8.4 Elastic Collisions in One Dimension

Let us consider various types of two-object collisions. These collisions are the easiest to analyze, and they illustrate many of the physical principles involved in collisions. The conservation of momentum principle is very useful here, and it can be used whenever the net external force on a system is zero.

We start with the elastic collision of two objects moving along the same line—a one-dimensional problem. An **elastic collision** is one that also conserves internal kinetic energy. **Internal kinetic energy** is the sum of the kinetic energies of the objects in the system. **Figure 8.6** illustrates an elastic collision in which internal kinetic energy and momentum are conserved.

Truly elastic collisions can only be achieved with subatomic particles, such as electrons striking nuclei. Macroscopic collisions can be very nearly, but not quite, elastic—some kinetic energy is always converted into other forms of energy such as heat transfer due to friction and sound. One macroscopic collision that is nearly elastic is that of two steel blocks on ice. Another nearly elastic collision is that between two carts with spring bumpers on an air track. Icy surfaces and air tracks are nearly frictionless, more readily allowing nearly elastic collisions on them.

Elastic Collision

An **elastic collision** is one that conserves internal kinetic energy.

Internal Kinetic Energy

Internal kinetic energy is the sum of the kinetic energies of the objects in the system.
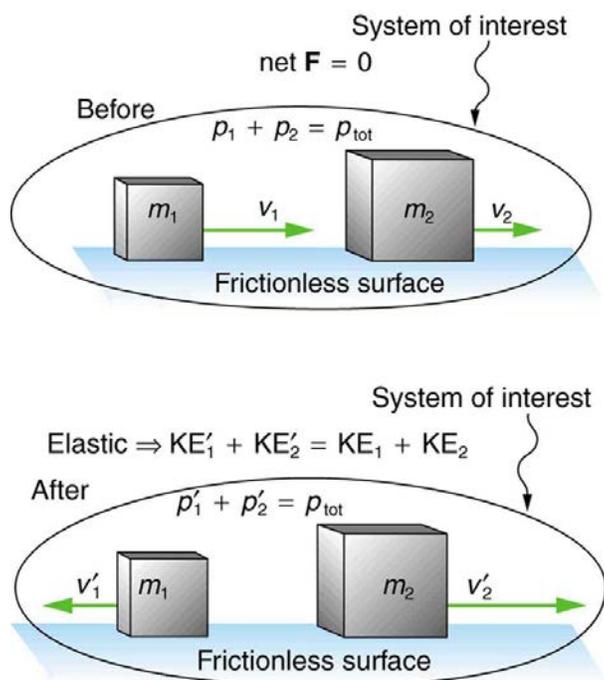
Figure 8.6 An elastic one-dimensional two-object collision. Momentum and internal kinetic energy are conserved.

Now, to solve problems involving one-dimensional elastic collisions between two objects we can use the equations for conservation of momentum and conservation of internal kinetic energy. First, the equation for conservation of momentum for two objects in a one-dimensional collision is

$$p_1 + p_2 = p'_1 + p'_2 \quad (F_{\text{net}} = 0) \tag{8.33}$$

or

$$m_1 v_1 + m_2 v_2 = m_1 v'_1 + m_2 v'_2 \quad (F_{\text{net}} = 0), \tag{8.34}$$

where the primes (') indicate values after the collision. By definition, an elastic collision conserves internal kinetic energy, and so the sum of kinetic energies before the collision equals the sum after the collision. Thus,

$$\tfrac{1}{2}m_1 v_1{}^2 + \tfrac{1}{2}m_2 v_2{}^2 = \tfrac{1}{2}m_1 v'_1{}^2 + \tfrac{1}{2}m_2 v'_2{}^2 \quad \text{(two-object elastic collision)} \tag{8.35}$$

expresses the equation for conservation of internal kinetic energy in a one-dimensional collision.

Example 8.4 Calculating Velocities Following an Elastic Collision

Calculate the velocities of two objects following an elastic collision, given that

$$m_1 = 0.500 \text{ kg}, \; m_2 = 3.50 \text{ kg}, \; v_1 = 4.00 \text{ m/s, and } v_2 = 0. \tag{8.36}$$

Strategy and Concept

First, visualize what the initial conditions mean—a small object strikes a larger object that is initially at rest. This situation is slightly simpler than the situation shown in **Figure 8.6** where both objects are initially moving. We are asked to find two unknowns (the final velocities v'_1 and v'_2). To find two unknowns, we must use two independent equations. Because this collision is elastic, we can use the above two equations. Both can be simplified by the fact that object 2 is initially at rest, and thus $v_2 = 0$. Once we simplify these equations, we combine them algebraically to solve for the unknowns.

