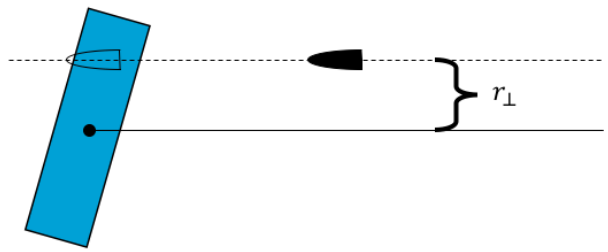
7.3 Angular Momentum of a Point Mass

We must think that a point mass can have no angular momentum because it has a radius of zero and thus a zero moment of inertia. But we can see that it *does* have angular momentum. Consider shooting a bullet into the center of a stationary wooden block anchored in the center with a low friction pivot, so the block is free to spin. If the bullet strikes the block off-center, the bullet-block combination is subsequently spinning about the center of mass: it has angular momentum! Because there are no outside torques on the bullet-block system angular momentum is conserved. The block was initially at rest, so the angular momentum came from the bullet. The bullet must have had this angular momentum before the impact and would also keep this angular momentum about the rotation point, even if the bullet was able to pass freely through the block.

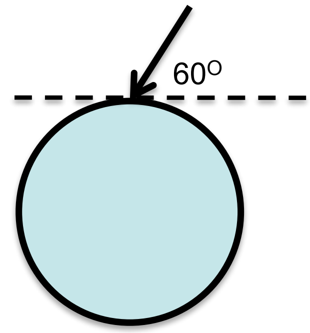


We can calculate the angular momentum of a bullet with respect to a center of rotation. The bullet is off center from the pivot by the *impact parameter*, shown as $r_\perp$ in the diagram above. We know that $\vec{l} = I\vec{\omega}$, and we can calculate both I and ω. Because the angular momentum doesn't change, we can take the bullet's angular momentum at any point in time. Let's choose when the bullet is closest to the pivot point so $r = r_\perp$and $\vec{v} \perp \vec{r}$. Then the bullet has $I = mr_\perp^2$ and $\omega = {}^{v}/_{r_\perp}$. Now we can see that $I = mr_\perp^2$ and $\omega = {}^{v}/_{r_\perp}$. Thus the angular momentum of the point mass about the center of mass of the system is $l = I\omega = mvr_\perp = \ pr_\perp$. And we can see that if $r_\perp = 0$, the object passes through the center and has no angular momentum with respect to that center of rotation.

What if the bar is not secured at the center, but is instead freely floating in outer space? Without the external force of the bar, we need to conserve momentum as well as angular momentum. So, the bar/bullet (whether or not it is spinning), will move off with the correct velocity to conserve linear momentum, as well as it conserves angular momentum. There is another more subtle change: instead of rotating about the center of the bar, the bar/bullet will rotate about the center of mass, which will be somewhere between the bullet and the center of the bar.

Example 1:

A child's carousel, initially at rest, is a 100 kg disk of uniform density, 3 m in diameter. A 40 kg point child runs as fast as she can (5 m/s), jumps onto and grabs the edge of the carousel as shown. Please find the following:



- The final angular velocity?
- Was kinetic energy conserved in this process? If so, why can you be sure? If not, please calculate the kinetic energy lost in the collision.
- If the carousel instead of being at rest, was slowly rotating into the paper (clockwise), would the collision increase, decrease, or not affect the rotation rate? How do you know?
- Did we conserve the girl's linear momentum? If not, where did the momentum go?
- If the carousel's rotation bearing were not mounted in the ground… if it were instead floating on a lake, what would be the final velocity of the carousel and girl system after the collision? In which direction?