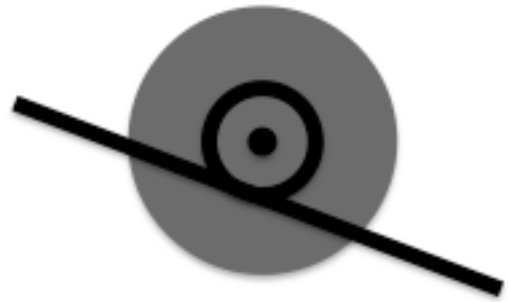
Problem Set #10 due beginning of class, Monday, Nov. 30

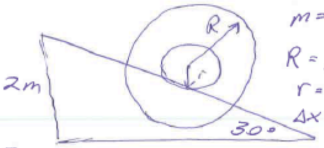
#1 Remember the flywheel from the first problem in PS #7?, now it has a hub on either side, rolling down two rails inclined at 30° as shown at right. The flywheel is a 3 kg flat disk of uniform thickness and has a radius of 30 cm. The hub is of radius = 10 cm.
The flywheel starts from rest and rolls without slipping along 4 m of rail.



a) $\Delta h = 2m$

$\Delta PE = mg\Delta h = 3kg\, 10\, m/s^2\, 2m$

$= 60\, \frac{kg\, m^2}{s^2} = 60J$



$m = 3kg$
$R = .3m$
$r = .1m$
$\Delta x = 4m$

b) Energy line –

$I = \frac{1}{2}MR^2$

$v = \omega r$

$PE \Rightarrow KE_{linear} + KE_{rotation}$

$= \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}m\omega^2 r^2 + \frac{1}{2}\,\frac{1}{2}mR^2\omega^2$

$\cancel{m}g\Delta h = \frac{1}{2}\cancel{m}\omega^2(r^2 + \frac{1}{2}R^2) = \frac{1}{2}\cancel{m}\omega^2 \underbrace{(10^{-2}m^2 + \frac{1}{2}.09m^2)}_{0.55\ m^2}$

$\omega^2 = \frac{2}{0.055\, m^2}\, 10\, m/s^2\, 2\cancel{m} \approx 727/s^2$

$\boxed{\omega_f \approx 27/s}$

$\omega_{Ave} = \frac{\omega_f}{2} \approx 13.5/s$

$V_{ave} = \omega_{ave} \cdot r = 1.35\, m/s = \frac{\Delta x}{t}$

$t = \frac{\Delta x}{V_{ave}} = \frac{4m}{1.35\, m/s} \approx 3.0s$

$\alpha = \frac{\Delta\omega}{\Delta t} \approx 9.1/s^2$

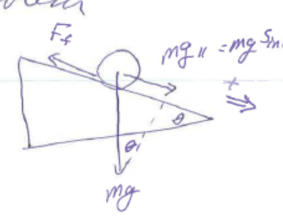
$a = \alpha r = 0.91\, m/s^2$

$\tau = \alpha I = \frac{9.1}{s^2}\,\frac{1}{2}mR^2 = \frac{9.1}{s^2}(\frac{1}{2}).3kg\,(.3m)^2 = \underline{1.23\, Nm}$

$F_f = ?$ $\tau = F_f r$ $F_f = \frac{\tau}{r} = \underline{12.3N}$ (up the hill)

This is now a dynamics problem

$\Sigma\vec{F} = m\vec{a} = mg_{\parallel} - F_f$



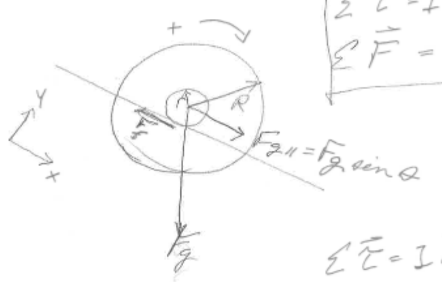
$= 15N - 12.3N \approx 2.7N$

$a = \frac{\Sigma\vec{F}}{m} = \frac{2.7N}{3kg} \approx 0.9\, m/s^2$

which is what we calculated previously.

below is a different to find the answers using a dynamics approach.

$$\boxed{\begin{array}{ll} \sum \vec{\tau} = I\vec{\alpha} & a = \alpha r \\ \sum \vec{F} = m\vec{a} & \end{array}}$$



