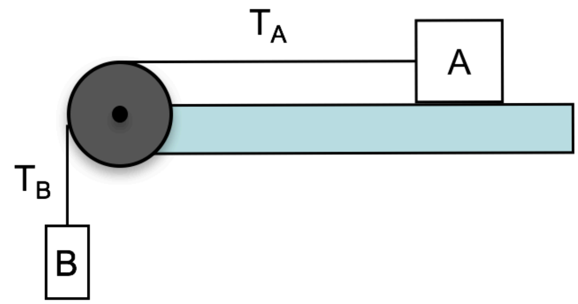
You will be graded on your _COMMUNICATION_ of physics understanding

Define a lens, and make a statement describing the motivation of the lens and its application

Distinguish rotational dynamics from uniform circular motion (which is linear dynamics).



#1 A system of masses has a pulley wheel _with considerable mass_ that turns freely on the axel with the rope stretched over it. That is, the rope does not slip on the wheel's surface. There is a coefficient of friction, μ_A between block A and the surface. I release the system from rest and it moves until mass *B* hits the floor *4 m* below.

FOR a) and b), if you've decided you have a dynamics problem, then you need to consider the sum of the forces ***on a mass***. and torque produces angular acceleration on a rotating body.

a) In order for the system to move and not just sit there after I let it go, what must be true? Explain why you know this, briefly and clearly. Dynamics lens because we are looking at forces and (potential) acceleration. The net force on the system must be greater than zero. How many external forces are acting on the system? There's just two: $F_{g(B)}$ and $F_{f(A)}$. The system will accelerate down to the left of $F_{g(B)} > F_{f(A)}$.

b) As mass *B* falls (and mass *A* slides), is T_A greater than, less than or equal to T_B? Briefly and clearly explain how you know this to be true. We normally assume that tension in a string is the same everywhere, but that assumes the string is massless and frictionless. However, the wheel has mass. How does that wheel accelerate? It rotationally accelerates because of an applied torque. Thus there must be a torque in the outward direction, requiring a tangential force… so $T_B > T_A$.

c) You need to calculate everything about the system: the final speed when it hits the ground, the acceleration, the angular acceleration of the wheel, and T_A. Explain how you would go about this. Be brief but clear, and a diagram is always good. You can use any of the three methods we tried in the problem set. Conservation of energy may be the most straight forward, but be sure to include all energy terms (E_{gB}, E_K, E_R, E_{Therm}) . Then you can use kinematics and dynamics lenses of a single mass to calculate the rest. You can also use a straight up hard core dynamics lens to solve the three simultaneous equations to find the three unknowns (T_A, T_B and the acceleration). Lastly, I show you a tricky dynamics systems way to look at this as a rotating system with two forces ($F_{g(B)}$ and $F_{f(A)}$) pulling on it.

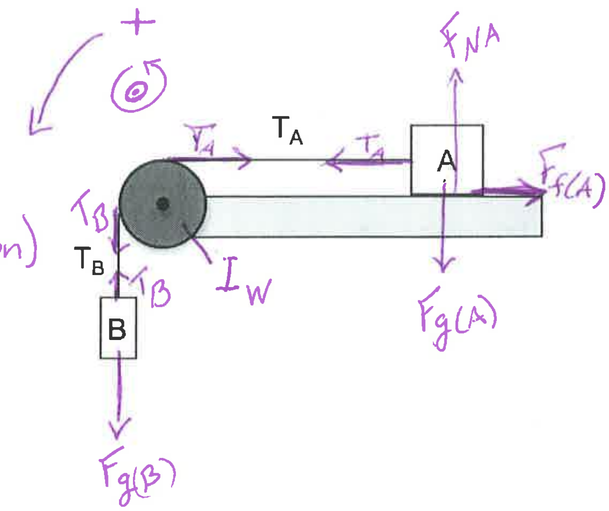
d) $m_A = 2$ kg, $m_B = 1$ kg, The wheel is a uniform solid disk of radius 20 cm and mass 2 kg, and the coefficient of friction is 0.1. Please find T_A, the torque on the wheel, the speed of block B when it hits the ground and anything else you want to find. (extra for Problem Set)

e) Do a very thorough analysis and ask yourself if all of your answers for d) above make sense. Just looking at the system, I'm sure that the acceleration must be well under 1/3 of a gravity. If the acceleration is low, T_B should be close to (but less than) $F_{g(B)}$; and T_A should be less than T_B but greater than $F_{f(A)}$.

