

# Problem Set #1

①

$$m_1 = 5 \text{ kg}$$

$$m_2 = 10 \text{ kg}$$

$$v_1 = 12 \text{ m/s}$$

perfectly inelastic

a) Yes momentum is conserved, because the velocity

$$P_1 = P_2$$

$$m_1 v_1 = (m_1 + m_2) v_2$$

$$(5 \frac{\text{kg}}{\text{s}})(12 \frac{\text{m}}{\text{s}}) = (5 \frac{\text{kg}}{\text{s}} + 10 \frac{\text{kg}}{\text{s}})(v_2 \frac{\text{m}}{\text{s}})$$

$$\frac{60}{15} = v_2$$

$$v_2 = 4 \text{ m/s}$$

of the two objects together after the collision will decrease. The velocity after the collision decreases because the mass after the collision increases. I guess I'm describing how  $P$  is conserved.  $P$  is always conserved. No external forces.

b) Yes energy is conserved. Kinetic Energy is not conserved.

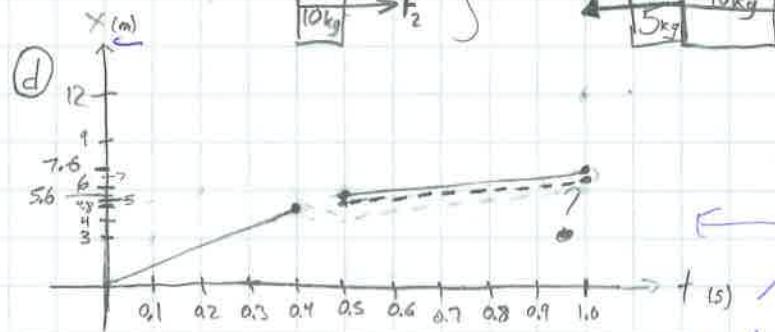
Upon collision, Kinetic Energy ~~can change~~ to heat.

c) Yes there are forces & therefore accelerations. If you look at the objects individually, each object has a  $\Delta V$  upon collision. See diagram:



& only during the collision.

acceleration in same direction as force b/c  $F = ma$ .



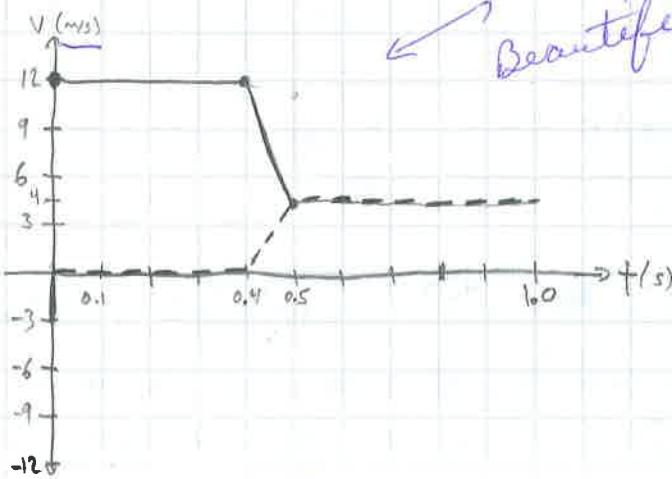
$$\Delta x = v_i t + \frac{1}{2} a t^2$$

A = 5kg obj B = 10kg obj

units

$$\Delta V_A = (4 \frac{\text{m}}{\text{s}} - 12 \frac{\text{m}}{\text{s}}) = -8 \text{ m/s}$$

$$\Delta V_B = (4 \frac{\text{m}}{\text{s}} - 0) = 4 \text{ m/s}$$



Beautiful!



