

Problem Set #3 due beginning of class, Tuesday April 24. Please state the lens you are using and why. Remember that you are graded on your communication of physics understanding.

1. *not a true story. You have a mass of 50 kg. On your trip to Dubai, you visit the tallest building in the world\*\* and select the “extreme” elevator. You bring a bathroom scale and stand on it. This is a dynamics lens because forces ( $F_g$  and  $N$ ) cause acceleration. Write  $\sum \vec{F} = m\vec{a}$ , identify the body (my body), the forces, and the direction of acceleration in a FBD.*
- a) You test the scale in the lobby by standing on it. What does it read? Why do you know it reads that? **Show that if  $a = 0$ , then the forces are = and opposite.**
- b) You **hear** that the acceleration on the elevator is  $15 \text{ m/s}^2$ . If this is the case, what should the scale read as the elevator begins its ascent? **Again, please follow the full protocol and show that I feel very “heavy” as the scale provides 1250 N of normal force upward on my body.**
- c) Near the end of the ascent (just before you come to your destination) you find yourself standing on the ceiling of the elevator, upside down, on your scale (scale against the ceiling), which now reads 300 N. Never mind how I got into this position, what must be my present acceleration? **Again, please follow the full protocol and show that the full force on my body is 800 N downward, corresponding to an acceleration of  $16 \text{ m/s}^2$ .**

① a) dynamics lens,  $a=0$ , standing still on flat ground  $\sum \vec{F} = m\vec{a}$   $m=50\text{kg}$

$$\begin{aligned}\sum \vec{F} &= m\vec{a} \\ \sum \vec{F} &= N + F_g \\ N &= -F_g \\ &= -(mg) \\ &= -(50\text{kg}) \cdot (10\text{m/s}^2) \\ N &= 500\text{N}\end{aligned}$$

We know this because the scale reflects what would be the Normal force ( $N$ ), and because you are in equilibrium  $N$  is equal and opposite to  $F_g$ .

b) dynamics lens,  $a=15$ , and forces cause acceleration  $m=50\text{kg}$   $a=15\text{m/s}^2$

$$\begin{aligned}\sum \vec{F} &= m\vec{a} \\ \sum \vec{F} &= N + F_g \\ N &= \sum \vec{F} + F_g \\ &= Ma + Mg \\ &= 50\text{kg} \cdot 15\text{m/s}^2 + 50\text{kg} \cdot 10\text{m/s}^2 \\ N &= 1250\text{N}\end{aligned}$$

Great!

c) Dynamics lens,  $a$  causes forces  $\Rightarrow \sum \vec{F} = m\vec{a}$   $m=50\text{kg}$   $a=?$   $N=300\text{N}$

$$\begin{aligned}\sum \vec{F} &= m\vec{a} \\ N + F_g &= ma \\ N - F_g &= ma \\ 300\text{N} - 500\text{N} &= 50\text{kg} \cdot a \\ -200\text{N} &= 50\text{kg} \cdot a \\ a &= -4\text{m/s}^2\end{aligned}$$

\*\* [https://en.wikipedia.org/wiki/Burj\\_Khalifa](https://en.wikipedia.org/wiki/Burj_Khalifa)

2. From an old midterm. Even if you've never heard of fusion, you have the basic skills to draw a picture and analyze this problem. Fusion is the process that powers the sun and hydrogen bombs: small nuclei are fused into larger nuclei. One fusion process involves a triton (two neutrons and a proton – recall that neutrons and protons have about the same mass) and a deuteron (one neutron and a proton) fusing to form a supercharged 5-nucleon nucleus, which gives off its energy by blasting apart into a single neutron and a helium nucleus (or alpha particle) at high speeds. I want to know which of the particles gets more of the energy. Let's simplify the problem to just the explosive breakup: Protons and neutrons have the same mass, so we can think of this process as **a 5-ball cluster (in space, at rest) breaking up into one ball and a 4-ball cluster. Do the two pieces equally share the kinetic energy or does one get all or more kinetic energy?** You will be graded not on your answer, but on your reasons, drawings, and lens descriptions.

