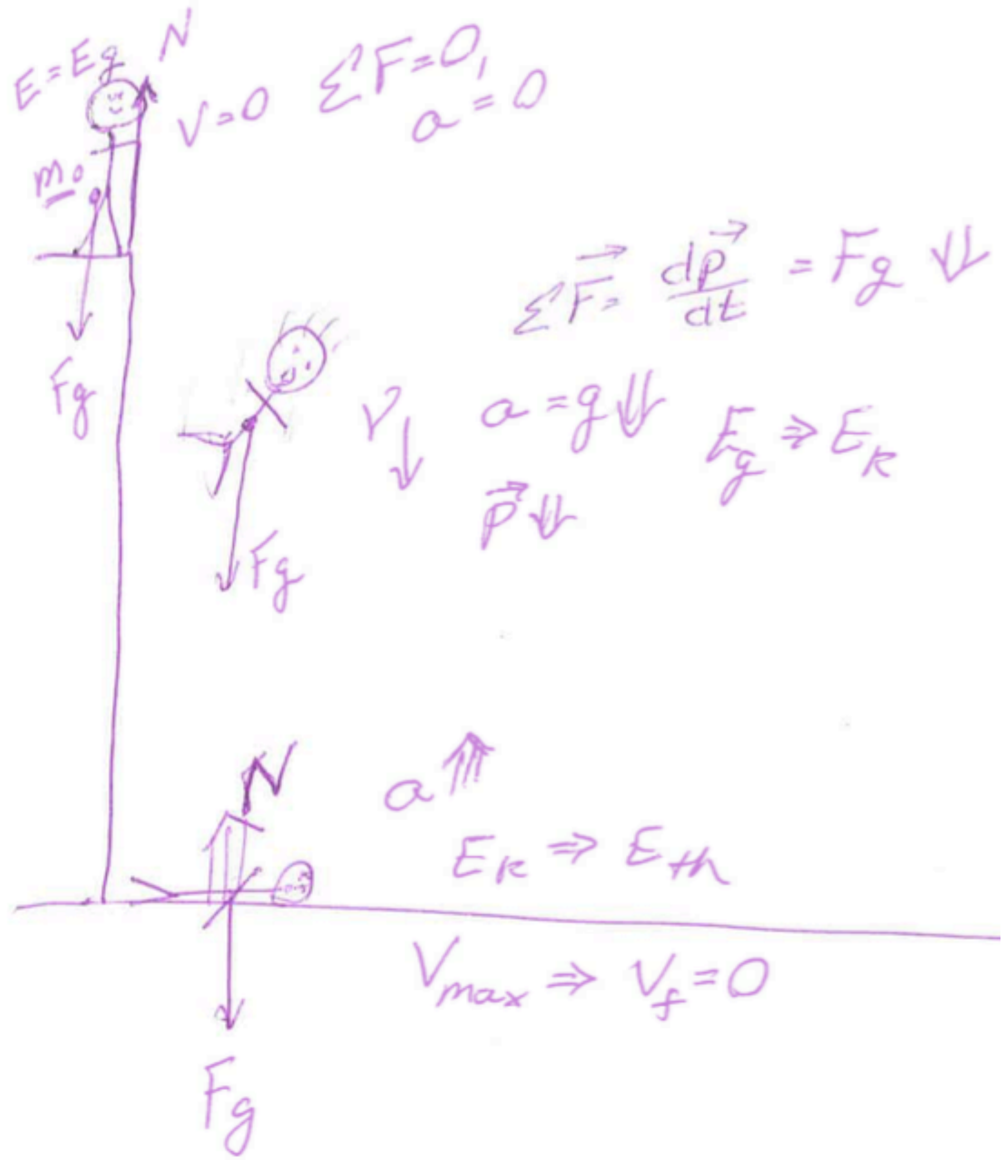
Problem Set #1 due beginning of class, Monday April 10

1. I inadvertently walk off a cliff. The process comes to a grim result 3 seconds later when I meet the ground. Please look at this process closely through all 4 lenses.
   I'm not going to allow the question to dictate how I look at this problem. I draw a picture as shown below and consider the lenses as I decide.



Energy Lens:

Gravitational potential energy transforms to kinetic energy and at the end is thermal energy in the slight increase in temperature to my body and rocks I land on.

Forces, Dynamics and Momentum:

Throughout the process, there is a force of gravity acting on me. We remember that when a net force acts on something, it accelerates (F=ma) and the momentum changes (F=dp/dt). However, at the beginning, there is a normal force provided by the ground, so there is no acceleration. After I step off the cliff, there is an "unbalanced force" of gravity because the normal force is gone, so my momentum increases due to this attractive force between me and the earth, and I accelerate. When I touch the ground, the normal force the earth provides is enough to prevent me from passing through the ground. There is huge acceleration as I come to rest (and die) in a very short time. Thus the normal force the earth provides must be very large to change my momentum so quickly.