(d) cont'd



Legend

— = 5 kg obj.  
--- = 10 kg obj.

$$A = 5 \text{ kg} \rightarrow b!$$

$$B = 10 \text{ kg} \rightarrow b!$$

$$a_A = \frac{\Delta v_B}{\Delta t} = \frac{40 \text{ m/s}}{0.5 \text{ s} - 0.4 \text{ s}} = 40 \text{ m/s}^2$$

$$a_B = \frac{\Delta v_A}{\Delta t} = \frac{40 \text{ m/s}}{0.1 \text{ s}} = 400 \text{ m/s}^2$$

$$F_A = -F_B \quad (\text{Same Force!})$$

$$m_A a_A = -m_B a_B$$

$$m_A = \frac{1}{2} m_B, \text{ so}$$

$$a_A = \frac{1}{2} a_B$$

(e)

$$\begin{aligned} P_1 &= m_1 v_1 \\ &= (5 \text{ kg})(12 \text{ m/s}) \\ &= 60 \text{ kg} \cdot \text{m/s} \end{aligned}$$

$$KE_1 = \frac{1}{2} m_1 v_1^2$$

$$\begin{aligned} &= \frac{1}{2} (5 \text{ kg})(12 \text{ m/s})^2 \\ &= 360 \text{ J} \end{aligned}$$

(f)

$$\begin{aligned} P_2 &= (m_1 + m_2)(v_2) \\ &= (5 \text{ kg} + 10 \text{ kg})(4 \text{ m/s}) \\ &= (15 \text{ kg})(4 \text{ m/s}) \\ &= 60 \text{ kg} \cdot \text{m/s} \end{aligned}$$

$$\begin{aligned} KE_2 &= \frac{1}{2} (m_1 + m_2)(v_2)^2 \\ &= \frac{1}{2} (15 \text{ kg})(4 \text{ m/s})^2 \\ &= 120 \text{ J} \end{aligned}$$

$$360 \text{ J} - 120 \text{ J} = 240 \text{ J}$$

(g) KE is not conserved. KE ~~dm~~ gets converted into Heat.

h)  $\Delta p = m \Delta v$  for A:  $\Delta p = 5 \text{ kg}(-8 \text{ m/s}) = -40 \text{ kg m/s}$   
 for B:  $\Delta p = 10 \text{ kg}(4 \text{ m/s}) = +40 \text{ kg m/s}$   
 $\underline{\text{total } \Delta \vec{p} = 0}$

i)  $\vec{F} = m \vec{a}$ , OR  $\frac{d \vec{p}}{dt}$  ~~dt~~  $\frac{d \vec{p}}{dt} dt = 0.1 \text{ s}$ , so  $F_A = -400 \text{ N}$   
 $F_B = +400 \text{ N}$

This is good because it's the same force!  
 (acting oppositely on the interacting bodies.)

let  $K = KE$

j)

$$P = mv$$

$$\frac{P}{m} = v$$

$$\left. \begin{array}{l} K = \frac{1}{2}mv^2 \\ K = \frac{1}{2}m\left(\frac{P}{m}\right)^2 \\ = \frac{1}{2}m\left(\frac{P^2}{m^2}\right) \\ = \frac{P^2}{2m} \end{array} \right\}$$

$$K_1 = \frac{P^2}{2m} = \frac{(60)^2}{2(5)} = \cancel{\frac{3600}{10}} = 360 \text{ J}$$

$$K_2 = \frac{60^2}{2(15)} = \frac{3600}{30} = 120 \text{ J}$$



$$\Delta K = 120 - 360 = -240 \text{ J}$$

$$-\frac{240}{360 \text{ J}} = -\frac{2}{3} \text{ J}$$

OR

$$KE = \frac{P^2}{2m}$$

p is conserved in the collision so the numerator stays the same, but mass is tripled, so KE  $\downarrow$  by  $\frac{1}{3}$

we write:

as  $m \Rightarrow 3m$  (mass is tripled)

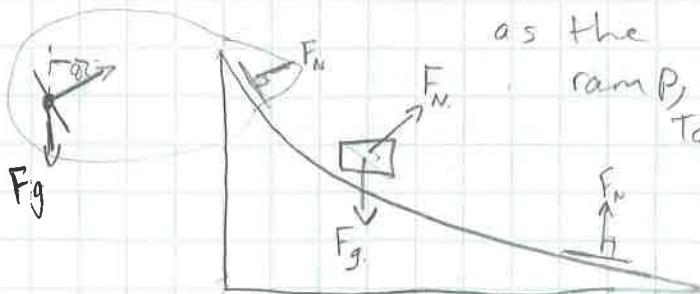
$P \Rightarrow 1 P$ , momentum is the same (conserved)

$$KE \Rightarrow \frac{1}{3} KE_0$$

②

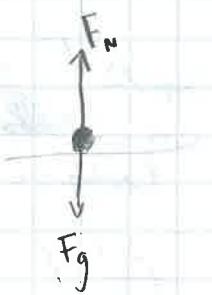
a) Dynamics: Gravity pulls the block down.

