

## Big Exam! #4

A

- 1) You are looking to put two identical satellites into orbit around two planets made of the same substance, except that planet A has twice the diameter as planet B. The satellites will be close to the surface of each planet, which we can do because neither planet has atmosphere. How does the speed of satellite B (orbiting planet B in low orbit) compare to the speed of satellite A? That is:

C.  $v_A = 2 v_B$

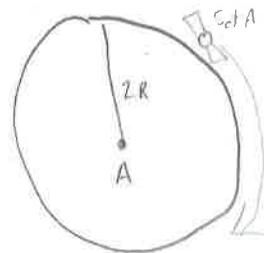
A correct answer with no explanation will earn you an F. Please explain what is going on.

- A. Along the way, it might be a good idea to explain what is the ratio of the masses of the planets:  
 $m_A = 8 m_B$

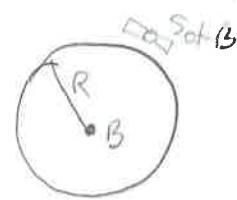
- B. As well as the ratio of the accelerations of the two satellites:  
 $a_A = 2 a_B$

This is a complicated problem. So please step back and consider how to look at this and what you know and don't know yet. You will be graded on your process. Start with a good picture and a lens/lenses.

rotational dynamics/kinematics (I'm not really sure which lens this is, it is using the equations to describe centripetal acceleration and circular motion.)



A



$$a = \frac{F}{m}$$

$$a \propto F \propto \frac{m_{\text{planet}}}{r^2}$$

$V_{\text{of } A} \text{ is proportional to } r^3$   
 $= (2R)^3 = 8R^3$

$V_{\text{of } B} \text{ is proportional to } r^3$   
 $= R^3$

$$a_c = \frac{v^2}{r}$$

$$= \frac{v^2}{2R}$$

$a_c$  also equals  
acceleration due  
to gravity because  $F_N = 0$   
excellent

$$M_{\text{sat}} a_g = \frac{M_{\text{planet}} M_{\text{sat}}}{(2R)^2} G$$

$$a_g = \frac{8M_B}{4R^2} G = 2 \frac{M_B}{R^2} G$$

$\therefore 2 a_g \text{ of } B$

$$a_c = \frac{v^2}{r}$$

$$= \frac{v^2}{R}$$

$a_c$  also equals  
acceleration due to  
gravity because  $F_N = 0$

$$M_{\text{sat}} a_g = \frac{M_B M_{\text{sat}}}{R^2} G$$

$$a_g = \frac{M_B}{R^2} G$$

$$a_{cA} = 2 a_{cB}$$

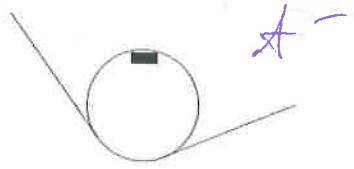
$$\frac{v_A^2}{2R} = 2 \frac{v_B^2}{R}$$

$$v_A^2 = 4 v_B^2$$

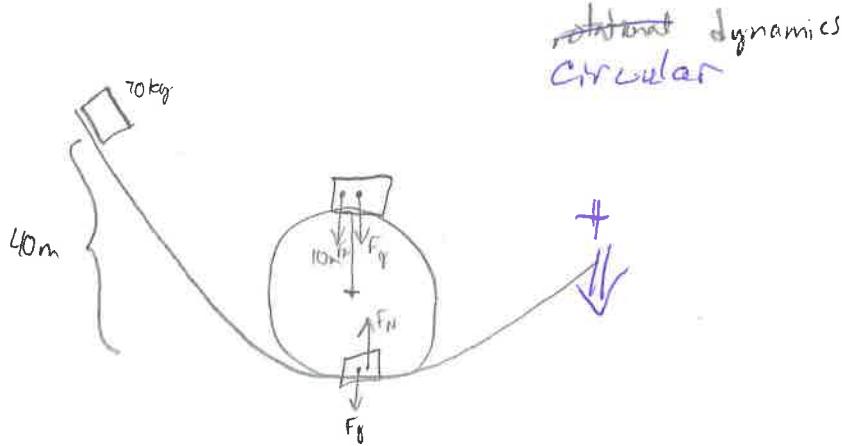
$$v_A = 2 v_B$$

C

- 2) You go on a  $R = 10\text{ m}$ , loop-de-loop ride at the carnival, but you have to choose how high to start the cart. Say you have a mass of 70 kg, like your instructor and you are sitting on a scale that reads in kg. Don't use this drawing... please make your own. If you start from a vertical height of 40 m,



- what does the scale under you read as you are at the top of the loop?
- What does it read at the bottom of the loop?
- Is this a good ride for pregnant women?
- How does it feel as you round the bottom of the loop?



+ ↓  
and a force equivalent  
to 9 "gs"

- a) First, find velocity at top by using energy laws:

$$PE_i = mgh = (70\text{kg})(10\text{m/s}^2)(40\text{m}) = 28000\text{J}$$

$$PE_{Top} = mgh = (70\text{kg})(10\text{m/s}^2)(20\text{m}) = 14000\text{J}$$

$$KE_{Top} = 14000\text{J} = \frac{1}{2}mv^2 = \frac{1}{2}(70\text{kg})v^2 \quad v = 20\text{m/s}$$

now find centripetal acceleration:

$$a_c = \frac{v^2}{r} = \frac{400\text{m/s}^2}{10\text{m}} = 40\text{m/s}^2 \text{ downward}$$

now find force necessary for this centripetal acceleration:

$$\sum \vec{F} = m\vec{a} = (70\text{kg})(40\text{m/s}^2) = 2800\text{N downward}$$

$$\sum \vec{F} = F_N + F_g = F_N + (70\text{kg})(10\text{m/s}^2) = 2800\text{N} \quad F_N = 2100\text{N} \downarrow$$

- b) repeat the steps from part A.

$$KE_{Bottom} = 28000\text{J} = \frac{1}{2}mv^2 = \frac{1}{2}(70\text{kg})v^2 = 1800\text{m/s}^2 = v_{Bottom}$$

$$a_c = \frac{v^2}{r} = \frac{1800\text{m/s}^2}{10\text{m}} = 180\text{m/s}^2 \text{ upward}$$

$$\sum \vec{F} = m\vec{a} = (70\text{kg})(180\text{m/s}^2) = 12600\text{N upward}$$

$$\sum \vec{F} = F_N + F_g = F_N - (70\text{kg})(10\text{m/s}^2) \quad F_N = 4900\text{N} \uparrow$$

$$F_N = F_g + ma = 700\text{N} + 5600\text{N} = 6300\text{N}$$