*Looking at this through an energy lens, we see that some kind of nuclear energy is transformed into kinetic energy. However, we don't know how much nuclear energy we started with, nor how the two pieces share that energy. But we know that there's no external forces, and we know it's at rest to begin with, so we can use a momentum lens and see that the final momentum = initial momentum = 0. Hence the two clusters must have equal and opposite momenta in order to have a sum momenta of zero. From there, please show that the single neutron must have 4 times the speed as the 4-ball cluster, in comparing the kinetic energies, the neutron should take 80% of the energy of the explosion, or 4 times that of the larger cluster.*

### Problem 2

before



$5M$

$V=0$

after



$M$

$V=?$

$4M$

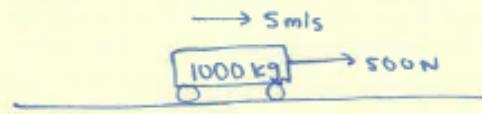
$V=?$

There appears to be a collision of sorts, so let's use the momentum lens for this problem. The two masses move in opposite directions, and their starting momentum was zero, so their momentums must cancel out. If the momentum of the  $4M$  mass was  $p = 4Mv$ , the smaller ball with mass  $m$  must have a momentum of  $p = -4Mv$  to cancel out. The velocity of the single particle  $m$  must be  $-4v$ , so the single particle gets more kinetic energy than the other 4 particles, by a factor of 4.

3. Exercise 6 in 2.4, Vectors pushing cars I would use an energy lens because my work turns the kinetic energy of the car to heat... or I am doing negative work on the car to be subtracted from its kinetic energy. I do  $-500\text{J}$  of work on the car, so the final kinetic energy of the car is  $7,500\text{ J}$ , corresponding to a speed of square root of  $15\text{ m/s}$  or just shy of  $4\text{ m/s}$ ...  $\sim 3.9\text{ m/s}$

3. Ex 6 in 2.4

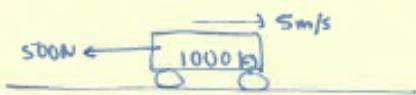
$$V_f = ?$$



Lens: Energy Lens because work is being done on the car

$$W = F \cdot \Delta x = \Delta E_{K, \text{car}}$$

$$E_{K, \text{car}} = W + E_{K_0}$$



$$\frac{1}{2}mv_f^2 = (F \cdot \Delta x) + (\frac{1}{2}mv_0^2)$$

$$500v_f^2 = (500)(20) + 500(25)$$

$$\frac{1}{2}mv_f^2 = (F \cdot \Delta x) + \frac{1}{2}mv_0^2$$

$$500v_f^2 = (500)(-10) + 500(25)$$

$$v_f = \sqrt{25 - 10}$$

$$v_f = 3.87 \text{ m/s} \approx 4 \text{ m/s}$$

$$v_f = \sqrt{20 + 25}$$

$$v_f = 3\sqrt{5} \text{ m/s} \approx$$

$$v_f = 6.71 \text{ m/s} \approx 7 \text{ m/s}$$

4. Exercise 5 in 2.7, potential energy graph.

7. Bt 4) Graph of PE as function of displacement. M = 2 kg. (pos. x is to the right).

Mass starts at x=0 moving @ 2 m/s to the left. (List all correct ans.)

a) Label stable equilibrium w/ "S"

b) Label unstable equilibrium w/ "U"

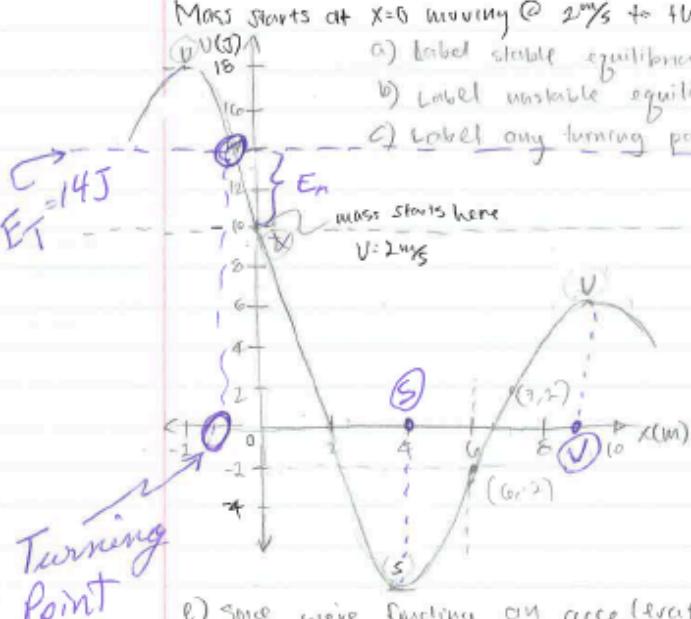
c) Label any turning point w/ "T"  $\rightarrow E_{P0} = 10 \text{ J}$

d) Where does block attain highest

speed? & what is that speed?

