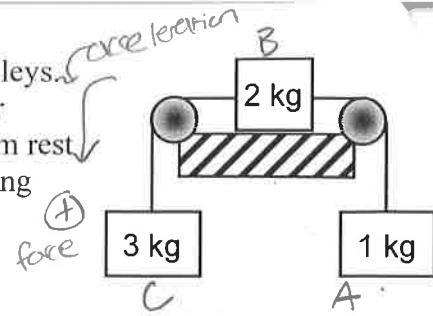


- 1) At right, you see a system of masses with massless, frictionless pulleys. However, there is a coefficient of friction of 0.4 between the center block and the surface it is on. The system of masses is released from rest with each of the hanging masses 4 m off the floor. Find the following three quantities by any means you like *in any order* you like:

  - a) The speed when a mass hits the ground. (optional)
  - b) The acceleration of the system
  - c) The tension in the string connected to the 1 kg mass.

(A)  
for



dynamics; the vector sum of the forces due to friction, gravity, and tension result in a net force. This force causes an acceleration of the system, which is what we are trying to find in addition to different components of force.

b)

$$F_g = F_{\text{Normal}}$$

$$F_{\text{friction}} = \mu N \\ = (0.4)(20N) \\ = 8N \rightarrow$$

c)

$$\sum \vec{F} = m\vec{a}$$

$$= (1\text{kg})(2\text{m/s}^2)$$

$$\sum \vec{F} = 2\text{N} \uparrow$$

$$\sum \vec{F} = T - F_g$$

$$T = \sum \vec{F} + F_g$$

$$= 2\text{N} + 10\text{N}$$

tension = 12N

$$\vec{F}_{\text{system}} = F_g(c) - F_g(A) - F_{\text{friction}}(B)$$

$$F_g = ma$$

$$3mg = m \cdot 8 \text{ m/s}^2$$

$$30N$$

$$F_g = 10N$$

↓ force

$$\vec{F}_s = 30N - 10N - 8N$$

$$= 12N$$

$$\vec{F} = m \vec{a}$$

$$\vec{a}_{\text{sys}} = \frac{\vec{F}_s}{M_s}$$

$$= \frac{12N}{6 \text{ kg}}$$

$$\vec{a}_{\text{sys}} = 2 \text{ m/s}^2 \downarrow$$

$$M_s = 1 \text{ kg} + 2 \text{ kg} + ?$$

$$= 6 \text{ kg}$$

$$\sqrt{k} = 2 \text{ N/m}$$

$$\sqrt{s} = 4 \text{ m}$$

$$\vec{v}_s \nearrow$$

break

a) accelerates  $2m/s^2$  for  $4m$

potential energy converted to kinetic energy  $\Delta X = V_{av}(t)$

kinetic energy is constant  $V_{av} = \frac{4m}{2s} = 2m/s$

so what is kinetic energy since final PE of  $3kg$  block is  $88J$  the initial PE is zero and the final PE is  $88J$

but mass  $PE_f = 0$   
 $m_{block} = m_{cylinder} = 325$

$W = F \cdot dx$   
 $= (8N)(4m)$   
 $= 32J$

$KE_f + PE_f + W_{in} = KE_f + PE_f + W_{in}$

$(120J) = KE_f + 0 + 32J$

$KE_f = 88J$

$KE = \frac{1}{2}mv^2$

$88J = \frac{1}{2}(3kg)v^2$

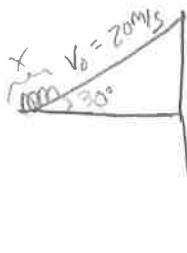
$v^2 = 58.67$

~~$V = 7.66m/s$~~

2. I compress a spring ( $k = 3200 \text{ N/m}$ ) and use it to launch a 2 kg mass at a speed of 20 m/s off the edge of a 25 m cliff, at an angle of  $30^\circ$  above the horizon. Please find as many as possible of the following:

- a) How far do I have to compress the spring in order to get the 20 m/s launching velocity?