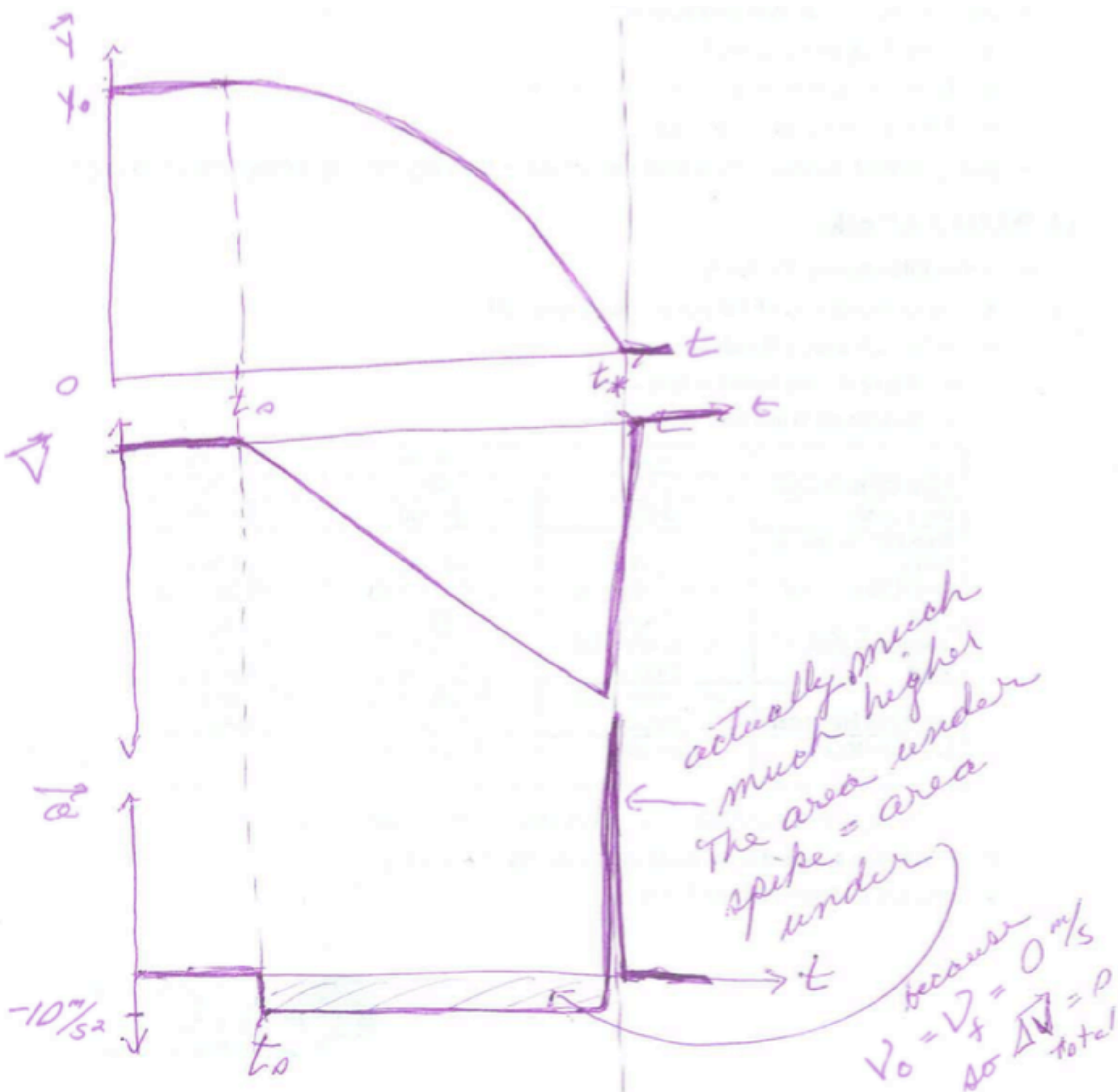
Momentum:

Momentum is conserved. I am at rest in the beginning and end, but there is lots of momentum downward when I'm falling. My momentum increases and decreases due to the unbalanced forces provided by the earth. Thus we must consider the earth as part of the system. Our momentum together must be constant. In the beginning, our momentum is zero. As I fall downward, the earth is falling upward (due to the force of gravity attracting us), thus the vector sum or our momenta is still zero. When I hit the ground we both stop with final system momentum of zero.