The normal force changes in magnitude as the block goes down the ramp  $P_1$ , because of the  $\Delta\theta$ . To be clear I am referring to the part of the normal force that is opposite to gravity.



$$F_N \neq F_g$$

$$\sum F = 0 = m(a)$$



### Kinematics

Non

- uniform acceleration due to gravity + normal force

so,

$$x = x_i + v_i t + \frac{1}{2} a t^2$$

$\uparrow g$  changes, changes w/  
that describes the motion different angle

### Energy

block has PE at top of ramp  
converts to KE as block moves to bottom

### Momentum

is conserved

$$\sum \vec{\Delta p} = 0, \text{ so } \vec{\Delta p}_{\text{ball}} = -\vec{\Delta p}_{\text{Earth}}$$

But the  $m_{\text{Earth}} \ggg m_{\text{Ball}}$ , so we don't notice a  $\Delta V_{\text{Earth}}$

$$b) \Delta E = 0 = \Delta PE + \Delta KE = 0$$

$$\left. \begin{array}{l} PE = mgh \\ \Delta PE = mg\Delta h \\ \quad = mg(h_f - h_i) \end{array} \right\} \left. \begin{array}{l} KE = \frac{1}{2}mv^2 \\ \Delta KE = \frac{1}{2}m\Delta v^2 \\ \quad = \frac{1}{2}m(v_f - v_i)^2 \end{array} \right\}$$

$$\Delta E = 0 = mg(0 - h_i) + \frac{1}{2}m(v_f + 0)^2$$

$$0 = -mg h_i + \frac{1}{2}m v_f^2$$

$$mgh_i = \frac{1}{2}mv_f^2$$

$$v_f = \sqrt{\frac{2mgh_i}{m}} \quad \text{look! mass cancels, so mass doesn't matter.}$$

$$= \sqrt{2(10)(50)} \quad \text{units!}$$

$$= \sqrt{1000} \frac{m}{s}$$

$$= 31.6 \text{ m/s}$$

This means that big things fall the same as small things?

c) All 3 same final speed. Look @ b, which uses energy lens and shows the final velocity, regardless of the path. All 3 paths start w/ the same PE

d) C has the greatest velocity at the middle because C also has the lowest PE &  $\therefore$  the highest KE.  $KE = \frac{1}{2}mv^2$

e) Path C reaches the bottom first because the ball reaches a greater velocity before the other two paths. For example at halfway down path C had the greatest velocity.

we don't know this kinematics formula yet

Energy

Yeah!

(3) a)

$$\cancel{V_F^2 = V_i^2 + 2ah}$$

$$KE = PE$$

$$\frac{1}{2}mv^2 = mgh$$

$$\begin{aligned} V_F &= \sqrt{0 + 2(10)(50)} \\ &= \sqrt{1000} \\ &= 31.6 \text{ m/s} \end{aligned}$$

$$V = \sqrt{\frac{2mgh}{m}}$$

$$V = 31.6 \text{ m/s}$$

b)

Momentum is conserved.