This is an energy and dynamic problem. The spring potential energy is converted to kinetic energy, and the force of the spring is what accelerates the mass.



$$\begin{aligned} KE &= \frac{1}{2} MV^2 \\ &= \frac{1}{2} (2\text{kg})(20\text{m/s})^2 \\ &= 400\text{J} \\ KE &= PE_S = 400\text{J} \\ PE_S &= \frac{1}{2} K X^2 \end{aligned}$$

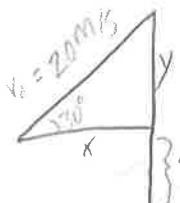
$$400\text{J} = \frac{1}{2} (3200\text{N/m}) X^2$$

$$X^2 = 25$$

$$X = 5\text{m}$$

- b) Please find the time that the 2 kg mass is in the air as it falls to the ground 25 m below.

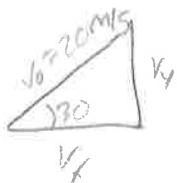
Kinematics by separating the movement over time into x and y components I can find the time before the ball hits the ground - displacement over time based on its time derivatives



$$\begin{aligned} V_{0(y)} &= 20 \sin(30^\circ) = 10\text{m/s} \\ a_g &= 10\text{m/s}^2 \\ y &= y_0 + V_{0(y)}(t) + \frac{1}{2} a_g t^2 \\ 25\text{m} &= 0 + 10\text{m/s}(t) + \frac{1}{2} (10\text{m/s}^2)t^2 \end{aligned}$$

$$\begin{aligned} 5t^2 - 10t - 25 &= 0 \\ t^2 - 2t - 5 &= 0 \\ t &= \frac{2 \pm \sqrt{4+20}}{2} \\ &= \sqrt{6} + 1, -\sqrt{6} + 1 \\ &= 3.4495 \end{aligned}$$

- c) How far from the base of the cliff does the mass land?



$$V_{0(x)} = 20 \cos(30^\circ) = 10\sqrt{3} = 17.32\text{m/s}$$

$V_{0(x)}$  is constant

$$\frac{17.32\text{m}}{s} = \frac{x\text{m}}{3.4495}$$

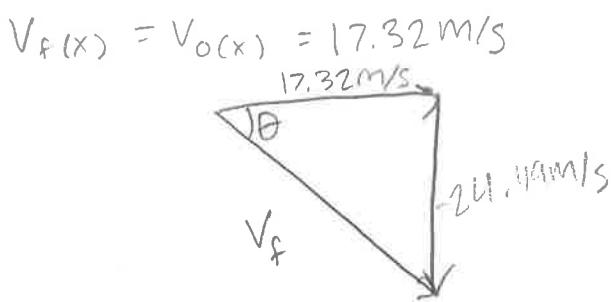
$$x = (V)(t) = 17.32\text{m/s} (3.4495) = 59.74\text{m}$$

- d) Please find the final velocity of the 2 kg mass when it hits the ground, 25 m below - provide the answer in terms of speed and angle.

$$\begin{aligned} V_{f(y)} &= V_{0(y)} + a_g t \\ &= 10\text{m/s} + (-10\text{m/s}^2)(3.4495) \\ &= -24.49\text{m/s} \end{aligned}$$

$$\begin{aligned} V_f &= \sqrt{(V_{nf})^2 + (V_{yr})^2} \\ &= \sqrt{(17.32)^2 + (24.49)^2} \\ &= 29.995\text{m/s} \end{aligned}$$

kinematics = finding the velocity as a vector sum of the x and y components (and their relative change over time)



$$\theta = \tan^{-1} \left( \frac{24.49}{17.32} \right)$$

$$54^\circ \text{ below horizon}$$

3. There are two planets made from the same substance. Planet B is three times as large as planet A, that  $r_B = 3r_A$ . Imagine that I visit each planet.

an answer alone is worth zero points – please explain your logic:

- a) What is the ratio of the masses of the two planets?  $m_B = \underline{\hspace{2cm}} m_A$ . This question particularly doesn't really fall under one of the 4 lenses, but show your logic.

$$V = \frac{4}{3} \pi r^3$$

$$V_A = \frac{4}{3} \pi r^3$$

$$V_B = \frac{4}{3} \pi (3r)^3 \\ = 27(\frac{4}{3})\pi r^3$$

$$V_B = 27V_A$$

$$\text{Density } A = D_B$$

$$D = \frac{m}{V}$$

$$m = DV$$

$$m_A = DV_A$$

$$m_B = DV_B$$

$$m_B = D(27V_A)$$

$$= 27(DV_A) \\ = 27m_A$$

$$m_B = 27m_A$$

- b) Where do I weigh more, and what is the ratio of my weight on each planet?  $F_B = \underline{\hspace{2cm}} F_A$ . This question particularly doesn't really fall under one of the 4 lenses, but show your logic.

$$F_g = \frac{m_1 m_2}{r^2} G_r$$

$$F_{gA} = \frac{m_A m_{(\text{me})}}{r^2} G_r$$

$$F_{gB} = \frac{m_B m}{r_B^2} G_r$$

$$= \frac{(27m_A)m}{(3r)^2} G_r$$

$$= \frac{27(m_A m)}{9r^2}$$

$$= 3 \left( \frac{m_A m}{r^2} \right)$$

$$F_{gB} = 3(F_{gA})$$

I weigh more on the planet with more mass, because the force of gravity between objects with greater mass is larger. Therefore my weight is greater on planet B.

- c) If the planets were allowed to fall together from rest, what would be the ratio of the speeds of the planets right before they hit?  $v_B = \underline{\hspace{2cm}} v_A$