$\sum \vec{\tau} = I\vec{\alpha}$ $\qquad \sum F = ma$

$F_f\, r = I\alpha$ $\qquad F_{g\parallel} - F_f = ma$

$F_{g\parallel} r - m\alpha r^2 = I\alpha$ $\qquad F_{g\parallel} - ma = F_f$

$F_{g\parallel} - m\alpha r = F_f$

$F_{g\parallel} r = I\alpha + mr^2\alpha$

$$\boxed{\frac{F_{g\parallel}\, r}{\underbrace{I + mr^2}_{I_{PA}}} = \alpha}$$

$$\alpha = \frac{(\cancel{m}g \sin 30^\circ)(0.1\,m)}{\frac{1}{2}\cancel{m}(0.3\,m)^2 + \cancel{m}(0.1\,m)^2}$$ mass cancels

$$= \frac{5\,m/s^2\,(.1\,m)}{.055\,m^2}$$

$$= 9.1/s^2$$

$$a = \alpha r = 0.91\,m/s^2$$

$\tau_{\text{Pt of contact}} = I_{\text{Parallel axis}}\,\alpha$,

$F_{g\parallel} \cdot r = I_{PA}\,\alpha$ $\qquad \alpha = \frac{F_{g\parallel} \cdot r}{I_{PA}}$

#2 A disk of uniform mass and radius, R is secured to a wall with a frictionless pivot that allows rotation as shown at right. It is started in the higher position where the center of the circle is at the same height as the pivot and allowed to drop and swing. When the disk is at the bottom of the swing (dotted line), please find:

a) Omega, the angular velocity of the disk about the pivot.
b) The angular momentum of the disk about the pivot.
c) The force that the pivot is providing to the disk. Include direction.

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a. We can use energy to solve this, knowing energy is conserved. PE is converted all into KE. Let $h = r$, so:

$$mgr = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$$
$$mgr = \frac{1}{2}I\omega^2 + \frac{1}{2}mr^2\omega^2$$
$$mgr = \frac{1}{2}\omega^2\left[I + mr^2\right]$$
$$mgr = \frac{1}{2}\omega^2\left[\frac{1}{2}mr^2 + mr^2\right]$$
$$\cancel{m}gr = \left(\frac{1}{2}\omega^2\right)\left(\frac{3}{2}\cancel{m}r^2\right)$$
$$g = \frac{3}{2}\cdot\frac{1}{2}\omega^2 r$$
$$\omega = \sqrt{\frac{4g}{3r}} = \boxed{\frac{2\sqrt{g}}{\sqrt{3}\sqrt{r}}\ \frac{\text{rad}}{\text{sec.}}}$$

b. $I = \frac{1}{2}mr^2$, $L = I\omega$, so $L = \frac{1}{2}mr^2\left(\frac{2\sqrt{g}}{\sqrt{3}\,r}\right) = \frac{1}{\sqrt{3}}\frac{mr^2\sqrt{g}}{\sqrt{r}}$

$$\boxed{L = \frac{m\sqrt{r^3 g}}{\sqrt{3}}}$$

c.

→ dynamics, because we know there is a centripetal acceleration from the nail, as well as a force of gravity and a force from the nail.

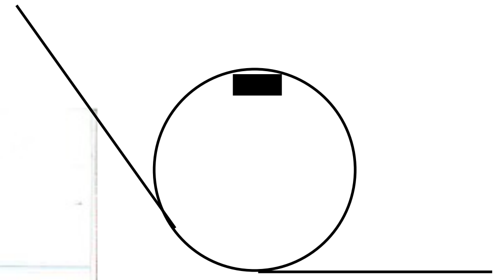
$\Sigma F_c = ma_c$ $\quad F_{nail} = ma_c + F_g$

$= \frac{m4g}{3\cancel{r}}\cancel{r}$ $\quad = m\omega^2 r + F_g$

$\quad = m\frac{4}{3}g + mg$

$= mg\left(\frac{4}{3}\right)$ $\quad \boxed{F_{nail} = \cancel{\frac{1}{3}mg}} \rightarrow 2\frac{1}{3}(mg)$

#3 *If you think you've already done this problem before, just skip down to part d*. You go on a R = 10 m, loop-de-loop ride at the carnival, but you have to choose how high to start the cart. Say you have a mass of 70 kg, like your instructor and you are sitting on a scale that reads in kg.



