends on the coefficient b - which depends only on how the mass of the object is distributed with respect to the axis of rotation.



Also notice that when a ball rolls down an incline, it does not gain speed as quickly as it would if it could just slide down the same incline without friction. But also note that it cannot roll without slipping *unless* there is sufficient friction to create the torque about the axis of rotation to cause the angular acceleration that is consistent with the linear acceleration down the incline. That is, a ball cannot roll down an incline *unless* there is friction. (On a frictionless plane, it would just slip. The acceleration of the ball down the incline can be calculated from the net force, that is:

$$mg\sin\theta - f_s = ma$$

where f_s is whatever static friction is required to keep the ball from slipping. (Notice that substituting $f=\mu N$ is not proper here, since the frictional force is *not* necessarily equal to the limiting static friction.) The only *torque* about the center of mass is just due to the frictional force. So

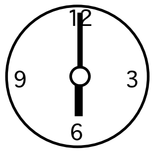
$$\tau = R\,f_s = I\alpha = I\frac{a}{R} \quad \text{or} \quad f_s = \frac{I}{R^2}a$$

assuming the ball rolls without slipping so that $a=R\alpha$. Combining these relationships yields the acceleration of the ball down the incline:

$$a = \frac{mg\sin\theta}{m + I/R^2}$$

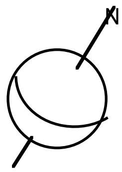
That is, if there is no slipping between the rolling object and the incline on which it rolls, then no energy is "lost" – *i.e.,* the friction does no negative work on the ball as it rolls. The effect is that the friction that acts on the ball that keeps it from slipping, transfers some of the kinetic energy gained from the potential energy lost as it goes down the incline into rotational kinetic energy. Furthermore, if it rolls down an incline without slipping, it would *have* to accelerate! That is, the frictional force *up* the incline will necessarily be less than the gravitational component down the incline. That is required since if there is friction acting, there is only a clockwise torque acting on the ball - there is no mechanism for a counter-clockwise torque. Given a net torque, there will also be an angular acceleration about the axis of rotation. But it cannot have an angular acceleration unless it also has a linear acceleration if it rolls without slipping. That is, the linear and angular accelerations are always proportional given that constraint.

ROTATIONAL MOTION - QUESTIONS AND PROBLEMS

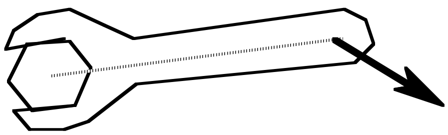
1. 

Consider a clock which reads 6 o'clock. What is the magnitude and direction of the angular velocity vector of each hand?

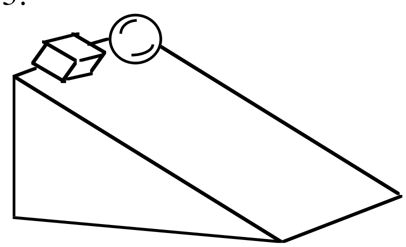
Write the equation that describes the position of the second hand as a function of time if it is straight up at $t=0$.



What is the magnitude and direction of the angular velocity vector for the earth?

2. 

Suppose you tighten a bolt by applying a force **F** to a wrench as shown. Define what you mean by "torque" in this problem and show that the applied torque is maximized when the force is perpendicular to the handle axis.

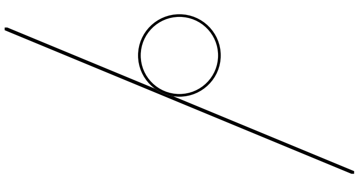
3. 

Consider that a ball and a block have a race down an incline. Assume that the ball rolls without slipping and the block slides without friction (do not worry about how that is done). Assume they have the same mass.

Determine which object would reach the bottom of the incline first if they are released at the same time. Explain your answer carefully.

Determine how the masses of the objects would affect the answer. Determine how the radius of the ball would affect the answer.

If there <u>were</u> friction acting on the block equal to the amount which prevented the ball from sliding, compare the final velocities. Compare the final energies.

4. 

Suppose an object (cylinder, ball, disk, hoop) of mass M and radius R rolls a distance S down an incline which makes an angle θ with respect to the horizontal.

Draw the force diagram (including friction)

a. Write expressions for Newton's second law:

i. Relating the forces on the object and the acceleration of the center of mass.

ii. Relating the torque about the center of mass due to the frictional force and the angular acceleration of the rolling object about the center of mass.

b. If the moment of inertia of the object is I, determine the frictional force and the acceleration of the center of mass (in terms of I, M, R, g, and θ) assuming that the object rolls but does not slip (*ie*, so that $a=R\alpha$).

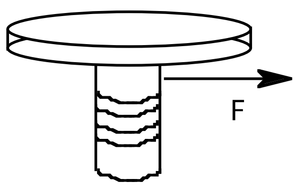
5. Consider the object which rolls down the incline of problem 4.

   a. If the object rolls a distance S down the incline, obtain an expression for the final velocity of the object. [This problem can either be done using kinematics or by using energy ideas.]

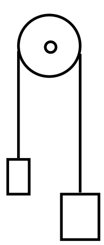
   Assuming the moment of inertia can be expressed in the form $I=bMR^2$ (where b is some fraction between 0 and 1), show that the final speed of the object does not depend on either the mass or the radius, but does depend on how the mass is distributed.

   b. Justify whether a solid disk, a solid sphere, or a hoop would be fastest down the incline.

6. Consider a disk and spindle with a string which wraps around the spindle (radius r) as shown. The moment of inertia of the disk/spindle is I.

