

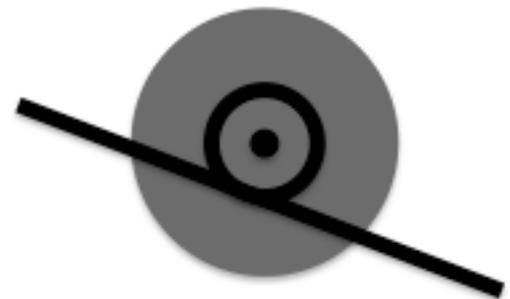
PS#11 Due Thursday, Nov. 30 in class. Remember to start each question with a description of the lens and method.

1) A bicycle is a beautiful thing to me! This question is largely addressed through the “bicycle transmission” in Week 11. Please see the video again if you are so inclined. However, I will address right here question H) What happens when you change the rear gear to twice the original radius? This is downshifting by a factor of two. At that moment, there is no immediate change of speed, so the rear wheel has the same rotational velocity. With twice the radius on the rear cog (gear), the chain must move twice the speed to keep up with the rotating wheel. Because the chain has the same tension on it (you are pushing with the same force on your feet), the power supplied by the chain ( $P = F \cdot v$ ) is doubled. Because the chain tension acts on the rear wheel at twice the radius, the chain’s torque on the rear wheel doubles, doubling the force to the earth’s surface (and the force of the earth’s surface on the bike doubles). By doubling the force to the rear wheel, the bike will accelerate, and you’ve doubled the power delivered to the rear wheel. In order to move the chain twice as fast, you will need to spin your legs twice as fast, doubling your power output if you are able to continue pushing with the same force on your legs:  $P = \tau\omega = Fv$ . This is what we experience every day... if you are cruising at constant velocity on the freeway, your engine is not putting out very much power. But then you want to pass someone. You downshift to a lower gear (or your automatic transmission does that). The engine spins much faster, which you can hear. The power to the wheels increases greatly and you accelerate increasing the kinetic energy of your car. Same thing on a bicycle. Now can you answer the questions:

- h) At some time, I change gears, putting the chain on a rear gear cluster doubling the radius, and I am able to continue putting the *same amount of force* on the pedals. What change to I experience? What do I notice in my pedaling? what would be the new:
- i) The torque on the rear wheel?
  - ii) The power to the rear wheel?
  - iii) The speed of the chain?
  - iv) Omega of my legs?
  - v) What will happen to the motion of my bike?
  - vi) What will happen to the feeling in my body? (will I relax or do I have to work harder?)

2) Remember the flywheel from the first problem in PS #7?, now it has a hub on either side, rolling down two rails inclined at  $30^\circ$  as shown at right. The flywheel is a 3 kg flat disk of uniform thickness and has a radius of 30 cm. The hub is of radius = 10 cm.

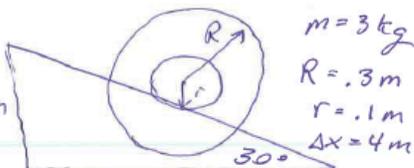
The flywheel starts from rest and rolls without slipping along 4 m of rail.



- a) What is the loss of potential energy?
- b) Find the final velocity and rotational velocity. *hint: you have two unknowns and only one equation! Poop! Ah, but there is a relationship between the speed of the disk and how fast it is spinning. Is this a helpful relationship?*
- c) Use the above to find the average velocity, the time taken, and the angular and linear acceleration.
- d) Use the above to find the torque on the wheel, and therefore find the frictional force that must have been applied by the rails.

e) Now that you know the frictional force on the wheel, and the force of gravity, can you find the acceleration the wheel should have and see if it matches your value for (c) above?

a)  $\Delta h = 2\text{m}$   
 $\Delta PE = mg \Delta h = 3\text{kg} \cdot 10\frac{\text{m}}{\text{s}^2} \cdot 2\text{m} = 60\frac{\text{kg} \cdot \text{m}^2}{\text{s}^2} = 60\text{J}$



b) Energy balance -  $I = \frac{1}{2}MR^2$   
 $v = \omega r$   
 $PE \Rightarrow KE_{\text{linear}} + KE_{\text{rotation}}$   
 $= \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}m\omega^2 r^2 + \frac{1}{2} \cdot \frac{1}{2}mR^2\omega^2$

$mg \Delta h = \frac{1}{2}m\omega^2(r^2 + \frac{1}{2}R^2) = \frac{1}{2}m\omega^2(0.1^2 + \frac{1}{2} \cdot 0.3^2)$   
 $0.55\text{m}^2$

$\omega^2 = \frac{2}{0.055\text{m}^2} \cdot 10\frac{\text{m}}{\text{s}^2} \cdot 2\text{m} \approx 727/\text{s}^2$

$\omega \approx 27/\text{s}$

$\omega_{\text{ave}} = \frac{\omega_f}{2} \approx 13.5/\text{s}$

$v_{\text{ave}} = \omega_{\text{ave}} \cdot r = 1.35\frac{\text{m}}{\text{s}} = \frac{\Delta x}{t}$

$t = \frac{\Delta x}{v_{\text{ave}}} = \frac{4\text{m}}{1.35\frac{\text{m}}{\text{s}}} \approx 3.0\text{s}$

$\alpha = \frac{\Delta \omega}{\Delta t} \approx 9.1/\text{s}^2$        $a = \alpha r = 0.91\frac{\text{m}}{\text{s}^2}$