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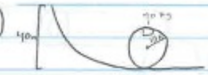


a) $PE_I = (70\ kg)(10\ m/s^2)(40\ m) = 28000\ J$

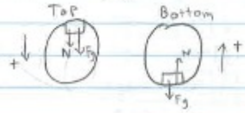
$PE_{\text{Top of Loop}} = (70\ kg)(10\ m/s)(20\ m) = 14000\ J$

$28000\ J = \frac{1}{2}mv^2,\ V_{bottom} = 28.28\ m/s$

$14000\ J = \frac{1}{2}mv^2,\ V_{top} = 20\ m/s$

$a_{c_{bottom}} = \frac{(28.28\ m/s)^2}{10\ m} = 80\ m/s^2$

$a_{c\ top} = \frac{(20\ m/s)^2}{10\ m} = 40\ m/s^2$



$\Sigma F_c = ma_c$

$N + F_g = ma_c$

$N = ma_c - F_g$

$N = (70kg)(40\ m/s^2) - (70kg)(10\ m/s^2) = \boxed{2100\ N}$

$\Sigma F_c = ma_c$

$N - F_g = ma_c$

$N = ma_c + F_g$

$N = (70kg)(80\ m/s^2) + (70kg)(10\ m/s^2) = \boxed{6300\ N}$

→ As you round the bottom of the loop, you feel more force pushing down on you. This is not a good ride for a pregnant woman!

b) If you start at the height as the top of the loop, you won't have enough velocity to make it around the top of the loop, because if energy is conserved, then KE will be 0 as you approach the top of the loop.

c) $PE_I = PE_T + KE_T$

$mgh = mg(20m) + \frac{1}{2}mv^2$

$gh = g(20m) + \frac{1}{2}v^2$

$(10\,m/s^2)h = (10\,m/s^2)(20m) + \frac{1}{2}(10.01\,m/s)^2$

$\boxed{h > 25\ m}$

$\Sigma F = ma_c$

$F_g < ma_c$

$mg < ma_c$

$a_c > g$

$\frac{v^2}{r} > g$

$v > \sqrt{gr}$

$V > 10\ m/s$

d) $KE = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$

$= \frac{1}{2}I\frac{v^2}{r^2} + \frac{1}{2}mv^2$

$= \frac{1}{2}\cdot\frac{2}{3}m r^2\frac{v^2}{r^2} + \frac{1}{2}mv^2$

$= \frac{2}{6}mv^2 + \frac{1}{2}mv^2$

$= \frac{5}{6}mv^2$

PE = 28000 J at top

So, $KE_{bottom} = 28000\,J = \frac{5}{6}(70kg)v^2$, $v = 21.9\ m/s$, $a_c = 48\ m/s^2$

$KE_{top} = 14000\,J = \frac{5}{6}(70kg)v^2$, $v = 15.49\ m/s$, $a_c = 24\ m/s^2$

Bottom

$\Sigma F = ma_c$

$-F_g + F_N = (70kg)(48\,m/s^2)$

$F_N = 3360\,N + 700\,N = \boxed{4060\,N}$

Top

$\Sigma F = ma_c$

$F_g + F_N = ma_c = (70kg)(24\,m/s^2)$

$F_N = 1680N - 700N = \boxed{980\,N}$

#4 Remember PS #8, question #4? We are doing a variation of this. Instead of pulling on the string with 100 N, we are putting a 10 kg mass on the end of the 2 m string and letting it fall. Solution for this is in video for Week 11, day 2: “Solving flywheel with hanging mass”

5. This is from class Week 10, Day 2: Where does the scale read more? This is a dynamics problem because it deals with forces (gravity and normal force) as well as acceleration (at the pole, a = 0. At the equator, we have circular motion, so there is centripetal acceleration inward). I know it’s a dynamics problem, so I know what to do!

Dynamics! ... protocol

$\Sigma\vec{F} = m\vec{a}$ $F_g + F_N$

Pole $\Sigma\vec{F} = m\vec{a} = 0$, $\vec{F}_g = -\vec{F}_N$

Equator a is inward, so $\Sigma\vec{F}$ must be inward

$|F_g| > |F_N|$

or $F_g - F_N = ma$ ($\hat{r}$) direction

so $F_g = F_N + ma_c$