Solution

For this problem, note that $v_2 = 0$ and use conservation of momentum. Thus,

$$p_1 = p'_1 + p'_2 \tag{8.37}$$

or

$$m_1 v_1 = m_1 v'_1 + m_2 v'_2. \tag{8.38}$$

Using conservation of internal kinetic energy and that $v_2 = 0$,

$$\tfrac{1}{2}m_1 v_1{}^2 = \tfrac{1}{2}m_1 v'_1{}^2 + \tfrac{1}{2}m_2 v'_2{}^2. \tag{8.39}$$

Solving the first equation (momentum equation) for v'_2, we obtain

$$v'_2 = \frac{m_1}{m_2}(v_1 - v'_1). \tag{8.40}$$

Substituting this expression into the second equation (internal kinetic energy equation) eliminates the variable v'_2, leaving only v'_1 as an unknown (the algebra is left as an exercise for the reader). There are two solutions to any quadratic equation; in this example, they are

$$v'_1 = 4.00 \text{ m/s} \tag{8.41}$$

and

$$v'_1 = -3.00 \text{ m/s}. \tag{8.42}$$

As noted when quadratic equations were encountered in earlier chapters, both solutions may or may not be meaningful. In this case, the first solution is the same as the initial condition. The first solution thus represents the situation before the collision and is discarded. The second solution $(v'_1 = -3.00 \text{ m/s})$ is negative, meaning that the first object bounces backward. When this negative value of v'_1 is used to find the velocity of the second object after the collision, we get

$$v'_2 = \frac{m_1}{m_2}(v_1 - v'_1) = \frac{0.500 \text{ kg}}{3.50 \text{ kg}}[4.00 - (-3.00)] \text{ m/s} \tag{8.43}$$

or

$$v'_2 = 1.00 \text{ m/s}. \tag{8.44}$$

Discussion

The result of this example is intuitively reasonable. A small object strikes a larger one at rest and bounces backward. The larger one is knocked forward, but with a low speed. (This is like a compact car bouncing backward off a full-size SUV that is initially at rest.) As a check, try calculating the internal kinetic energy before and after the collision. You will see that the internal kinetic energy is unchanged at 4.00 J. Also check the total momentum before and after the collision; you will find it, too, is unchanged.

The equations for conservation of momentum and internal kinetic energy as written above can be used to describe any one-dimensional elastic collision of two objects. These equations can be extended to more objects if needed.

8.5 Inelastic Collisions in One Dimension

We have seen that in an elastic collision, internal kinetic energy is conserved. An **inelastic collision** is one in which the internal kinetic energy changes (it is not conserved). This lack of conservation means that the forces between colliding objects may remove or add internal kinetic energy. Work done by internal forces may change the forms of energy within a system. For inelastic collisions, such as when colliding objects stick together, this internal work may transform some internal kinetic energy into heat transfer. Or it may convert stored energy into internal kinetic energy, such as when exploding bolts separate a satellite from its launch vehicle.
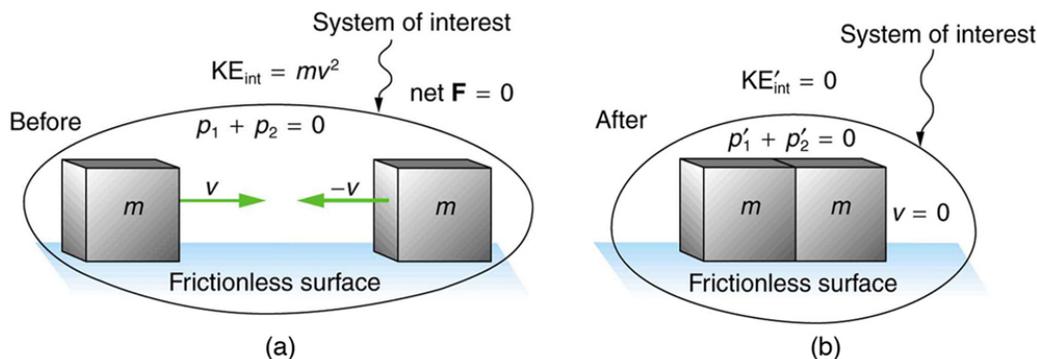
Inelastic Collision

An inelastic collision is one in which the internal kinetic energy changes (it is not conserved).