Kinematics:
The velocity and acceleration are zero until I step off the cliff. After that, my acceleration is ~ 10 m/s^2 downward. My downward velocity increases smoothly until I hit the ground, and then rapidly comes to zero when I hit.
The acceleration when I hit the ground must be very high, as this is the rate of change of velocity. Also, as I fall, there may be increasing wind resistance, so the increasing upward force it provides will slightly lower my acceleration, and therefore the velocity time graph may have a slope that drops slowly in time as the velocity increases.
The velocity is the slope of the position ⇔ time graph. Thus, the position starts out at the top and stays there until I step off. Then it drops at an increasing slope, until the end, when the velocity is again zero, so the position stays constant thereafter.

In the graphs below, it is easiest for me to start with the acceleration graph because I know the acceleration should be about -10 m/s^2 while I am falling. Because wind resistance increases, my acceleration may drop a little at higher speeds… of course for sky divers, the wind resistance is enough to bring their acceleration to zero at over 50 m/s, so after about 5 seconds, they fall at almost constant velocity. We know that dv = a*dt, which is the area under the a⇔t graph. Because $v_o = v_f = 0$, the (negative) area under the curve while speeding up under gravitational acceleration must equal the (positive) area of slowing down under the spike. Thus the spike must be very high owing to the large acceleration and force that kills me at the end of the fall.

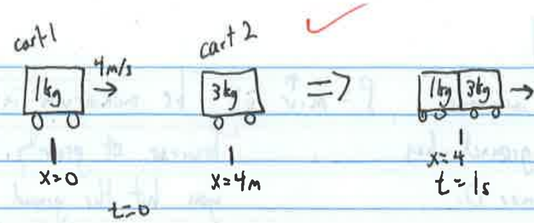
Then I can draw the velocity graph, with a constantly decreasing slope until I hit and come to rest.
Then I can draw the position ⇔ time graph.



2. A 1 kg cart moving at 4 m/s hits a 3 kg cart at rest. The two carts stick together. The 1 kg mass started at t = 0 at x = 0, moving in the positive x direction, and the 3 kg mass started at rest at x = 4 m.

Perfect #2

2



a) $KE = \frac{1}{2}mv^2 \quad \frac{1}{2}(1kg)(4\,m/s)^2 = 8\,kg\,m^2/s^2$

Total kinetic energy of the system before the collision is 8 J.

b) $P_0 = m_1 v_1 \quad (1kg)(4\,m/s) = 4\,kg\,m/s$ The total momentum of the system before the collision is $4\,kg\,m/s$.

c) $P_0 = P_1 \quad (1kg)(4\,m/s) = (4\,kg)(v_2) \quad v_2 = 1\,m/s$

The final speed of the two masses is 1 m/s

d) $KE = \frac{1}{2}mv^2 \quad \frac{1}{2}(4\,kg)(1\,m/s)^2 = 2\,kg\,m^2/s^2$

$P = mv \quad (4\,kg)(1\,m/s) = 4\,kg\,m/s$

Final kinetic energy is 2 J

Final momentum is $4\,kg\,m/s$

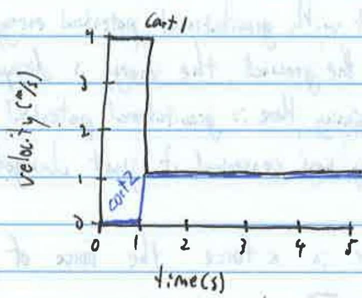
e)

Cart $1_i = 4\,kg\,m/s$ Cart $2_i = 0\,kg\,m/s$

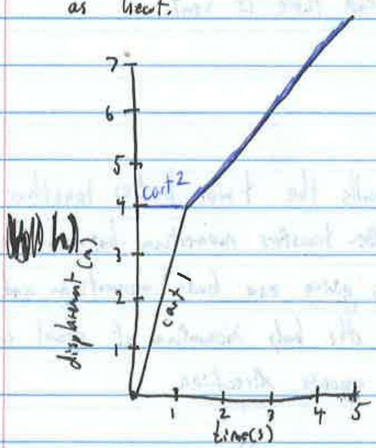
Cart $1_f = 1\,kg\,m/s$ Cart $2_f = 3\,kg\,m/s$

f) Yes, 6 J of energy were liberated as heat.

g)



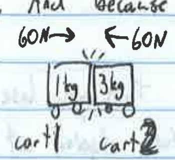
h)



Using Dynamics lens because Forces cause accelerations.

i) $F = ma \quad a = \frac{dv}{dt} \quad \frac{1\,m/s}{0.05\,s} = 20\,m/s^2 \quad (3\,kg)(20\,m/s^2) = 60\,kg\,m/s^2$ The average magnitude of the force was 60 N