~~$$PE_g = -\frac{m_1 m_2}{r} G_r$$~~

~~$$PE_{g(A)} = -\frac{m_A m}{r} G_r$$~~

~~$$PE_{g(B)} = -\frac{m_B m}{r_B} G_r$$~~

~~$$= -\frac{(27m_A)m}{3r} G_r$$~~

~~$$= \frac{27}{3} \left( -\frac{m_A m}{r} \right) G_r$$~~

~~$$PE_{g(B)} = 9 PE_{g(A)}$$~~

~~momentum conservation~~  
~~PE converted to kinetic energy~~

~~$$PE = KE_f$$~~

~~$$KE_A = \frac{1}{2} m v^2$$~~

~~$$KE_A = \frac{1}{2} m_A (v_A)^2 \rightarrow (v_A)^2 = \frac{2KE_A}{m_A}$$~~

~~$$KE_B = \frac{1}{2} m_B (v_B)^2$$~~

~~lens is why?~~

~~$$v_A = \sqrt{\frac{2KE_A}{m_A}}$$~~

~~$$9KE_A = \frac{1}{2} (27m_A)(v_B)^2$$~~

~~$$(v_B)^2 = \frac{18KE_A}{27m_A}$$~~

~~$$= \frac{9}{27} \left( \frac{2KE_A}{m_A} \right)$$~~

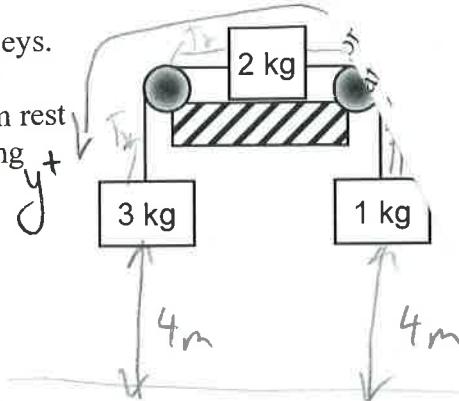
~~$$v_B^2 = \frac{1}{3} (v_A^2)$$~~

$$v_B = \sqrt{\frac{1}{3} (v_A)^2}$$

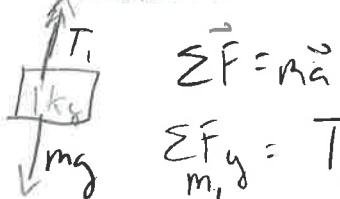
$$= (\sqrt{\frac{1}{3}}) (v_A)$$

$$v_B = .577 v_A$$

- 1) At right, you see a system of masses with massless, frictionless pulleys. However, there is a coefficient of friction of 0.4 between the center block and the surface it is on. The system of masses is released from rest with each of the hanging masses 4 m off the floor. Find the following three quantities by any means you like *in any order* you like:
- The tension in the string connected to the 1 kg mass.
  - The speed when a mass hits the ground.
  - The acceleration of the system

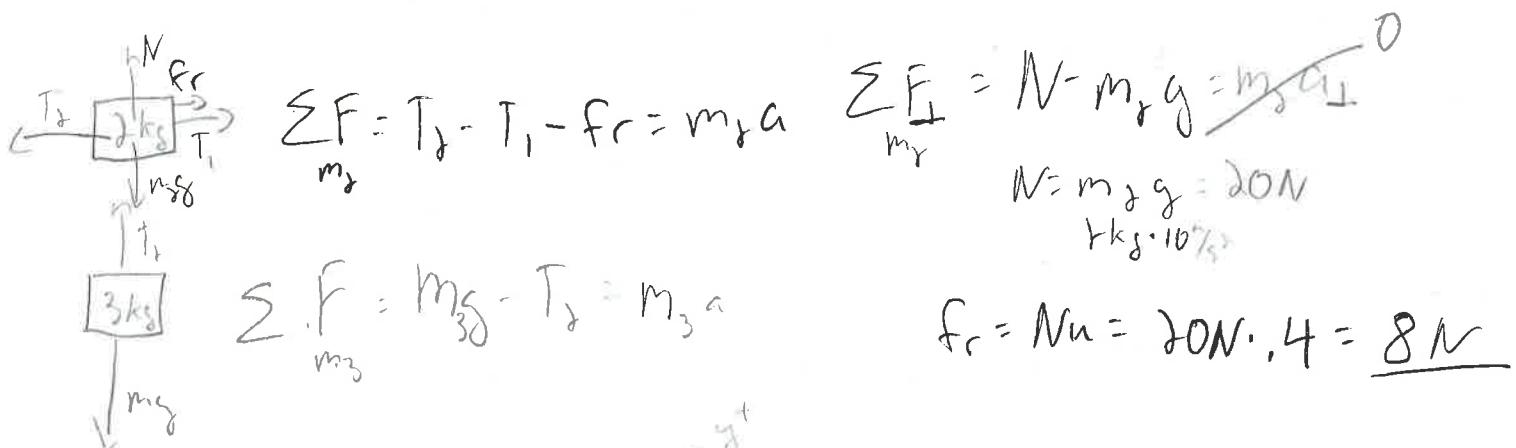


*lens: dynamics b/c we are looking @ forces & acceleration*



$$\sum \vec{F} = m\ddot{a}$$

$$\sum F_y = T_1 - m_1 g = m_1 a$$



$$\sum F = m_2 g - T_2 = m_2 a$$

$$f_r = N \mu = 20N \cdot 0.4 = 8N$$

$$\sum F_{\text{system}} = m_3 g - m_1 g - f_r = (m_1 + m_2 + m_3) a$$

$$a = \frac{m_3 g - m_1 g - f_r}{m_1 + m_2 + m_3} = \frac{20N - 10N - 8N}{1kg + 2kg + 3kg} = \frac{12N}{6kg} = 2m/s^2$$

c)