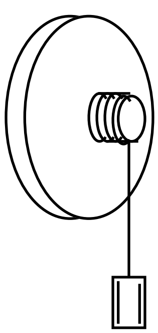


If a constant force F is used to pull the string, which in turn causes the spindle to rotate, obtain an expression for the angular velocity of the disk as a function of time if it is initially at rest when the force is applied at t=0. Show the direction of the angular velocity vector.

7. 

Suppose two masses M_1 and M_2 are connected over a pulley of mass m and radius R, as shown. If there is no friction in the system, and it is released so that M_2 (which is greater than M_1) falls, determine the accelerations of the masses and the tension in the string on either side of the pulley.

Determine the final velocity after the heavier mass has fallen a distance h.

8. 

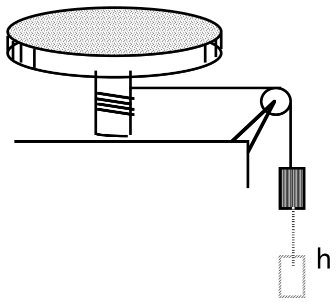
Consider a mass m which is attached to a solid disk flywheel of mass M and radius R by a string which is wrapped around a spindle attached to the flywheel (and which has a radius r). (Assume you know the values of m, M, R, r, and g.)

   a. When released, determine the acceleration of the falling mass and the tension in the string.

   b. Determine the velocity of the mass after it has fallen a distance h.

[NOTE: In part a., you need to set up force and torque equations in order to solve for the acceleration of the falling mass and tension in the string. In part (b), you could use that acceleration and the kinematics equations to find the final velocity – or, you could use energy conservation or the work-energy theorem to simply solve for the final velocity directly. It might be useful to see that both methods achieve the same result.]

9.



Consider a solid disk attached to a spindle of radius r which can spin freely about its axis. A string is wrapped about the spindle and suspends a mass m as shown (Recall the lab experiment). Assume the moment of inertia of disk is $I=(1/2)MR^2$

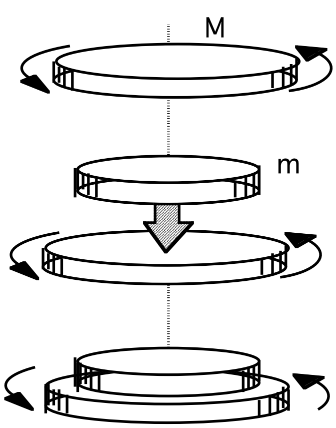
a. Write expressions that allow you to calculate the acceleration of the mass and the tension in the string in terms of *m, M, r, R*, and (of course) *g*.

b. Write the expression that allows you to determine the speed of m after it falls a distance *h*.

[Notice that parts (a) and (b) are essentially the same as the previous problem.]

c. Suppose after falling the distance *h*, the string becomes detached from the spindle and the disk spins freely. If a second identical disk is dropped onto the first, determine the angular velocity of the combined system.

10.



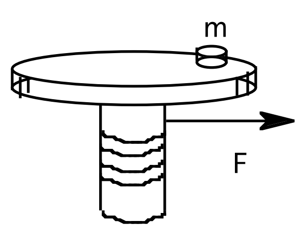
Consider a disk of mass *M* and radius *R* which is rotating at an angular velocity given by ω. If the disk has a "sense" of rotation as shown in the figure, show on the figure the <u>direction</u> of the angular velocity vector.

Now suppose a second disk (which is not spinning) of mass *m* and radius *r* is dropped directly onto the first. Obtain an expression for the new angular velocity of the system. Explain your reasoning carefully.

On the second figure, show the direction of the angular acceleration vector.

Obtain an expression for the energy lost in the "collision". Where does the energy go?

11.

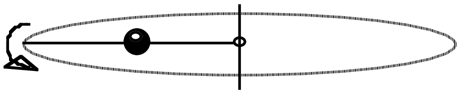


Suppose a horizontal disk has a force pulling on a string which is wrapped around its spindle as shown. An object of mass m is at rest with respect to the disk a distance *R* from the axis of rotation. Assume the moment of inertia of the rotating disk and spindle is *I* and the coefficient of friction between the disk and small mass is μ.

a. Obtain an expression for the angular acceleration of the disk in terms of the force F, the moment of inertia of the disk/spindle I, and the mass m and its location R assuming the mass does not slip on the disk's surface. Obtain an expression for the <u>maximum</u> force with which the string could be pulled and still have the mass moving with the rotating disk at a constant radius without slipping.

b. Explain why the force would have to be reduced as the disk increases its angular velocity if the mass is to continue to be at rest on the surface. [Consider the centripetal force as its speed increases.]

12. Consider that a mass M is on a massless rigid rod of length L that is pivoted at one end. Consider that the mass is a distance r from the pivot and that the rod rotates about the axis of rotation in time T (ie, its *rotation period* is T).



a. Write an expression for the kinetic energy of the mass M in terms of its distance from the pivot point r and the period of rotation.

   Show that this expression reduces to $K_{rot}=(1/2)I\omega^2$, where w is the *angular velocity* given by $2\pi/T$, where the *moment of inertia* or *rotational inertia* is given by $I=Mr^2$.

b. If the rod is set in rotation with the mass located at $L/2$ from the pivot and the mass slips out to the end ($r=L$) without changing the kinetic energy, how would the rotation rate change? By how much would it change?

c. Now consider that the mass M is distributed such that $M/2$ is at $r=L/2$ and $M/2$ is at the very end ($r=L$). Write an expression for the kinetic energy if the rotation rate is given by ω and compare with the energy when <u>all</u> of the mass is at the end and it rotates at the same rate.