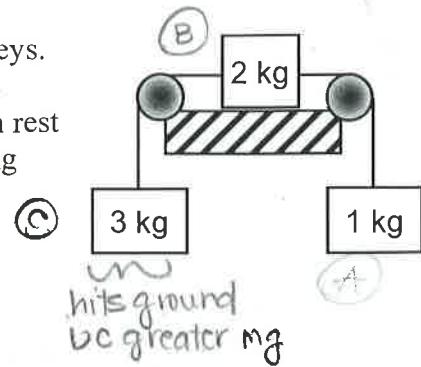
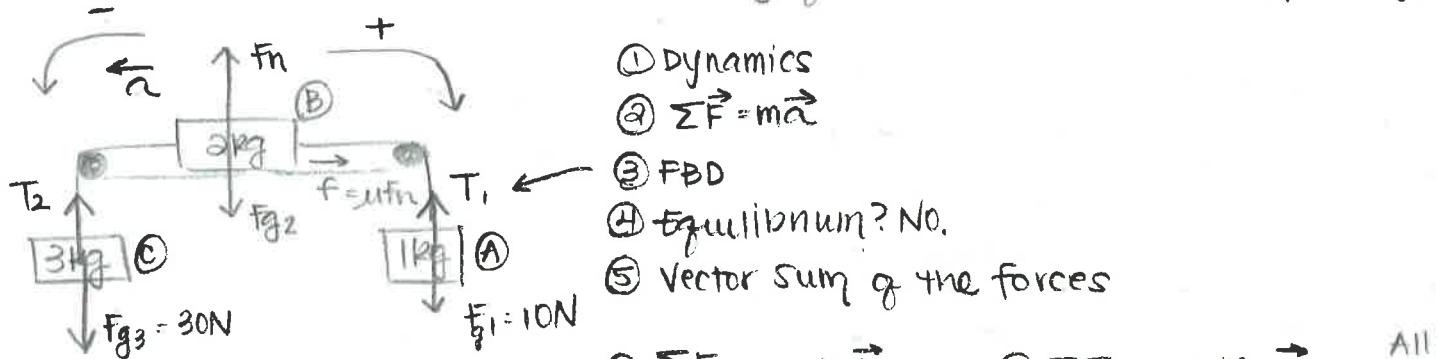


- 1) At right, you see a system of masses with massless, frictionless pulleys. However, there is a coefficient of friction of 0.4 between the center block and the surface it is on. The system of masses is released from rest with each of the hanging masses 4 m off the floor. Find the following three quantities by any means you like *in any order* you like:
- The tension in the string connected to the 1 kg mass. $12N$
 - The speed when a mass hits the ground. ~~7.4 m/s~~
 - The acceleration of the system -2 m/s^2



Dynamics: Tension, a force, is obviously acting on all the masses. Friction is also acting upon the 2kg block. We can use $\sum \vec{F} = m\vec{a}$ to determine the values of a and c .

Energy: Forces can find the acceleration of a body, but not the speed. Kinetic energy is $\frac{1}{2}mv^2$, and by using this concept and the law of energy conservation, the final velocity of a mass can be found (part b.).



① Dynamics

$$\textcircled{2} \quad \sum \vec{F} = m\vec{a}$$

③ FBD

④ Equilibrium? No.