$$a) \sum F_y = T_1 - m_1 g = m_1 a$$

$$T_1 = m_1 a + m_1 g = 1kg(2m/s^2) + (1kg)(10m/s^2) = 2N + 10N = 12N = T_1$$

$$b) \text{lens: kinetics b/c we are looking at motion/time (velocity)}$$

$$V_f^2 = V_i^2 + 2a\Delta x$$

$$V_f = \sqrt{2 \cdot (2m/s^2) \cdot (4m)} = \sqrt{16m^2/s^2} = 4m/s$$

$$= 2m/s^2$$

in ty direc

A

$k = \frac{1}{2} kx^2$   $.5m$   
 Press a spring ( $k = 3200 \text{ N/m}$ )  $50\text{ cm}$  and use it to launch a  $2\text{ kg}$  mass off the edge of a  $25\text{ m}$  cliff, an angle of  $30^\circ$  above the horizon. Use any method you like to find as many as possible of the following:

- The final velocity of the  $2\text{ kg}$  mass when it hits the ground,  $25\text{ m}$  below – provide the answer in terms of speed and angle.
- How far from the base of the cliff the mass lands.



lens = energy b/c EPE turns into KE when spring is released.  $\rightarrow$  use to find  $V_0$ .

$25\text{m}$

$$\text{EPE}_i = \text{KE}_f$$

$$\frac{1}{2}kx^2 = \frac{1}{2}mv_0^2$$

$$V_0 = \sqrt{\frac{kx^2}{m}} = \sqrt{\frac{3200 \text{ N/m} (.5\text{m})^2}{2 \text{ kg}}} = \sqrt{400 \frac{\text{N/m}}{\text{kg}}} = \sqrt{400 \frac{\text{kg m}^{-2}}{\text{kg s}^2}} = \frac{\sqrt{400} \frac{\text{kg m}^{-2}}{\text{s}^2}}{\sqrt{\text{kg}}} = \frac{20 \frac{\text{kg m}^{-2}}{\text{s}^2}}{\sqrt{\text{kg}}}$$

$$V_0 = 20 \text{ m/s}$$

$$\begin{array}{c} 20 \\ \swarrow \\ 17.32 \end{array} \quad \begin{array}{c} 10 \\ \searrow \\ 10 \end{array}$$

$$17.32 \text{ m/s}$$

$$V_{0,x} = V_0 \cos 30^\circ = 17.32 \text{ m/s}$$

$$V_{0,y} = V_0 \sin 30^\circ = 10 \text{ m/s}$$

- lens = kinematics - b/c looking at motion along  $x$ -axis.  
 - inset to find time it takes to reach ground.

$$y_f^0 = y_0 + V_{0,y} \Delta t + \frac{1}{2} a (\Delta t)^2$$

$$0 = 25\text{m} + V_{0,y} \Delta t + \frac{1}{2} (-10 \text{ m/s}^2) (\Delta t)^2$$

$$0 = -5 \text{ m/s}^2 t^2 + 10 \text{ m/s} (t) + 25$$

$$t = \frac{-10 \pm \sqrt{10^2 + 4(-5)(25)}}{-10} = \frac{-10 \pm \sqrt{100 + 500}}{-10} = \frac{-10 \pm \sqrt{600}}{-10} = \frac{-10 \pm 24.5}{-10} = 3.45 \text{ s}$$

$$V_{f,y} = V_{0,y} + a \Delta t = 10 \text{ m/s} + (-10 \text{ m/s}^2)(3.45 \text{ s}) = -24.5 \text{ m/s}$$

a)

A

$$V_f = \sqrt{V_{f,x}^2 + V_{f,y}^2} = \sqrt{(17.32 \text{ m/s})^2 + (-24.5 \text{ m/s})^2} = 30 \text{ m/s} @ 54.74^\circ \text{ below } +x\text{-axis}$$

$$\theta: \tan^{-1} \left( \frac{-24.5}{17.32} \right) = -54.74^\circ$$

(or below horizon)

no  $a$  in  $x$ -direction

lens = kinematics - use  $\Delta t$  found before to section for it from base of cliff goes

$$x_f = x_i^0 + V_{ix} \Delta t + \cancel{\frac{1}{2} a t^2}$$

$$x_f = (17.32 \text{ m/s})(3.45 \text{ s}) = 59.75 \text{ m} \approx 60 \text{ m}$$