The box has a  $\Delta P$ , but  $P$  is still conserved because gravity changes the box's  $P$  however gravity is the interaction between the Earth and the box. The Earth moves toward the box, w/ a very small velocity, but a big mass.

c)

$$m_b V_b = m_{\oplus} V_{\oplus}$$

$$V_{\oplus} = \frac{m_b V_b}{m_{\oplus}} = \frac{10 \text{ kg}}{6 \times 10^{24} \text{ kg}} \cdot 31.6 \text{ m/s}$$

$$V_{\text{earth}} \propto \frac{300}{6 \times 10^{24}}$$

$$= m_b V_b$$

$$V_{\oplus} = 4.96 \times 10^{-5} \text{ m/s}$$

up toward the box.  
 $5 \times 10^{-23} \text{ m/s}$  (pretty slow)

d)

$$K = \frac{1}{2}mv^2$$

$$\underline{10^{-23}}$$

$$\text{KE}_{\text{box}} = \frac{1}{2}(637100)(4.96 \times 10^{-5})^2 \cdot \frac{1}{2}(6 \times 10^{24} \text{ kg})(5 \times 10^{-23} \text{ kg})^2$$

earth has more mass than 0.078 J  
640 Tons!

$$= 15 \times 10^{-22} \text{ J} = 1.5 \times 10^{-21} \text{ J}$$

$$24 - 23 - 23 = -22$$

The KE of the earth is small enough to ignore.  
you bet!

(4)

$$a) m = 1000 \text{ kg} \quad P = 100 \text{ hp}$$

$$100 \text{ hp} \times \frac{745.699872 \text{ W}}{1 \text{ hp}} = \boxed{74569 \text{ Watts}}$$



#4 a)  $1 \text{ hp} = 746 \text{ watts}$

$$100 \text{ hp} \times \frac{746 \text{ watts}}{1 \text{ hp}} = 74,600 \text{ watts}$$

$$W = P\Delta t = \frac{1}{2}mv^2$$

↑  
constant function

b)  $P = \frac{\Delta E}{\Delta t}$   $\Delta T \neq \Delta E$   $E = \frac{1}{2}mv^2$  nice!

