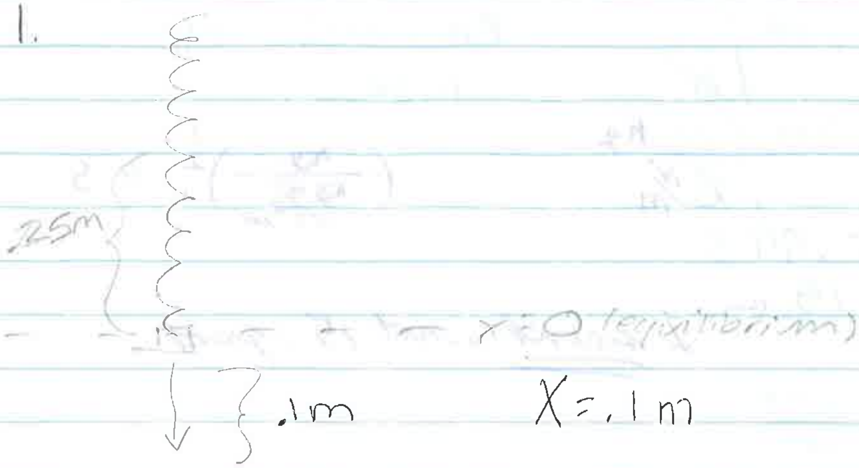
Problem set 1

1.



$X = .1\,m$

3) dynamics: no acceleration at equilibrium point = $\Sigma F = 0$

$F_g = mg$

$= (10\,m/s^2)(.2\,kg)$

$= 2\,N$

good start

$F_s = F_g \qquad \Sigma F = 0 \qquad \Sigma F = F_g + F_s$

$F_s = 2N$

$F_s = KX$

$\frac{2}{.25} = K$

$\boxed{K = 8\,N/m}$

great

1) $PE_{peak} = KE_{equilibrium}$

energy: potential energy from spring converted to kinetic energy at equilibrium point

$PE = \frac{1}{2}KX^2$

$= \frac{1}{2}(8\,N/m)(.1\,m)^2$

$= .04\,J$

$KE = \frac{1}{2}mv^2$

$V^2 = \frac{(.04\,J)(2)}{.2\,kg}$

$\left(\frac{m^2}{s^2}\right)^{\frac{1}{2}}$

$\boxed{V = .632\,m/s}$

2) $E_{tot} = PE + KE$

$.04\,J = \frac{1}{2}(8)(.05\,m)^2 + \frac{1}{2}(.2)v^2$

$\boxed{V = .548\,m/s}$

✓

4) $T = 2\pi\sqrt{\frac{m}{K}}$ Kinematics: motion based on time

$= 2\pi\sqrt{\frac{.2\ kg}{8\ N/m}}$ $\left(\frac{kg}{kg\,m/s^2\,m}\right)^{\frac{1}{2}} \Rightarrow s$

$= .993$

$\boxed{T \approx 1s}$ ← 1 second, not 15, great

5) $W_0 = \frac{2\pi}{T}$

$= \frac{2\pi}{1}$

$\boxed{W_0 = \frac{6.28}{s}}$

(6) $\Sigma F = ma$

$a = \frac{m}{\Sigma F}$

$\Sigma F_{(low)} = F_T + F_g$

$F_T = KX$

$= (8\ N/m)(.35m)$

$= 2.8\ N$

$F_g = 2N$

$\Sigma F = 2.8 - 2 = .8N \uparrow$

$\Sigma F = ma$

$a = \frac{.8}{.2} = \boxed{4\ m/s^2 \uparrow}$

$\Sigma F_{(high\ point)} = F_{Tension\ spring} + F_g$

$F_T = KX$

$= (8\ N/m)(.15m)$

$= 1.2N$

$F_g = mg$

$= 2N$

$\Sigma F = 1.2 - 2 = -.8N$

$.8N \downarrow$

$\Sigma F = ma$

$a = \frac{-.8}{.2} = \boxed{4\ m/s^2 \downarrow}$

↗ lots of different ways to do this

$a = \ddot{y} = A\omega^2 \cos(\omega t + \phi)$

max $= 10cm \cdot (6.28/s)^2$

$= 4\ m/s^2$

7) energy stored = sum of kinetic and potential energy : work done by stretching spring

$PE = \frac{1}{2}Kx^2$
$= \frac{1}{2}(8 N/m)(.1 m)^2$
$= .04 J$

← This is the energy of oscillation.