Figure 8.8 shows an example of an inelastic collision. Two objects that have equal masses head toward one another at equal speeds and then stick together. Their total internal kinetic energy is initially $\frac{1}{2}mv^2 + \frac{1}{2}mv^2 = mv^2$. The two objects come to rest after sticking together, conserving momentum. But the internal kinetic energy is zero after the collision. A collision in which the objects stick together is sometimes called a **perfectly inelastic collision** because it reduces internal kinetic energy more than does any other type of inelastic collision. In fact, such a collision reduces internal kinetic energy to the minimum it can have while still conserving momentum.

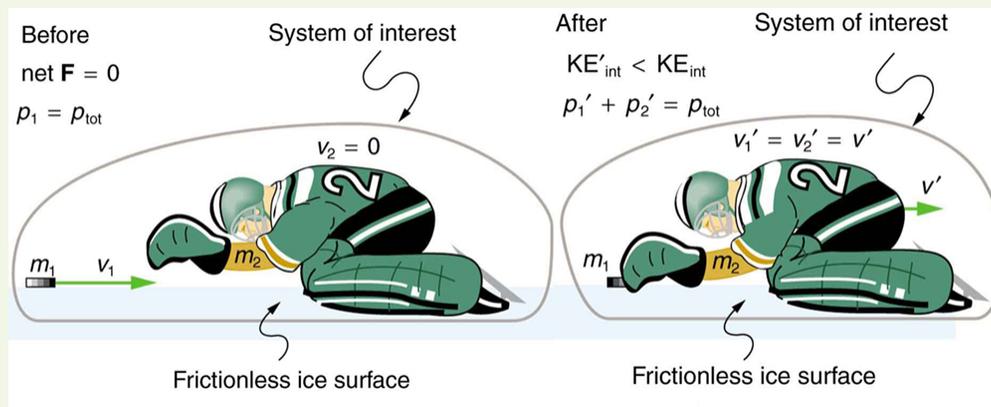**Perfectly Inelastic Collision**

A collision in which the objects stick together is sometimes called "perfectly inelastic."

Figure 8.8 An inelastic one-dimensional two-object collision. Momentum is conserved, but internal kinetic energy is not conserved. (a) Two objects of equal mass initially head directly toward one another at the same speed. (b) The objects stick together (a perfectly inelastic collision), and so their final velocity is zero. The internal kinetic energy of the system changes in any inelastic collision and is reduced to zero in this example.

Example 8.5 Calculating Velocity and Change in Kinetic Energy: Inelastic Collision of a Puck and a Goalie

(a) Find the recoil velocity of a 70.0-kg ice hockey goalie, originally at rest, who catches a 0.150-kg hockey puck slapped at him at a velocity of 35.0 m/s. (b) How much kinetic energy is lost during the collision? Assume friction between the ice and the puck-goalie system is negligible. (See **Figure 8.9**)

Figure 8.9 An ice hockey goalie catches a hockey puck and recoils backward. The initial kinetic energy of the puck is almost entirely converted to thermal energy and sound in this inelastic collision.

Strategy

Momentum is conserved because the net external force on the puck-goalie system is zero. We can thus use conservation of momentum to find the final velocity of the puck and goalie system. Note that the initial velocity of the goalie is zero and that the final velocity of the puck and goalie are the same. Once the final velocity is found, the kinetic energies can be calculated before and after the collision and compared as requested.

Solution for (a)

Momentum is conserved because the net external force on the puck-goalie system is zero.

Conservation of momentum is

$$p_1 + p_2 = p'_1 + p'_2 \tag{8.45}$$

or

$$m_1 v_1 + m_2 v_2 = m_1 v'_1 + m_2 v'_2. \tag{8.46}$$

Because the goalie is initially at rest, we know $v_2 = 0$. Because the goalie catches the puck, the final velocities are equal, or $v'_1 = v'_2 = v'$. Thus, the conservation of momentum equation simplifies to

$$m_1 v_1 = (m_1 + m_2) v'. \tag{8.47}$$

Solving for v' yields

$$v' = \frac{m_1}{m_1 + m_2} v_1. \tag{8.48}$$

Entering known values in this equation, we get

$$v' = \left(\frac{0.150 \text{ kg}}{70.0 \text{ kg} + 0.150 \text{ kg}} \right) (35.0 \text{ m/s}) = 7.48 \times 10^{-2} \text{ m/s}. \tag{8.49}$$

Discussion for (a)

This recoil velocity is small and in the same direction as the puck's original velocity, as we might expect.