(1)  $74,600 \text{ W} = \Delta E$   $74,600 = \Delta E$

$$v = \sqrt{\frac{2E}{m}}$$

$$v = \sqrt{\frac{2(74,600)}{1000}} = 12.215 \text{ m/s at 1s}$$

(2)  $74,600 = 149200 J$

$$v = 17.27 \text{ m/s at 2s}$$

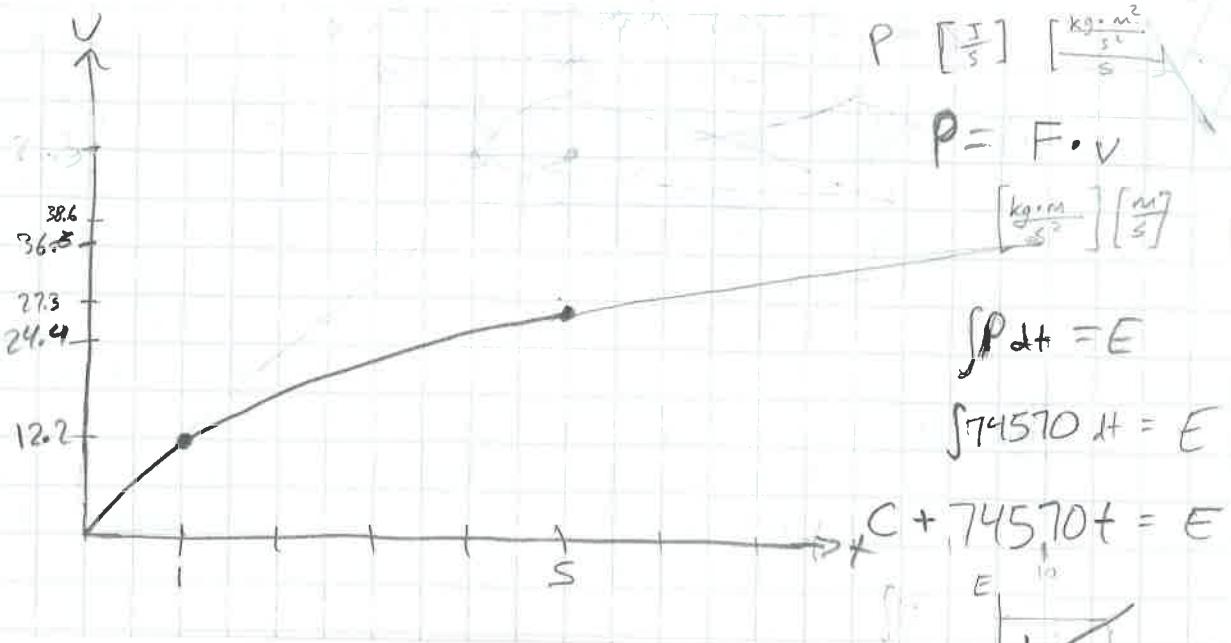
(5)  $(74,600) = 373000 J$

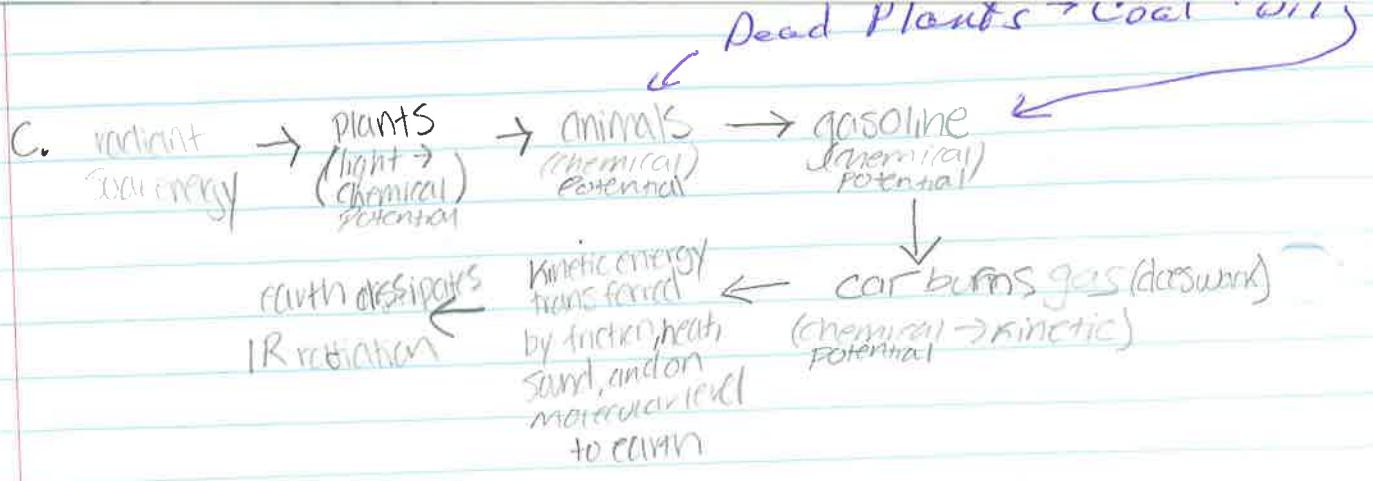
$$v = 27.31^3 \text{ m/s at 5s}$$

(10)  $(74,600) = 7460000 J$

$$v = 38.626 \text{ m/s at 10s}$$

b)





c. Sun  $\rightarrow$  Plants  $\rightarrow$  petrol  $\rightarrow$  KE  $\rightarrow$  Heat  $\rightarrow$  IR radiation

5. Our fundamental mistake is that we looked at the overall speed instead of the instantaneous acceleration and velocity. So we did not realise that the ball that accelerated the fastest at first will continue to have the highest velocity until the others catch up last it will always have an advantage from the initial boost.

c)

Sun  $\rightarrow$  plants  $\rightarrow$  food  $\rightarrow$  ATP  $\rightarrow$  chemical  $\rightarrow$  solar panel  $\rightarrow$  oil rig  $\rightarrow$  chemical  $\rightarrow$  KE  
heat  $\rightarrow$  IR

5) What happened was that ball C converted its potential energy into KE a lot faster than the other 3 balls allowing it to reach maximum velocity quicker. This allowed the ball to pass over the flatter part of the track quicker than the other 3.

Yes! it's not about momentum. Many people wrote about momentum. It's about recognizing that the average  $\Delta PE \Rightarrow \Delta KE$  isn't the same as the immediate  $\Delta PE \Rightarrow \Delta KE$ .