~~PE from weight = $\frac{1}{2}(8 N/m)(.25)^2$~~
~~$= .25 J$~~
~~$PE = .04 + .25 =$ .29 J~~

8) Kinematics : motion based on time

$x(t) = X\cos\frac{2\pi(t)}{T}$ → but $x(t=0) = -10 cm$

$T = 2\pi\sqrt{\frac{m}{K}}$
$= 2\pi\sqrt{\frac{.2 kg}{8}}$
$= 2\pi\sqrt{.025}$
$= .99$
≈ 1

$X = .1$

$x(t) = \cos\left[\frac{2\pi}{5}(t) + \pi\right]$ ← ϕ

or $= -\cos\left[\frac{2\pi}{5}t\right]$ or $\sin\left(\frac{2\pi}{5}t + \frac{3\pi}{2}\right)$

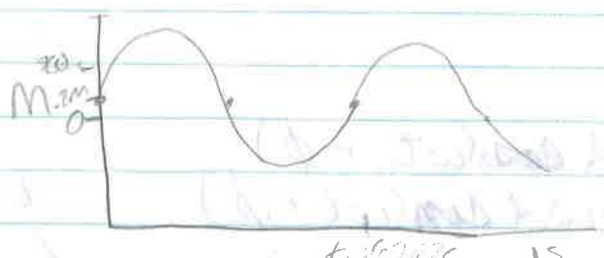
great → Kinematics: motion based on time - displacement and its time derivatives

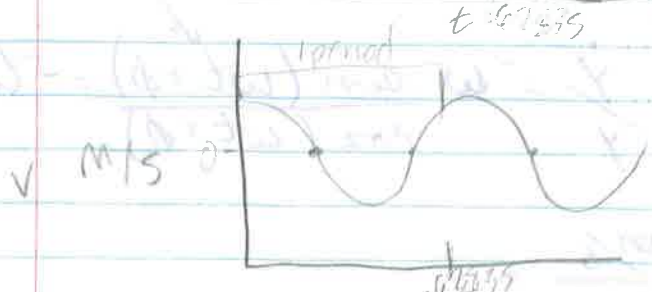
2) $w = \frac{10}{s}$ $X(0) = 20 cm = .02 m$ $V(0) = 2 m/s$

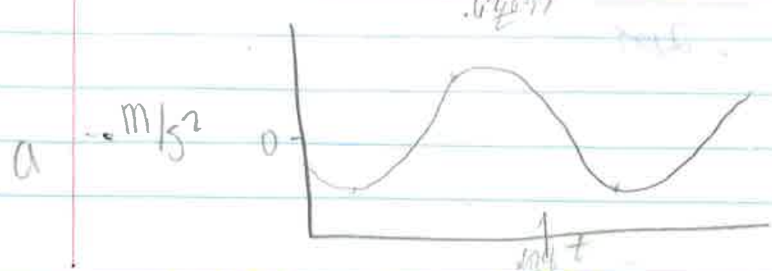
$T = \frac{2\pi}{w} = \frac{\pi}{5} = .6283 s$

$x(t) = X\cos\frac{2\pi t}{.6283}$

~~$V = $~~ ~~$\frac{2}{10}$~~ $= .2 m$







nice!

I notice that $\vec{a}$ looks like $\vec{x}$ upside down!

$\omega = \sqrt{\frac{k}{m}}$ $\omega^2 = \frac{k}{m}$ $k = m\omega^2 = ?$

oops!

c) $E_{tot} = PE + KE$

$KE_{(I)} = \frac{1}{2} m v^2$
$= \frac{m}{2} (4 m^2/s)$
$= 2m$ (J)

$PE_I = \frac{1}{2} k x^2 = \frac{1}{2} k x^2$

you can't find the energy of the system because you don't know the spring constant or the mass!

b) the cosine and sine functions would be shifted to the right if the system was in this state at $t = .2s$

2 There was a mistake in the way this question was asked:

We know that the maximum speed for an oscillator is this value. for our

$$\left[\dot{y} = -A\omega \sin(\omega t + \phi)\right]$$

question, $A\omega = V_{max} = 2\ m/s$, but that's the speed I gave you for the initial conditions when the initial displacement $\neq 0$! We know we have V_{max} when $PE = 0$, $Y = 0$ only, so let's solve a feasible real system where