Solution for (b)

Before the collision, the internal kinetic energy KE_{int} of the system is that of the hockey puck, because the goalie is initially at rest. Therefore, KE_{int} is initially

$$
\begin{aligned}
\text{KE}_{\text{int}} &= \tfrac{1}{2} m v^2 = \tfrac{1}{2} (0.150 \text{ kg})(35.0 \text{ m/s})^2 \\
&= 91.9 \text{ J.}
\end{aligned}
\tag{8.50}
$$

After the collision, the internal kinetic energy is

$$\text{KE}'_{\text{int}} = \frac{1}{2}(m+M)v^2 = \frac{1}{2}(70.15 \text{ kg})(7.48 \times 10^{-2} \text{ m/s})^2 \tag{8.51}$$
$$= 0.196 \text{ J}.$$

The change in internal kinetic energy is thus

$$\text{KE}'_{\text{int}} - \text{KE}_{\text{int}} = 0.196 \text{ J} - 91.9 \text{ J} \tag{8.52}$$
$$= -91.7 \text{ J}$$

where the minus sign indicates that the energy was lost.

Discussion for (b)

Nearly all of the initial internal kinetic energy is lost in this perfectly inelastic collision. KE_{int} is mostly converted to thermal energy and sound.

During some collisions, the objects do not stick together and less of the internal kinetic energy is removed—such as happens in most automobile accidents. Alternatively, stored energy may be converted into internal kinetic energy during a collision. **Figure 8.10** shows a one-dimensional example in which two carts on an air track collide, releasing potential energy from a compressed spring. **Example 8.6** deals with data from such a collision.
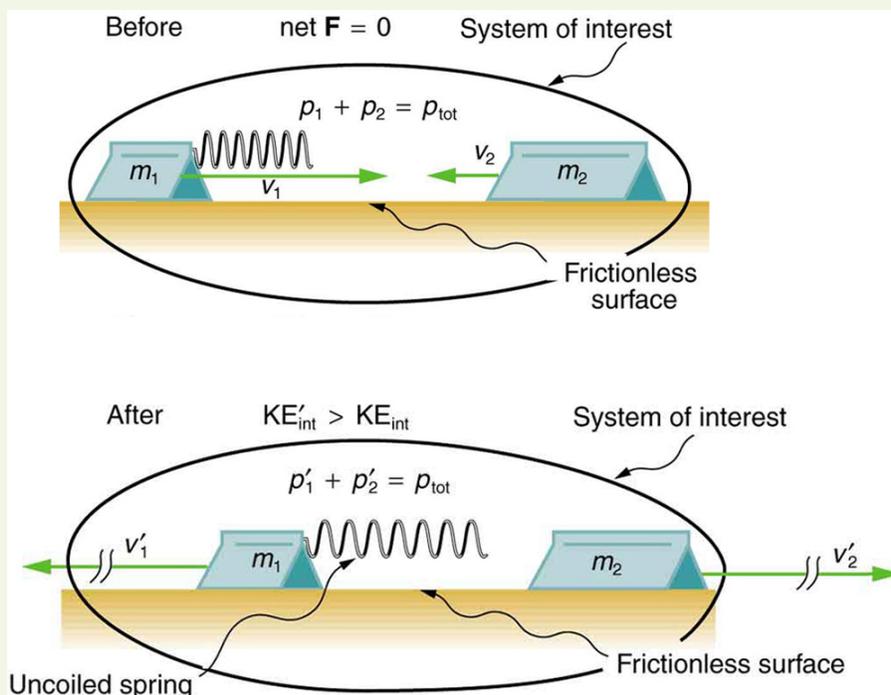
Figure 8.10 An air track is nearly frictionless, so that momentum is conserved. Motion is one-dimensional. In this collision, examined in **Example 8.6**, the potential energy of a compressed spring is released during the collision and is converted to internal kinetic energy.

Collisions are particularly important in sports and the sporting and leisure industry utilizes elastic and inelastic collisions. Let us look briefly at tennis. Recall that in a collision, it is momentum and not force that is important. So, a heavier tennis racquet will have the advantage over a lighter one. This conclusion also holds true for other sports—a lightweight bat (such as a softball bat) cannot hit a hardball very far.

The location of the impact of the tennis ball on the racquet is also important, as is the part of the stroke during which the impact occurs. A smooth motion results in the maximizing of the velocity of the ball after impact and reduces sports injuries such as tennis elbow. A tennis player tries to hit the ball on the "sweet spot" on the racquet, where the vibration and impact are minimized and the ball is able to be given more velocity. Sports science and technologies also use physics concepts such as momentum and rotational motion and vibrations.

