Problem Set #10 due beginning of last day of class, Dec. 4 or 5. 100 pts. total

2 pts extra credit per extra person in the group – up to 8 points possible!

3 pts extra credit if you don't use a calculator: if so, write and sign a statement at the top of the problem set: "I [your name] did not use a calculator for any part of this problem set."

1. The classic "infamous ladder problem: A 30 kg 5 m ladder leans up against a frictionless wall at an angle of 53° with respect to the ground. You are 50 kg, and the coefficient of static friction with the floor is a dangerous 0.50. At first, you are standing at the base of the ladder on the bottom rung, essentially 0 meters from the bottom.

a) I hope you already drew a great diagram! … and labeled all the forces? And thought about all the torques?

b) How much force can we depend on the friction to provide for us? Is this the actual force friction is providing, or don't we know? The actual amount of frictional force is going to depend on how hard we push on it. This will be equal to the normal force provided by the wall… as long as this normal force is less than the maximum force we calculate. Also remember that the frictional force increases when I get on the ladder.

c) Let's see if we need more than this… what force is pushing against the frictional force? Normal force of the wall – we don't know this force, but it is the amount of force that is necessary to keep the ladder from rotating into the building.

d) This is a statics problem… There are forces in the x direction, in the y direction, and there are torques. Three equations, three unknowns: The normal force, the required frictional force, and the actual frictional force. Set up the equilibrium equations for forces in the x and y direction, and consider what each tells us.

e) Please set up the torques. Which point to you use to be the center of rotation? Why do you choose that point? Can you use this equation to find the normal force of the wall on the ladder.

f) Can you use this information to show that the ladder does not slip? Can you show that you can "test" the ladder by bouncing up and down on it and it won't slip?

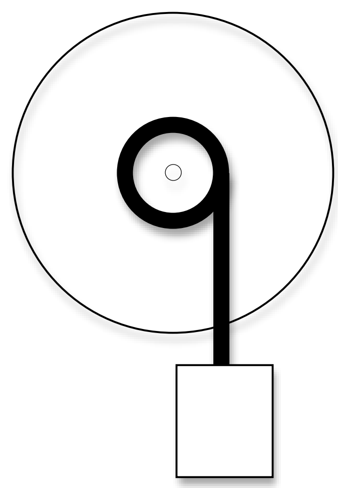
g) Now that you are confident about the security of the ladder, you start walking up the ladder. Which of the equations does this change? How does it change them? How does the situation become more dangerous?

h) Will I make it to the top of the ladder? Find out by doing an analysis with me at the top of the ladder, and see if the ladder will slide.

i) Please find my location when the ladder slides. Is it bad for me?

j) If we were to do this problem again, and we changed the inclination angle of the ladder to 60°, would this make the situation safer, or more dangerous? How do you know?

2. Remember PS #7, question #1? We are doing a variation of this. Instead of pulling on the string with 100 N, we are putting a 10 kg mass on the end of the 2 m string and letting it fall. Again, the flywheel is 3 kg flat disk of uniform thickness, is on a frictionless bearing, and has a radius of 30 cm. You have the string wrapped around the hub (or spindle, or pulley) of radius = 10 cm.



a) How is this different from the situation in PS #7 from a perspective of

i) energetics? Where does the 200 J go? What would it mean if it was?

ii) dynamics? Is the tension on the string still 100 N? What would it mean if it was?

***Now you are going to solve this problem 3 different ways.

b) Using energetics, please find the final angular velocity of the wheel after the block has fallen 2 m.

c) Using dynamics, please set up the torque and force equations on the wheel and mass respectively, to find the two unknowns: the tension in the string and the acceleration of the block.

d) Lastly there's a tricky way you can solve this as a system! Imagine that the length of the string is zero meters. Then the block is part of the wheel. This mass just adds to the wheel's moment of inertia. Because the block is offset, it provides torque. Use this to find the angular acceleration of the wheel at that moment. In reality, can you show that as the mass falls, it maintains this same rotational acceleration?

e) Verify that all three methods give you the same answers. You will need to use the velocity from b) to find accelerations and angular accelerations… or the other way around.