Energy lens because

$$E_{g(B)} \Rightarrow E_{K(A,B)} + E_{Rot} + E_{therm}$$

Also E_K Work (friction)



Let the system fall 4m.

$$\Delta E_{g(B)} = m_B g \Delta h_B =$$

$$\Delta E_{K_{AB}} = E_{K_f} - \cancel{E_{K_o}}^{\,0} = \frac{1}{2}(m_A + m_B)v_f^2$$

$$\Delta E_{Rot} = E_{Rot_f} - \cancel{E_{Rot_o}}^{\,0} = \frac{1}{2} I \omega_f^2 = \frac{1}{2}\left(\frac{1}{2} m_w \underbrace{r_w^2\right)\omega_f^2}_{v_f^2} = \frac{1}{4} m_w v_f^2$$

$$E_{therm} = W_{friction} = F_f \cdot \overset{4m}{\Delta h}$$

$$F_f = \mu N \qquad \sum F_{YA} = m_A \cancel{a_{YA}}^{\,0} = 0 \therefore F_{NA} = F_{gA} = m_A g$$

$$F_{f(A)} = \mu m_A g \qquad W_f = \mu m_A g \Delta h_B$$

From above

$$\Delta E_{g(B)} = \Delta E_{K(A,B)} + \Delta E_{Rot} + E_{therm}$$

$$m_B g \Delta h = \frac{1}{2} m_{A+B} v_f^2 + \frac{1}{4} m_w v_f^2 + \mu m_A g \Delta h_B$$

$$1\,kg\,10\,m/s^2\,4\,m = \frac{1}{2}(3\,kg) v_f^2 + \frac{1}{4}(2\,kg) v_f^2 + 0.1(2\,kg)\,10\,m/s^2\,4\,m$$

$$40\,J = 2\,kg\,v_f^2 + 8\,J$$

total loss of Potential Energy

"Lost to heat"

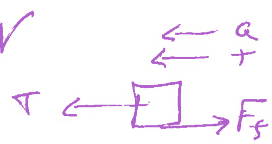
$$v_f^2 = \frac{32\,J}{2\,kg} = 16\,\frac{\cancel{kg}\,m^2/s^2}{\cancel{kg}} = 16\,m^2/s^2 \therefore \underline{v_f \approx 4.0\,m/s}$$

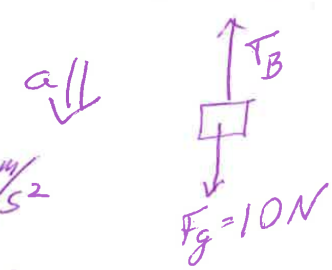
$$\underline{v_{ave} \approx 2.0\,m/s} = \frac{\Delta x}{\Delta t} \qquad \Delta t = \frac{\Delta x}{v_{ave}} = \frac{4\,m}{2.0\,m/s} = 2.0\,s$$

$$a = \frac{\Delta v}{\Delta t} \approx \frac{4.0\,m/s}{2.0\,s} \approx 2.0\,m/s^2 \checkmark \text{ (less than } \tfrac{1}{3} g\text{)}$$