$Y_{(t=0)} = 0.2m$, $\dot{Y}_{(t=0)} = 1.0\ m/s$ $\qquad T = \frac{2\pi}{\omega} \approx 0.63\ s$

$$y = A\cos(\omega t + \phi)$$
$$\dot{y} = -A\omega \sin(\omega t + \phi)$$

$$\frac{\dot{Y}}{Y} = -\omega \tan(\omega t + \phi)$$

@ $t = 0$

$$\frac{1.0\ m/s}{.2m} = -10/s \tan(\phi)$$

$$\therefore -0.5 = \tan\phi$$

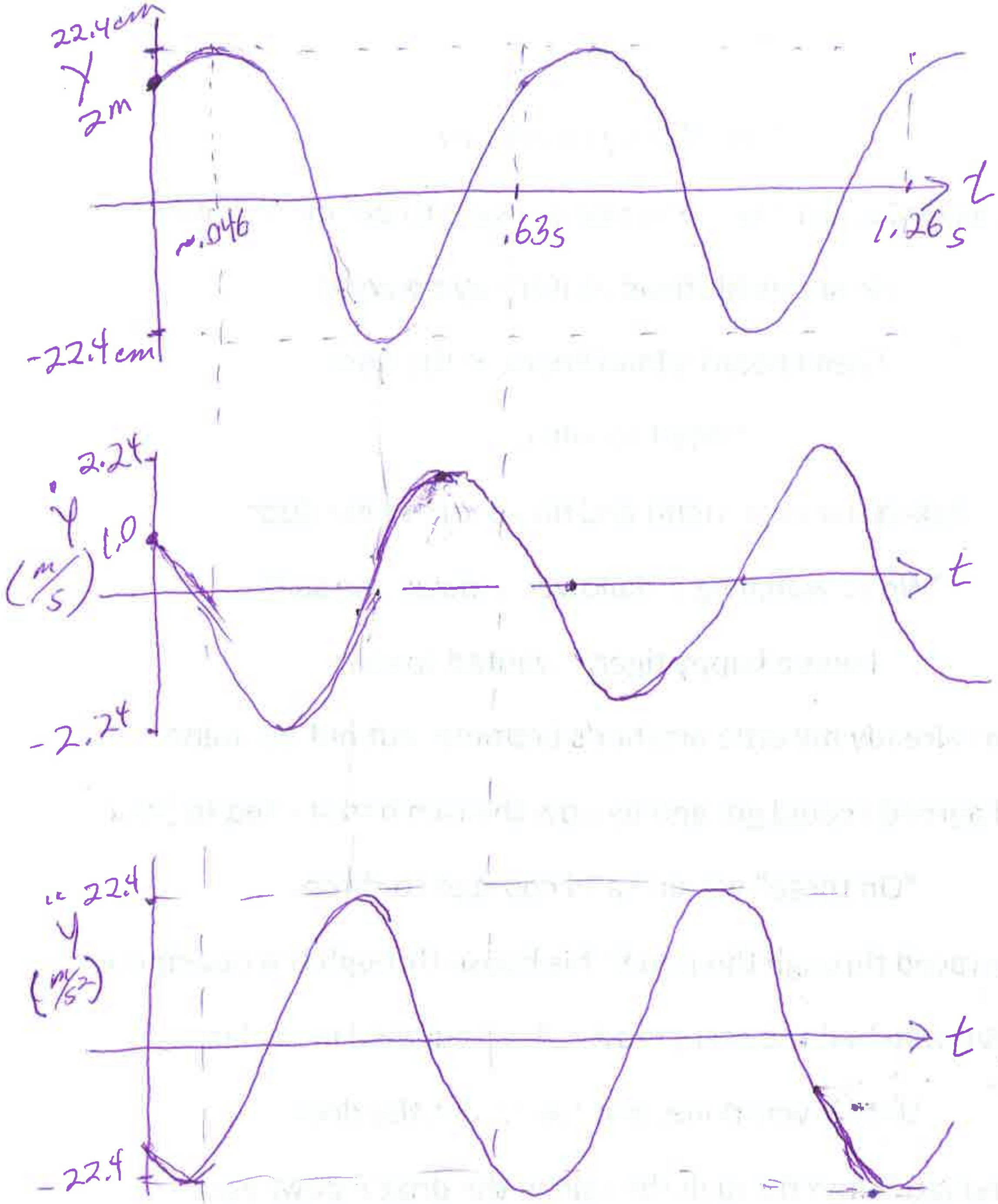
$$\phi = \tan^{-1}\left(\frac{-1}{2}\right) = -0.46\ \text{rad} \approx 27^\circ$$

$Y_{(t=0)} = 0.2m = A\cos(-0.46)$

$$A = \frac{0.2m}{\cos(0.46)} = 22.4\text{cm}$$

$A\omega = 2.24\ m/s = \dot{Y}_{max}$

$A\omega^2 = 22.4\ m/s^2 = \ddot{Y}_{max}$



IF instead of saying these initial conditions were @ t=0, we said they are at t=.2s, we would substitute t=0.2s into the equations of motion, which shift everything 0.2 s to the right on the time axis.