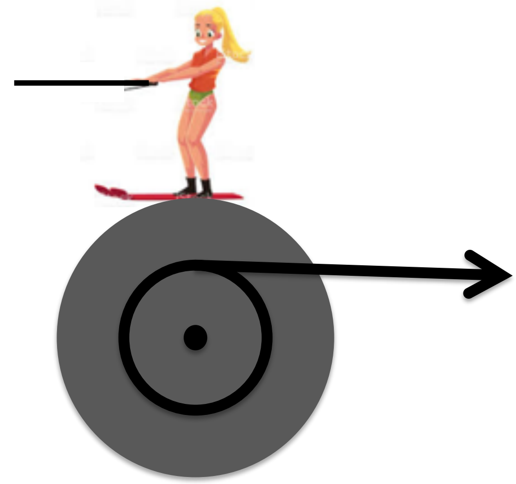
Big Exam #4

Keep in mind, these are not full solutions – you need to draw these out and show the work and carry the units. I'm only providing a roadmap on how to do the work.



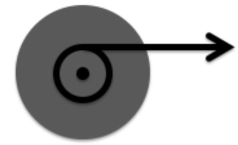
1) My friend (50 kg) is an amazing water skier, and I'm helping her train! She holds a rope tied to a wall while standing on a rotating wheel (50 kg disk, r = 2m). All I have to do is pull on the string tied around the (r = 1m) pulley attached to the wheel and run as fast as I can. The coefficient of friction between her skis and the wheel is only 0.1. Starting from rest, I pull with a force of 200 N

   a) When I'm running at 5 m/s, how fast is she skidding on the wheel? *I us a kinematics lens because we're just working with relationship of speeds. The tangential speed of a wheel is proportional to the radius ($v_t = \omega r$), and the whole wheel has the same rotational velocity, ω, so the wheel is moving at twice the speed under my friend than the speed I pull the string.*

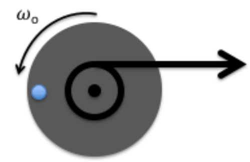
   b) Identify the forces acting on the wheel when I start pulling. (no lens needed) You of course made a large, wonderful, well labeled diagram… like a free body diagram. The tension in the string is 200 N. Also there is a force of gravity on the wheel of 500 N, and my friend of 500 N. But my friend is in equilibrium, held up with the normal force between the skis and the wheel. So, the normal force is 500 N acting upward on my friend and downward on the wheel. Lastly, there is a force of friction of 50 N between the skis (to the right) and the wheel (to the left). The sum of these forces is 1000 N downward and 150 N to the right, but the wheel doesn't accelerate because it's held in place in the axel, so the axel must provide a force of 1000 N upward and 150 N to the left.

   c) When I'm pulling, what is the wheel's angular acceleration? I'll you an angular dynamics lens because angular acceleration is caused by torques! I already have a good free body diagram and I write down: $\sum \vec{\tau} = I\vec{\alpha}$, and then identify the torques… I examine each force above and see that only the tension and the frictional forces are perpendicular to the radius and provide a torque. The normal force and force of gravity are oriented radially, so provide no torques. And the force on the axel has a radius of zero, so there is no torque there. Be careful to identify a positive direction. I'd call into the board as positive, so adding all the torques, I should get + 100 Nm, yielding angular acceleration of 1 s^{-2}.

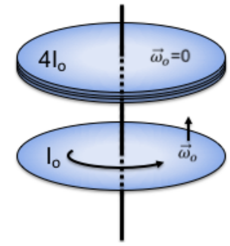
In each of the following, aim for a “C”: draw the picture, and outline how you’d solve it. Setting up the problem correctly for “B” is not required.



1) I pull on a string wrapped around the pulley of this flywheel (from the problem on the other side) with 100 N. Describe the motion of the wheel as I pull the string. Using a dynamics lens, I can see that the torque causes angular acceleration. Make a good FBD.



2) After I pull the string 2 meters with this force of 100 N, what is the speed of the wheel? We could use a dynamics lens because torques cause acceleration. We have everything we need to find rotational acceleration. However, we want final rotational velocity. Using a rotational kinematics lens, we can find the total angular displacement and then time, but it’s a lot of work and some graph making. But we also see that given force and distance I move the string, we can see I do 200 J of work. Using an energy lens, we can see that this work turns into rotational kinetic energy! Using this, we can calculate the final rotational velocity in a single step!



3) If the wheel was initially rotating the other way at ω_o and pulling the string gives it angular acceleration of α, what is the position of the dot, θ and rotation speed, ω after 2 seconds? We have all the kinematic variables to calculate this using a kinematics lens.

4) If this motionless disk is dropped onto my spinning disk, and they stick together, what is their final ω? I would try an angular momentum lens because angular momentum is conserved inside of a system, in a collision if there are no outside torques. Then I would calculate the initial angular momentum and set it equal to the final angular momentum. Kinetic energy would be lost to thermal energy in the collision through the friction of the disks rubbing together.