⑤ Vector sum of the forces

$$\textcircled{1} \quad \sum F_{A(y)} = m_A \vec{a}_{(y)}$$

$$\textcircled{2} \quad \sum F_{B(x)} = m_B \vec{a}_{(x)}$$

$$F_{g1} - T_1 = m_A \vec{a}_{(y)}$$

$$T_1 + f - T_2 = m_B \vec{a}_{(x)}$$

All the same acceleration

$$T_1 = F_{g1} - m_A \vec{a}_{(y)}$$

$$\sum F_{B(y)} = m_B \vec{a}_{(y)}$$

$$\textcircled{3} \quad \sum F_{C(y)} = m_C \vec{a}_{(y)}$$

$$F_n - F_{g2} = 0$$

$$T_2 - F_{g3} = m_C \vec{a}_{(y)}$$

$$F_n - F_{g2} = 20N$$

$$T_2 = m_C \vec{a}_{(y)} + F_{g3}$$

$$\text{a. } T_1 = F_{g1} - m_A \vec{a}_{(y)}$$

$$T_1 = 10N - (1\text{ kg})(2\text{ m/s}^2)$$

$$= 10N + 2N = 12N$$

$$T_1 + f - T_2 = m_B \vec{a}_{(x)}$$

$$(F_{g1} - m_A \vec{a}_{(y)}) + f - (m_C \vec{a}_{(y)} + F_{g3}) = m_B \vec{a}_{(x)}$$

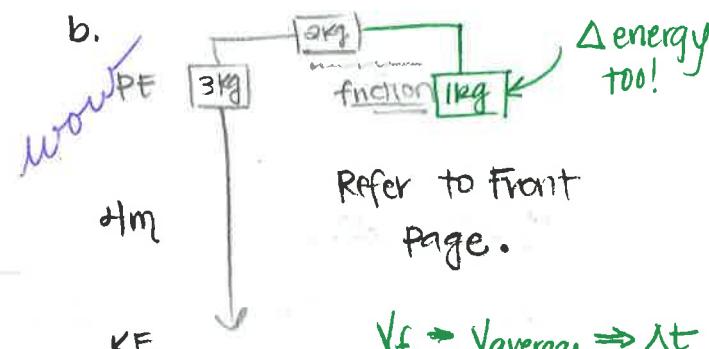
$$F_{g1} - m_A \vec{a}_{(y)} + f - m_C \vec{a}_{(y)} - F_{g3} = m_B \vec{a}_{(x)}$$

$$\frac{F_{g1} + f - m_C \vec{a}_{(y)} - F_{g3}}{m_A + m_B + m_C} = m_B \vec{a}_{(x)}$$

$$\frac{m_A + m_B + m_C}{m_A + m_B + m_C} = m_B \vec{a}_{(x)}$$

$$a = 10N + (0.4)(2\text{ kg})(10\text{ m/s}) - 30N$$

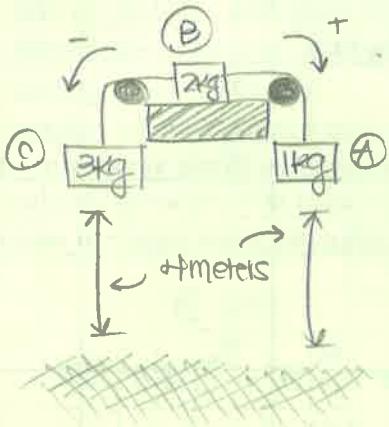
$$\text{c. } \frac{a = -12N/10\text{ kg}}{10\text{ kg}} = -2\text{ m/s}^2$$



$$V_f \rightarrow V_{average} \Rightarrow \Delta t$$

Midterm #2
Corrections

- ① b. Find the speed when a mass hits the ground.



The 3kg mass will hit the floor because the force of gravity acting upon it is greater than the 1kg mass. (Dynamics), assuming that we are not at equilibrium.

This problem can be solved using Dynamics and Kinematics, or energy. In this case, I will use energy. If I calculate the total mechanical energy at rest and set it equal to the final mechanical energy plus the work liberated by the 2kg mass, then we can use the kinetic energy final ($\frac{1}{2}mv^2$) to get the final speed of the whole system!

$$PE_i + KE_i @ \text{rest} + W_{\text{in}}^0 = PE_f @ h=0 + KE_f + W_{\text{out}}$$

no work added

\downarrow and \circlearrowleft have PE!

\downarrow \circlearrowleft \circlearrowright have KE!