$\tau = \alpha I = \frac{9.1}{\text{s}^2} \cdot \frac{1}{2} \cdot 3\text{kg} \cdot (0.3\text{m})^2 = 1.23\text{Nm}$

$F_f = ?$      $\tau = F_f r$      $F_f = \frac{\tau}{r} = 12.3\text{N}$  (up the hill)

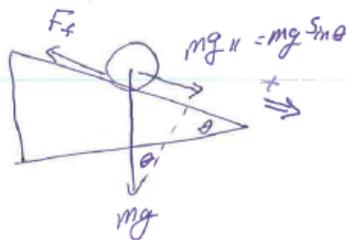
This is now a dynamics problem

$\sum \vec{F} = m\vec{a} = mg_{\parallel} - F_f$

$= 15\text{N} - 12.3\text{N} \approx 2.7\text{N}$

$a = \frac{\sum \vec{F}}{m} = \frac{2.7\text{N}}{3\text{kg}} \approx 0.9\frac{\text{m}}{\text{s}^2}$

which is what we calculated previously.



Below is a different way to find the answers using a dynamics approach and solving the simultaneous equations. **BUT at the very end, I show you how to solve it in one line** by just saying that at this instant in time, the wheel is pivoting around the point of contact and finding the torque  $= F_{g(\text{parallel})} \cdot r$ , and using the parallel axis theorem to find the moment of inertia of the wheel about this point.

$\sum \vec{\tau} = I \vec{\alpha} \quad a = \alpha r$   
 $\sum \vec{F} = m \vec{a}$

$F_f r = I \alpha$   
 $F_{g\parallel} - F_f = ma$

$F_{g\parallel} r - m r^2 \alpha = I \alpha$   
 $F_{g\parallel} - ma = F_f$   
 $F_{g\parallel} - m r \alpha = F_f$

$F_{g\parallel} r = I \alpha + m r^2 \alpha$

$\frac{F_{g\parallel} r}{I + m r^2} = \alpha$

$I_{PA}$

$\alpha = \frac{(mg \sin 30^\circ)(0.1 \text{ m})}{\frac{1}{2} m (0.3 \text{ m})^2 + m (0.1 \text{ m})^2}$  *mass cancels*

$= \frac{5 \text{ m/s}^2 (0.1 \text{ m})}{0.55 \text{ m}^2}$

$= 9.1 / \text{s}^2$

$a = \alpha r = 0.91 \text{ m/s}^2$

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$\sum \tau_{\text{Pt of contact}} = I_{\text{parallel axis}} \alpha$

$F_{g\parallel} \cdot r = I_{PA} \alpha$

$\alpha = \frac{F_{g\parallel} \cdot r}{I_{PA}}$

3) The classic "notorious ladder problem": Please see the dedicated video, Tuesday of Week 11.

4) In class I give you two ways to find a coefficient of friction between the masses and the spinning turntable: by measuring how far the disks move horizontally as they fall, and by measuring the inclination when the disks slip. Please get the measurements in class and do both calculations and see if you get reasonably close answers for the two different ways. Dreadfully embarrassing, we never did this demonstration in class. I apologize, although I can't regret spending the class time as we have. In any case, here you go:

You know that freely rotating (no torque) platform that I can spin around on? I did two experiments with is allowing me to measure coefficient of friction between the surface and a metal mass:

- 1) I slowly raised one side of the turn table increasing the slope of the inclined plane of the surface. When the surface is inclined at an angle of about 30 degrees to the horizontal, the mass slid off the inclined surface.
- 2) Keeping the surface of the rotating table horizontal, I slowly increased the rotational velocity with the mass at a radius of 20 cm. At some point, the speed was great enough to cause the mass to slide off the rotating surface with a tangential velocity. The mass fell about 1 m to earth and I was able to measure that the mass landed 0.5 m away from where it left the rotating surface.

.... See if you can do these... Solutions below.

- 1) This is a simple dynamics (statics) problem where the coefficient of static friction is broken when the parallel component of gravitational force (pushing the mass down the incline) = the force of static friction. Remember that the normal force is not =  $mg$ , because we are in equilibrium in the perpendicular direction. You should get a coefficient of friction of about 0.6.
- 2) This is a glorious combination of kinematics (projectile motion) and (circular) dynamics. It requires two drawings, which I expect of you, but I am not supplying right now:

Using dynamics, make a good FBD and follow the protocol and you can see that the force of friction is the only radial force, so you set it equal to  $mass \cdot centripetal\ acceleration$ . This would provide us with the coefficient of friction, except that we don't know the speed... but we know how far it moves horizontally when it falls, so we go to the kinematics lens.

Please show the parabolic trajectory as the coin falls from the edge of the spinning turntable. We want the initial horizontal velocity =  $\Delta x / \Delta t$ .  $dx = 0.5\text{ m}$ , but how about  $\Delta t$ ? This is revealed in the vertical component because the coin is falling from rest (vertically speaking) and hits the ground. I could use energy to find  $v_{final}$  and then use  $v_{average} = \frac{1}{2}v_{final} = \Delta x / \Delta t$ . Or I could just use kinematics that  $\Delta x = \frac{1}{2}at^2$ . You should find that it takes about 0.45 s to fall 1 m from rest, so the initial horizontal speed is about 1.1 m/s.

Now we can go back to our circular dynamics lens and see that the centripetal acceleration at this speed at a radius of 20 cm is about  $6\text{ m/s}^2$ , requiring a frictional force of  $m \cdot 6\text{ m/s}^2$ . Knowing the vertical acceleration = 0, the normal force =  $mg$ ; yielding a coefficient of friction of 0.6.