I would use an energy lens for this since it involves a transformation of energy. The block has highest speed where PE is the lowest & KE is the highest; at 4m.

calculate...



e) Since we're finding an acceleration, I would use a dynamics lens since I would need to consider a force that causes that acceleration. I would also use an energy lens since I can also consider that force is the negative gradient of potential energy;  $F = -\frac{dE}{dx}$ . So,

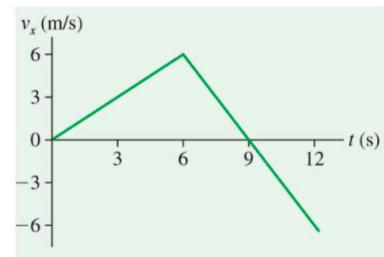
$$\text{the force } x=6 \text{ m, the force is approximately } \vec{F} = \left[ \frac{2 \text{ J} - (-2 \text{ J})}{7 \text{ m} - 6 \text{ m}} \right] = -2 \frac{\text{J}}{\text{m}} = -4 \text{ N.}$$

Since  $\vec{F} = m\vec{a}$ , then;  $\vec{a} = \frac{\vec{F}}{m} = \frac{-4 \text{ N}}{2 \text{ kg}} = -2 \frac{\text{m/s}^2}{\text{kg}} = \boxed{-2 \frac{\text{m/s}^2}{\text{kg}}}$  in the negative direction. nice!

5. Please see the velocity time graph at right for an object that starts at  $x = -10$  m. Please:

- Write a narrative – what is happening in the graph.
- Make an acceleration vs time graph.
- Make a position vs time graph.

This is all kinematics because I have motion (velocity) as an explicit function of time, without considering the causes.



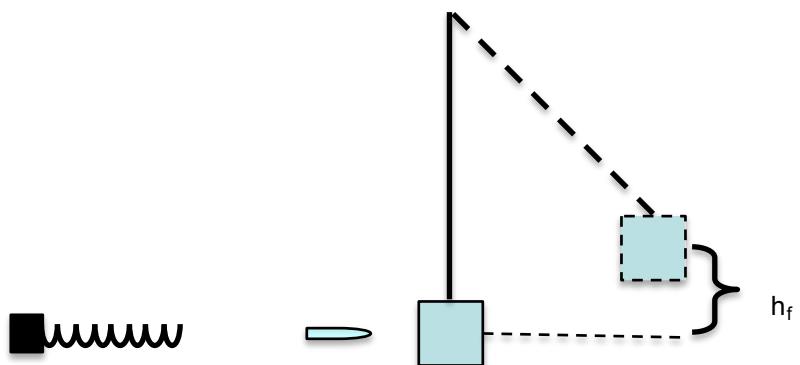
a) Starting from rest, I increase in speed with constant acceleration (slope of v-t graph) for 6 seconds. From 6 to 9 seconds, I continue forward to slow down until, at 9 seconds I change directions and start speeding up in the opposite (negative) direction.

b) Acceleration is the slope of the v-t graph. The acceleration is  $+1 \text{ m/s}^2$  for the first 6 seconds, and  $-2 \text{ m/s}^2$  after 6 seconds.

c) velocity (the values of the graph) is the slope of the x-t graph, so the x-t graph is flat at 0s and 9s. The slope increase for the first 6 seconds and decreases to flat between 6 and 9 s. After 9 seconds, the displacement curves increasingly downward. Also, the displacement is the area under the v-t curve, so if we start at  $-10$  m, then at 3 seconds we've moved  $+4.5$  m to  $-5.5$  m, at 6 s, we are at  $+8$  meters, at 9s, we are at  $-17$  m, and at 12 seconds, we are back at 8 meters again.

6. A loaded gun is cocked by compressing a spring of  $k = 10^4 \text{ N/m}$ . and then releasing it behind a  $20 \text{ g}$  bullet. The bullet strikes and sticks inside of a  $0.5 \text{ kg}$  ballistics pendulum and swings upward to a final height of  $50 \text{ cm}$ . Presume the spring is massless and there is no friction in the system. Please find:

- The bullet's speed.
- how far the spring was compressed.
- Does the bullet have constant acceleration in the gun, or does the acceleration change over time? Please explain your answer... identify a lens.
- Please find the maximum acceleration of the bullet in the gun.
- Did you identify the lenses at the very beginning, or one at a time for each question? Which do you think would be a better approach?



As soon as we see this, we are tempted to use an energy lens equating the initial spring potential energy to the final gravitational potential energy. However, the great majority of the bullet's kinetic energy is converted to thermal energy in the inelastic collision. Thus, we can find the kinetic energy of the bullet/block immediately after collision using an energy lens. However, to find the bullet's speed, we need to use the momentum lens because there is negligible outside forces so the momentum is conserved in the collision. The bullet's kinetic energy does come from the spring potential energy.