T_B Dynamics: $\Sigma F_B = m a_B = 1\,kg(2.0\,m/s^2) = 2.0\,N$

$T_B = 8.0\,N$





T_A: $\Sigma F_A = -F_f + T_A = m_A a_A = \underbrace{2\,kg \cdot 2.0\,m/s^2}$

$$T_A = 4\,N + F_f = 4\,N + 2\,N = 6\,N$$

$$\mu \cdot m_A \cdot g = .1(2\,kg)(10\,m/s^2) = 2\,N$$

$T_B - T_A = 8\,N - 6\,N = 2\,N$ Provides a $\vec{\tau} = F \cdot r_\perp = 2\,N \cdot .2\,m = 0.4\,Nm$

$$I_w = \frac{1}{2} m r_w^2 = \frac{1}{2}(2kg)(.2m)^2 = 0.04\,kg\,m^2$$

$$\alpha = \frac{\tau}{I} = \frac{0.4\,N\cancel{m}}{0.04\,kg\,m^{\cancel{2}}} = 10\ \frac{\cancel{kg}\,\cancel{m}/s^2}{\cancel{kg}\,\cancel{m}} = 10/s^2$$

also $a = r_w \alpha_w = 0.2\,m\ \frac{10}{s^2} = \underline{\underline{2\,m/s^2}}$ it agrees

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2nd way w/ 3 simultaneous equations, Paying strict attention to the signs on my FBD:

$\Sigma F_B = m_B a_B$ ① $F_{gB} - T_B = m_B a_B$ $\quad \underline{a_B = a_a \equiv a}$

$\Sigma F_A = m_A a_A$ ② $T_A - F_{fA} = m_a a_a$ $\quad \underbrace{\alpha_w = a/r}$

$\Sigma \tau_w = I_w \alpha_w$ ③ $T_B(r) - T_A(r) = I_w \alpha_w$

and $= \left(\frac{I}{r}\right) a$

$v = \omega r,\ a = \alpha r$

adding ① + ②

$$F_{gB} \underbrace{- T_B + T_A}_{\text{from ③ this } = -\frac{I}{r^2}a} - F_{fA} = m_{A+B}\, a \qquad \frac{I}{r^2} = \frac{0.04\,kg\,m^2}{(.2m)^2} = 1\,kg$$

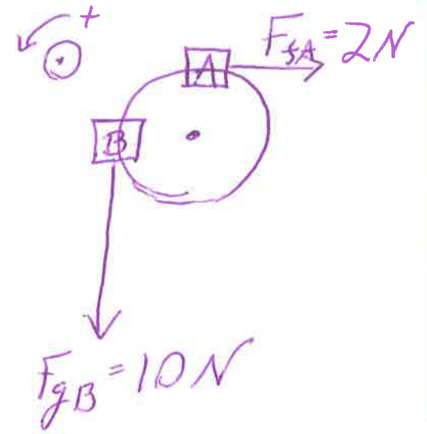
$$F_{gB} - \frac{I}{r^2}a - F_{fA} = m_{A+B}\,a \Rightarrow F_{gB} - F_{fA} = \left(m_{A+B} + \frac{I}{r^2}\right) a$$

$$10N - 2N = (3kg + 1kg)\,a$$

$$a = \frac{8N}{4kg} = 2\,m/s^2$$

it agrees!

This is the way I like best!
Let the length of the strings
$\Rightarrow 0$, just for this moment



This is a rotational system
that will accelerate!

$$\Sigma \tau = I \alpha \qquad \left[m_B r^2 + m_A r^2 + \tfrac{1}{2} m_w r^2\right] = 0.16\,kgm^2$$

$$F_{gB}\, r - F_{fA}\, r = I_s\, \alpha$$

$2Nm \quad 0.4Nm$

$$\alpha = \frac{\tau_B - \tau_A}{I_{System}} = \frac{1.6\ Nm}{0.16\ kgm^2} = 10\ \frac{\cancel{kg}\frac{\cancel{m}}{s^2}\cancel{m}}{\cancel{kgm^2}}$$

$$= 10/s^2$$ ✓ it agrees!