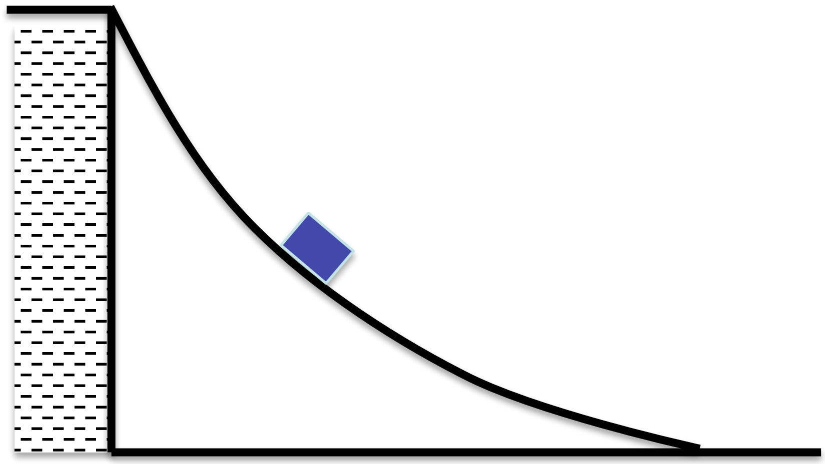
j) The lense I will use is force, because we know the force on one, so we know the force is equal and opposite on the other. And because we know both masses we can calculate the acceleration on each.



$60N = (1kg)(a_1)$ $\quad$ $60N = (3kg)(a_2)$

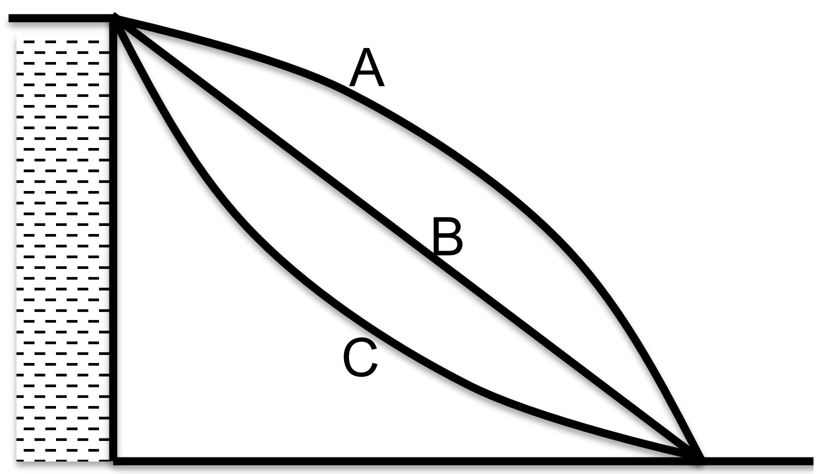
Acceleration Cart 1 = $\overleftarrow{60}\,m/s^2$ $\quad$ Acceleration Cart 2 = $\overrightarrow{20}\,m/s^2$

3 Imagine a 5 kg box sliding down a frictionless curved track at the edge of a 60 m high cliff as shown at right. We would like to know how fast it's going at the bottom. Neglect air friction.



a) Describe using each of the four lenses, what is happening in this process.
   <u>Momentum:</u> the block is exchanging momentum with the earth: They both start out at rest, and the two of them maintain equal and opposite momenta.
   <u>Energy:</u> See below
   <u>Forces:</u> The force of gravity and the normal force act on the block. This causes an acceleration of the block. The problem here is that the force on the block changes as the block moves down the slope according to the slope of the surface. As the line levels off, so does the acceleration of the block. We don't know the slope, and the slope is always changing.
   <u>Kinematics:</u> The acceleration along the slope causes the block's speed to increase, so the displacement of the block continues along at a rate that is increasing. This will prove to be an unhelpful lens because we don't havce any time dependent information about the block's position, speed, or acceleration.
b) Which lens is the most helpful to find the final speed of the block at the end? Most useful one is energy because energy is conserved. Potential energy is changed to kinetic energy
c) Please find out the speed at the bottom of the track. Conserving energy, I get $v_f = 34.6$ m/s

Now imagine that there are two other tracks that the box could use as shown at right, bottom.



d) Which track should we use for the fastest final speed, or would all three tracks yield the same final speed? Which lens do you look at this problem through? Please explain your answer. Energy lens shows us that all three lose the same amount of potential, and thus gain the same amount of kinetic, so same final speed.
e) How about if we wanted to know which was going the fastest *half way* down the total length of its path? C, by using the same conservation of energy logic above.
f) If three identical frictionless boxes were released at the top of each track, which would get to the bottom first, or would it be the same for all of them? Please explain your answer in terms of which lens you used. This we can use the energy lens and the kinematics lens. The box on C is always below the bock on A, so it will move faster, dropping even further, going that much faster than A. C gets there first, then B, then A.

4) See solutions for Big Exam #1