\downarrow friction! $\mu f_n \cdot d$

\downarrow from a.) we know $f_n = 20N$!

$$m_Agh + m_Bgh = \frac{1}{2}m_{A+B+C} V_f^2 + \text{Joules from friction}$$

$$(1\text{kg})(-10\text{m/s}^2)(4\text{m}) + (3\text{kg})(+10\text{m/s}^2)(+4\text{m}) - \frac{1}{2}(4\text{kg})V_f^2 + (0.4)(20\text{N})(4\text{m})$$

$$-40 \text{ Joules} + 120 \text{ Joules} = 13\text{kg} (V_f^2) + 32 \text{ Joules}$$

$$48 \text{ Joules} = 3\text{kg} (V_f^2)$$

$$(10 \text{ m}^2/\text{s}^2 = V_f^2)^{1/2}$$

$$\boxed{V_f = 4\text{m/s}}$$

3. There are two planets made from the same substance. Planet B is three times as large as planet A, that is $r_B = 3r_A$. Imagine that I visit each planet.

an answer alone is worth zero points – please explain your logic:

- a) What is the ratio of the masses of the two planets? $m_B = 27 m_A$. This question particularly doesn't really fall under one of the 4 lenses, but show your logic.

$$r_B = 3r_A$$

$$V_A = \frac{4}{3} \pi r_A^3$$

$$V_B = \frac{4}{3} \pi (3r_A)^3$$

$$= \frac{4}{3} \pi 27r_A^3 \leftarrow \boxed{1:27 \text{ ratio}}$$

I know that $\text{volume} \cdot \text{density} = \text{mass}$, and since density is constant, volume is the factor that determines the ratio of masses. The volume of a sphere is $\frac{4}{3} \pi r^3$.

- b) Where do I weigh more, and what is the ratio of my weight on each planet? $F_B = 3 F_A$. This question particularly doesn't really fall under one of the 4 lenses, but show your logic.

The greater the mass of the object, the greater its gravitational pull. Refer to the universal gravity formula

$$F_g = \frac{m_1 m_2}{r^2} G$$

$$F_{gA} = \frac{m_{me} \cdot m_A}{r^2} G$$

$\boxed{1:3 \text{ ratio}}$

$$F_{gB} = \frac{m_{me} \cdot 3m_A}{r^2} G = \boxed{\frac{m_{me} \cdot 3m_A}{r^2} G}$$

- c) If I need to escape from the planet into deep space, what is the ratio of my escape velocities from

③ d) Ratio of the speeds of the planets right before they hit?

The two planets will experience a collision, so momentum is conserved. There is initially no momentum because the system is at rest.

$$m_A V_A + m_B V_B = m_A V_A(f) + m_B V_B(f)$$

$$-m_A V_A(f) = m_B V_B(f)$$

$$-m_A V_A(f) = 27 m_A V_B(f)$$

$$\therefore V_B = \frac{1}{27} V_A$$

c) Ratio of my space velocities from the planets?

To escape the planet, I must have kinetic energy equal to the gravitational potential energy between the mass and I after escaping.

$$KE = PE$$

$$\frac{1}{2} m_{me} V^2 = \frac{m_{me} M_{\text{planet}} G}{r}$$

$$V = \sqrt{\frac{2 M_{\text{planet}} G}{r}}$$

$$V_A = \sqrt{\frac{2 V_A G}{r_A}} \quad V_B = \sqrt{\frac{2 (27 V_A) G}{r_B}}$$

$$\therefore V_B = 3 V_A$$

d) Ratio of my distance from each planet.

The only force acting between myself and the planets is gravity.

The universal gravity equation is $F_g = \frac{m_1 m_2}{r^2} G$.

$$F_A = F_B$$

$$\frac{m_{me} M_A G}{(r_A)^2} = \frac{m_{me} 27 M_A G}{(r_B)^2}$$

$$\frac{M_A}{(r_A)^2} = \frac{27 M_A}{(r_B)^2}$$

$$\frac{M_A (r_B)^2}{M_A} = 27 \frac{M_A (r_A)^2}{M_A}$$

$$r_B = \sqrt{27} r_A$$

$$\therefore r_B = \sqrt{27} r_A$$