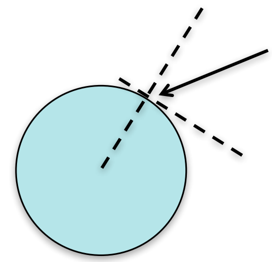
Big Exam #6:

A child's carousel, initially at rest, is a 100 kg disk of uniform density, 3 m in diameter. A 40 kg point child runs as fast as she can (5 m/s), jumps onto and grabs the edge of the carousel as shown. Please find the following:



- The final angular velocity?
- Was kinetic energy conserved in this process? If so, why can you be sure? If not, please calculate the kinetic energy lost in the collision.
- If the carousel instead of being at rest, was slowly rotating into the paper (clockwise), would the collision increase, decrease, or not affect the rotation rate? How do you know?
- Did we conserve the girl's linear momentum? If not, where did the momentum go?
- If the carousel's rotation bearing were not mounted in the ground… if it were instead floating on a lake, what would be the final velocity of the carousel and girl system after the collision? In which direction? There's some variation in answers, but I find the girl has angular momentum about the rotation point of about 160 kgm^2/s. Her moment of inertia as a point mass at r = 1.5 m (ignoring the contribution from her own body rotating about her center of mass as well) is about 90 kgm^2. The disk has a moment of inertia of ~113 kgm^2. So the final rotational velocity is ~ 0.8/s. The majority of the original kinetic energy is changed to thermal energy.

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a) I use an angular momentum lens because I see that with no outside torques in this collision, angular momentum must be conserved $\Rightarrow L_i = L_f$.

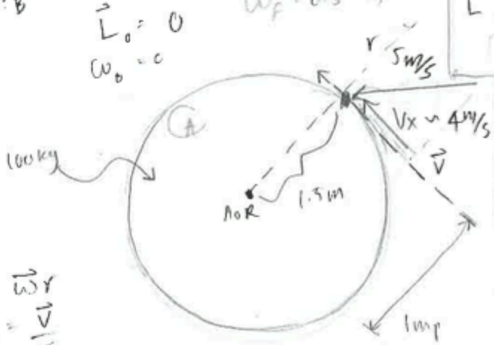
carousel : A

child : B

$\vec{L}_0 = 0$

$\omega_0 = 0$

$\omega_f = 0.5\ rad/s$



$\vec{L}_{carousel} + \vec{L}_{child} = \vec{L}_{carousel + child}$

$I_A \vec{\omega}_A + \frac{1}{2} m_B \vec{V}_B r_B = I_{A+B} \vec{\omega}_f$

$\dfrac{m_B \vec{V}_B r_B}{(\frac{1}{2} m_A r_A^2 + m_B r_B^2)} = \omega_f$

V_{tan} ~3 m/s.

seems a little high to me...

$\omega_f = \dfrac{(40\ kg)(4\ m/s)(1.5\ m)}{[(0.5)(100\ kg)(1.5\ m)^2 + (40\ kg)(1.5\ m)^2]} = \dfrac{240}{202.5}$

$\omega_f = 1.2\ rad/s$

$\vec{V} = \vec{\omega} r$

$\omega = \dfrac{\vec{V}}{r}$

$I = \frac{1}{2} m r^2$

$\vec{L} = I\omega$

$\vec{L} = (m r^2)\left(\dfrac{\vec{V}_\perp}{r}\right)$

$\vec{L} = m v_\perp r$

b) I use an energy lens since I see energy transformations. $E_i = E_f$

$KE_i = \frac{1}{2}(40\ kg)(5\ m/s^2)^2 = 500\ J$

$KE_f = \frac{1}{2}\left[\frac{1}{2}(100\ kg)(1.5\ m)^2 + (40\ kg)(1.5\ m)^2\right](1.2\ rad/s)^2$

$= 122\ J$

$\therefore$ KE is not conserved since

$KE_i > KE_f$ nice!

• It would decrease, because the direction of the rotation is opposite. Carosal is into the paper ⊗, but the child is out of the paper ⊙.

Angular momentum lens

c) The collision would decrease the rotation rate, since the carousel is rotating in the opposite direction that the child would hit and provide angular momentum.

$\vec{L}_o = \vec{L}_f$

$mvr_\perp - I\vec{\omega} = L_f$

d) momentum lens. Momentum is always conserved since there are no outside forces. The girl's momentum is transferred to the Earth, but since the Earth's mass is so large, velocity is negligable.

e) momentum lens because no outside forces.

$p_o = p_f$

$\cancel{p_{co}} + p_{po} = p_f$ (0)

$mv_{po} = m_{c+p} v_f$

$40kg(5 m/s) = 140kg\, v_f$

$v_f = 1.4 m/s$ southwest