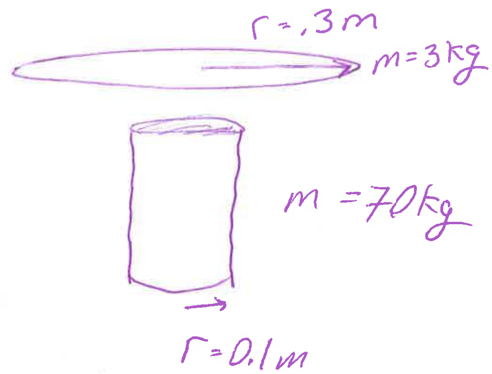
you can use Kinematics +
dynamics to get all the other v, a, T, etc.

#2 I hold a spinning wheel over my head with a vertical axis of rotation. I step onto a rotating table so I am free to rotate. With my body motionless and the wheel spinning, I grab the rim of the wheel and hold it tightly, only hurting my hand slightly. Model my body as a solid cylinder of radius 10 cm and mass 70 kg. The wheel has a radius of 30 cm and mass of 3 kg and it originally spins around 3 times per second.

a) After I grab the wheel, find my angular velocity in radians per second. Clearly explain your thought process.
b) Is kinetic energy conserved in this process? If so, please explain how you know this to be true. If not, please calculate how much kinetic energy is lost (extra for problem set), and explain where it went.

#2 Free to rotate means $\Sigma \tau = 0$, so, $\tau = \frac{d\vec{l}}{dt}$, so $dl = 0$ or angular momentum is conserved / $\Delta \vec{l} = 0$

$l_o = l_f$



$l_{wheel_o} = l_{pete_f} + l_{wheel_f}$

$f_o = 3/s \quad \omega = \frac{3 \cdot 2\pi}{s} \approx 19/s$

$l = I\omega$ so $\omega \Downarrow$ by the same factor as $I \Uparrow$

$I_{whel} = mr^2 = 3kg(0.3m)^2 = 0.27\, kg\, m^2$

$I_{pete} = \frac{1}{2}mr^2 = \frac{1}{2}(70kg)(0.1m)^2 = 0.35\, kg\, m^2$

$I_{wheel} + I_{pete} = 0.62\, kg\, m^2$

$I \Rightarrow \frac{0.62}{0.27} I_o \approx 2.3 I_o \qquad \omega \Rightarrow \frac{1}{2.3}\omega_o \approx 8/s \left(\frac{radians}{s}\right)$

$$E_{rot_0} = \frac{1}{2} I_0 \omega_0^2 = \frac{1}{2}\left(0.27\,\text{kg}\,\text{m}^2\right)\left(\frac{3 \cdot 2\pi}{s}\right)^2 \approx 48\,\text{J}$$

$$E_{rot_{final}} = \frac{1}{2} I_{Pete+Wheel}\, \omega_f^2 = \frac{1}{2}\left(0.62\,\text{kg}\,\text{m}^2\right)\left(\frac{8}{s}\right)^2 \approx 20\,\text{J}$$

$\therefore$ "lost" to heat $\sim 28\,\text{J}$

I could also look at it this way:

$$E_{rot} = \frac{L^2}{2I}$$

$\vec{L}$ is conserved + $I \Uparrow$ by 2.3

$I \Rightarrow \underline{\underline{2.3}}\, I_0$, so

$E_{rot} \Rightarrow \underline{\underline{\frac{1}{2.3}}}\, E_{rot_0}$

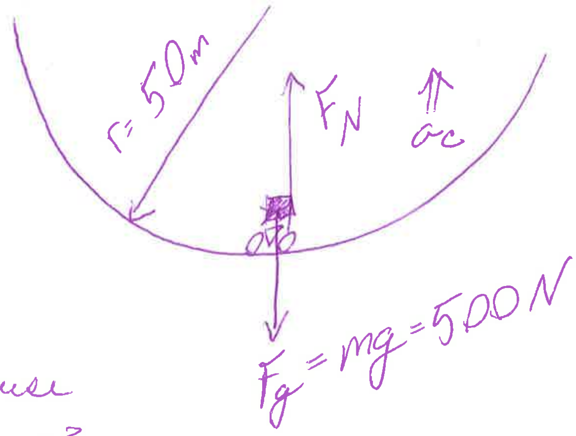
#3 You are riding your bike with a scale between you and your bicycle seat at a speed of 10 m/s and you have a mass of 50 kg and you have standard 700 mm wheels (diameter = 700 mm). You ride through a dip approximated by a circular arc of radius 50 m. Briefly and clearly explain your reasoning with a drawing.