3. A bicycle is a beautiful thing to me! Imagine that I can put a constant force of 200 N onto the pedal that is 20 cm long, and am able to maintain that force for some time as I pedal along. Let's say that I am rotating the pedals at 60 rotations per minute. Imagine that I am riding up at constant speed against wind friction.

A) Find the torque my legs put on the pedals and the omega of the pedals.
B) Find the power I'm putting out.
C) I'm in my highest gear, so the diameter of the pedal gear is 20 cm, and the diameter of the gear driving the rear wheel is 4 cm. Please find the tension in the chain, and the torque the chain produces on the rear wheel.
D) Given the speed of the chain and the tension in the chain, what is the power I deliver to the chain?
E) What is omega of the rear wheel? What is the power the torque of the chain delivers to the rear wheel?
F) If the diameter of the rear wheel is 700 mm, what is the force that the torque on the rear wheel delivers to the road (assume that there is no slipping). This is equal to the force of the air friction if I am not accelerating.
G) What must be the speed of the surface of the rear tire surface (which is equal to the speed of the bike)? And what is the power that this surface delivers to the bicycle?
H) At some time, I change gears, putting the chain on a rear gear cluster on a gear that is 8 cm in diameter (doubling the diameter of the rear gear), and I am able to continue putting the same amount of force on the pedals. What change to I experience? What do I notice in my pedaling? what would be the new:
   i) The torque on the rear wheel?
   ii) The power to the rear wheel?
   iii) The speed of the chain?
   iv) Omega of my legs?
   v) What will happen to the motion of my bike?
   vi) What will happen to the feeling in my body? (will I relax or get more tired?)

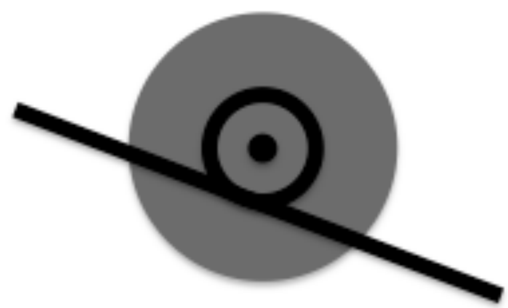
4. I spin a 2 kg rock over my head about a vertical axis (like David and Goliath) from a 2m string such that it makes an angle of 30° below the horizontal.

a) Find the radius of the rock's trajectory, the speed of the rock and the tension in the string.

****Now that you know the answers, you write the next three questions****

b) Make the corresponding question for a (miniature) bicyclist rounding a corner. What must be the angle that the bicyclist's body makes with the pavement?
c) Make the corresponding question for a toy car rounding a banked turn on a frictionless surface. What must be the bank of the curve?
d) Make the corresponding question for a (toy) airplane baking a turn. What must be the bank of the wings?

5. Remember the flywheel from PS#9, questions 3? It's rolling down two rails inclined at 30° as shown at right. The flywheel is a 3 kg flat disk of uniform thickness and has a radius of 30 cm. The hub is of radius = 10 cm.
The flywheel starts from rest and rolls without slipping along 4 m of rail.



a) You already solved this by using energy considerations. Please see the solutions so you understand how this worked.
b) Now, please solve for the angular acceleration and acceleration by using three equations: rotational dynamics, linear dynamics, and the relationship between acceleration and angular acceleration. See if this gives you the same answer as using energy.
c) Now, please solve for angular acceleration using the sneaky way I showed you in class after SuperQuiz 2, by recognizing that at this instant, the wheel is actually rotating about the point of contact. Please verify that this gives you the same answer as a)
d) Using your value for angular acceleration, please find the final velocity of the wheel.