Take-Home Experiment—Bouncing of Tennis Ball

1. Find a racquet (a tennis, badminton, or other racquet will do). Place the racquet on the floor and stand on the handle. Drop a tennis ball on the strings from a measured height. Measure how high the ball bounces. Now ask a friend to hold the racquet firmly by the handle and drop a tennis ball from the same measured height above the racquet. Measure

how high the ball bounces and observe what happens to your friend's hand during the collision. Explain your observations and measurements.

2. The coefficient of restitution (c) is a measure of the elasticity of a collision between a ball and an object, and is defined as the ratio of the speeds after and before the collision. A perfectly elastic collision has a c of 1. For a ball bouncing off the floor (or a racquet on the floor), c can be shown to be $c = (h/H)^{1/2}$ where h is the height to which the ball bounces and H is the height from which the ball is dropped. Determine c for the cases in Part 1 and for the case of a tennis ball bouncing off a concrete or wooden floor ($c = 0.85$ for new tennis balls used on a tennis court).

Example 8.6 Calculating Final Velocity and Energy Release: Two Carts Collide

In the collision pictured in **Figure 8.10**, two carts collide inelastically. Cart 1 (denoted m_1 carries a spring which is initially compressed. During the collision, the spring releases its potential energy and converts it to internal kinetic energy. The mass of cart 1 and the spring is 0.350 kg, and the cart and the spring together have an initial velocity of 2.00 m/s . Cart 2 (denoted m_2 in **Figure 8.10**) has a mass of 0.500 kg and an initial velocity of -0.500 m/s . After the collision, cart 1 is observed to recoil with a velocity of -4.00 m/s . (a) What is the final velocity of cart 2? (b) How much energy was released by the spring (assuming all of it was converted into internal kinetic energy)?

Strategy

We can use conservation of momentum to find the final velocity of cart 2, because $F_{net} = 0$ (the track is frictionless and the force of the spring is internal). Once this velocity is determined, we can compare the internal kinetic energy before and after the collision to see how much energy was released by the spring.

Solution for (a)

As before, the equation for conservation of momentum in a two-object system is

$$m_1 v_1 + m_2 v_2 = m_1 v'_1 + m_2 v'_2. \tag{8.53}$$

The only unknown in this equation is v'_2 . Solving for v'_2 and substituting known values into the previous equation yields

$$\begin{aligned} v'_2 &= \frac{m_1 v_1 + m_2 v_2 - m_1 v'_1}{m_2} \\ &= \frac{(0.350 \text{ kg})(2.00 \text{ m/s}) + (0.500 \text{ kg})(-0.500 \text{ m/s})}{0.500 \text{ kg}} - \frac{(0.350 \text{ kg})(-4.00 \text{ m/s})}{0.500 \text{ kg}} \\ &= 3.70 \text{ m/s}. \end{aligned} \tag{8.54}$$

Solution for (b)

The internal kinetic energy before the collision is

$$\begin{aligned} KE_{int} &= \tfrac{1}{2}m_1 v_1^2 + \tfrac{1}{2}m_2 v_2^2 \\ &= \tfrac{1}{2}(0.350 \text{ kg})(2.00 \text{ m/s})^2 + \tfrac{1}{2}(0.500 \text{ kg})(-0.500 \text{ m/s})^2 \\ &= 0.763 \text{ J}. \end{aligned} \tag{8.55}$$

After the collision, the internal kinetic energy is

$$\begin{aligned} KE'_{int} &= \tfrac{1}{2}m_1 v'^2_1 + \tfrac{1}{2}m_2 v'^2_2 \\ &= \tfrac{1}{2}(0.350 \text{ kg})(-4.00 \text{ m/s})^2 + \tfrac{1}{2}(0.500 \text{ kg})(3.70 \text{ m/s})^2 \\ &= 6.22 \text{ J}. \end{aligned} \tag{8.56}$$

The change in internal kinetic energy is thus

$$\begin{aligned} KE'_{int} - KE_{int} &= 6.22 \text{ J} - 0.763 \text{ J} \\ &= 5.46 \text{ J}. \end{aligned} \tag{8.57}$$

Discussion

The final velocity of cart 2 is large and positive, meaning that it is moving to the right after the collision. The internal kinetic energy in this collision increases by 5.46 J. That energy was released by the spring.