a) What is the rotational velocity of your bicycle wheels?
b) How do you feel when you are at the bottom of the dip? Why?
c) What does the scale read as you are at the bottom of the dip? Assume you bear no weight on your hands and feet.

Kinematics lens

a) $v_t = \omega r$

$\omega = \frac{v}{r} = \frac{10\,m/s}{.3m} = 33/s$



b) This is clearly a dynamics lens because we have forces + $a_c = \frac{v^2}{r}$

so $\Sigma F = ma$ I'll call $\uparrow +$

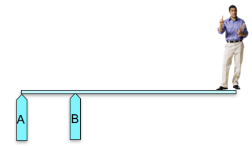
$F_N - F_g = ma$ $a_c = \frac{v^2}{r} = \frac{(10\,m/s)^2}{50m} = 2\,m/s^2$

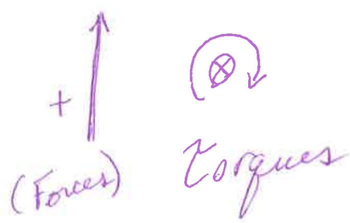
$F_N = ma + F_g =$

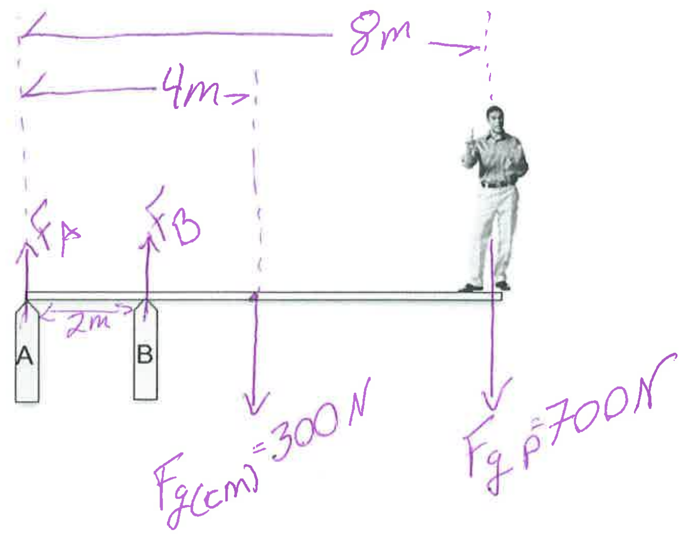
$= 100N + 500N = 600N$

I feel heavy, about $\frac{1}{5}$ heavier than usual.

#4 I stand at the end of an 8 m diving board with 2 m between pylons A and B. I have a mass of 70 kg and the board is a uniform plank of mass 30 kg. Find the force provided by Pylon A and Pylon B (including direction) while explaining your reasoning with a drawing *of your own.*







This is a statics Problem because I'm looking for Forces and nothing is moving

So $a = 0$, $\alpha = 0$

$\Sigma F = 0$

$\Sigma \tau = 0$

Pick a center of rotation... I'll pick A

$F_A + F_B - F_{g(cm)} - F_{g_{Pete}} = 0$

$F_A + F_B = 1000\,N$

$\cancel{\tau_A} - \tau_B + \tau_{cm} + \tau_p = 0$

0

$-2m\,F_B + 4m(300N) + 8m(700N) = 0$

$F_B = \dfrac{-1200\,Nm - 5600\,Nm}{-2m}$

$F_B = 3400\,N \Uparrow$

so $F_A = 1000N - 3400N$, or $2400N \Downarrow$

Name ______________________________________