3. There are two planets made from the same substance. Planet B is three times as large as planet A, that is  $r_B = 3r_A$ . Imagine that I visit each planet.

an answer alone is worth zero points – please explain your logic:

- a) What is the ratio of the masses of the two planets?  $m_B = \underline{\hspace{2cm}} m_A$ . This question particularly doesn't really fall under one of the 4 lenses, but show your logic.

$$m = \rho \cdot V$$

$$\frac{m_B}{m_A} = \frac{\rho \frac{4}{3}\pi r_B^3}{\rho \frac{4}{3}\pi r_A^3} = \frac{4(3r_A)^3}{4(r_A)^3} = \frac{27r_A^3}{r_A^3} = \boxed{m_B = 27 m_A}$$

- b) Where do I weigh more, and what is the ratio of my weight on each planet?  $F_B = \underline{\hspace{2cm}} F_A$ . This question particularly doesn't really fall under one of the 4 lenses, but show your logic.

$$F = ma \quad a = \frac{Gm}{r^2} \quad a_g = \frac{Gm}{r^2}$$

$$\frac{F_B}{F_A} = \frac{(m_B)(a_g)}{(m_A)(a_g)} = \frac{27m_A}{m_A} = \boxed{27}$$

$$\boxed{F_B = 27 F_A}$$

what about  $r^{-2}$

- c) If I need to escape from the planet into deep space, what is the ratio of my escape velocities from the planets:  $v_B = \underline{\hspace{2cm}} v_A$ .

$$\frac{1}{2}mv_p^2 + \frac{1}{2}mv_r^2 = \cancel{Gm \frac{v_p}{r}}$$

$$\frac{1}{2}mv_r^2 = \frac{1}{2}Gm \frac{v_p}{r}$$

$$V = \sqrt{\frac{2Gm}{r}}$$

$$\frac{V_B}{V_A} = \sqrt{\frac{2Gm_B}{3r_A}} \quad \frac{2Gm_B}{3r_A} = \frac{2Gm_A}{r_A}$$

$$\sqrt{\frac{2Gm_A}{r_A}} = \sqrt{\frac{2Gm_A}{3r_A}} = \frac{1}{\sqrt{3}} = \boxed{V_B = 3V_A}$$

- d) If the planets were allowed to fall together from rest, what would be the ratio of the speeds of the planets right before they hit?  $v_B = \underline{\hspace{2cm}} v_A$



lens: momentum

$$m_B v_B + m_A v_A = m_B v_B' + m_A v_A' = 0$$

super direction doesn't matter

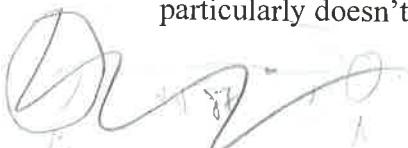
$$m_B v_B' = -m_A v_A'$$

$$v_B' = -\frac{m_A v_A}{m_B} = -\frac{m_A v_A}{27m_A} = -\frac{1}{27} v_A$$

$$V_B = \frac{1}{27} V_A$$

$$\boxed{V_B = \frac{1}{27} V_A}$$

- e) If I want to put myself between the two planets so that I am not attracted to either one more than the other, estimate the ratio of my distance from each planet:  $x_B = \underline{\hspace{2cm}} x_A$ . This question particularly doesn't really fall under one of the 4 lenses, but show your logic.



$$F_A = F_B$$

$$\frac{Gm_A m_B}{x^2} = \frac{Gm_A m_B}{x_A^2}$$

$$\frac{m_A}{x_A^2} = \frac{m_B}{x_B^2}$$

$$x_B^2 = x_A^2 \cdot 27$$

$$\boxed{x_B = \sqrt{27} x_A}$$



yes!