For letter "c" and "d", constant acceleration would be the result of a constant force. However, the force of the spring is proportional to the spring's compression. This the maximum acceleration would be when the spring is maximally compressed. This acceleration comes out to be 6000 times the acceleration of gravity, but so is the life a bullet! In fact, this acceleration is small compared to the acceleration the bullet experiences when it hits the target!

b). Lens: Energy lens because energy is being converted from potential to kinetic and momentum

$$E_{\text{spring}} \rightarrow KE_{\text{bullet}} \rightarrow E_{\text{therm}} + KE_{\text{block}} \rightarrow PE_g$$

$$\frac{1}{2} Kx^2 \rightarrow \frac{1}{2} mv^2 \rightarrow mg h$$

$$\frac{1}{2} (10000 \text{ N})^2 \rightarrow \frac{1}{2} (.02 \text{ kg})v^2 \rightarrow (.5 \text{ kg})(.5 \text{ m})(10 \text{ m/s}^2) h_g = .5 \text{ m}$$

inelastic collision of bullet / box

$$m_b v_b = m_{\text{box}} v_{\text{box}}$$

$$(.02 \text{ kg})v_b = (.005 \cdot .52 \text{ kg})(3.16 \text{ m/s})$$

$$v_{\text{bullet}} = \frac{(.52 \text{ kg})(3.16 \text{ m/s})}{(.02 \text{ kg})} = 82.16 \text{ m/s} \quad \boxed{a}$$

$$\frac{1}{2} (10000 \text{ N})^2 = \frac{1}{2} (.02 \text{ kg})(82.16 \text{ m/s})^2$$

$$x = \frac{(.02 \text{ kg})(82.16 \text{ m/s})^2}{10000 \text{ N}} = 0.12 \text{ m} \quad \boxed{b}$$

*lens? P because  $\frac{1}{2} mv^2$*

*Footnote = 0*

*$m_b v_b = m_{\text{box}} v_{\text{box}}$*

*$(.52 \text{ kg})v^2 = 2.6 \text{ J}$*

*$v = \sqrt{\frac{5.2 \text{ J}}{.52 \text{ kg}}}$*

*$v = 3.16 \text{ m/s}$*

*box*

*Energy lens, because*

*$E_s \Rightarrow E_k$*

*$\frac{1}{2} Kx^2 \rightarrow \frac{1}{2} mv^2$*

*$(\frac{1}{2} \cdot \frac{m^2}{s^2}) = (m^2)^{\frac{1}{2}} = m$*

c) The force of the bullet comes from the force of the spring ( $F = kx$ ). To have a constant acceleration, there must be a constant force. The force of the spring is not constant because as the displacement of compression changes, the spring reaches equilibrium, the  $F_{\text{spring}}$  decreases. Thus the bullet does not have constant  $a$ .

Please include units! *Mile!*

d)  $F_s = F_a = ma$

$$\frac{1}{2} (10000 \text{ N}) \cdot (1.2 \text{ m}) = (.02 \text{ kg}) a \quad a = 58000 \text{ m/s}^2$$

$$F_s = kx = 10000 \text{ N} \cdot 0.12 \text{ m} \approx 1200 \text{ N}$$

e) Lenses are the way to go!

$$a = \frac{F}{m} = \frac{1200 \text{ N}}{0.02 \text{ kg}} = 60000 \text{ m/s}^2$$

$$= 6 \times 10^4 \text{ m/s}^2 = 6000 \text{ gravities!}$$

7. Using an energy lens, please show that if you drop a 5 kg box from 60 m, it hits the ground at ~35 m/s. But then, you throw the box downward from 60 meters height with an initial speed of 35 m/s.

- Find the speed that it has when it hits the ground.
- What if I throw it upwards at 35 m/s, what is the speed when it hits the ground?
- What if I throw it straight off the cliff at 35 m/s horizontally, what speed does it have when it hits the ground now?
- Can I throw a 5 kg box at 35 m/s? Please back up your answer.

I solve this in a video "throwing box off cliff" for Tuesday, April 24.

8. According to the hydrodynamic flow equations you'll learn in PHYS-132, the speed of water coming from a 200 PSI fire house is about 45 m/s (~100 mph!). Wikipedia claims these hoses are 25 mm in diameter. Imagine if you were hit with water by one of these hoses, like if you were protesting the Dakota Access Pipeline, and the fire department was called to clear the area (please see some drama: <https://www.youtube.com/watch?v=K3Iv9okL4QU>). I'd like to know the force that this water puts on someone's body. Let's model the water as a moving column that hits you and disperses all directions perpendicular to its original direction of travel, as in the figure of the demonstrator at right.



- a) Clearly map out why this problem should be solved with conservation of momentum.
- b) What is the volume, mass and momentum of a 1-meter column of water *before* it hits your body?
- c) What is the momentum of water *after* it hits your body?
- d) How long did it take the water to change momentum?
- e) Find the force that this water puts on your body. Could it knock you over?

I solved this in the firehose video last Thursday of week 3.