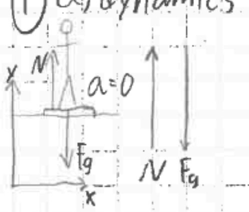
Problem Set #3 due beginning of class, Monday Jan. 29. Please state the lens you are using and why. Remember that you are graded on your ***communication*** of physics understanding.

1. *not a true story. You have a mass of 50 kg.* On your trip to Dubai, you visit the tallest building in the world** and select the "extreme" elevator. You bring a bathroom scale and stand on it. *This is a dynamics lens because forces (F_g and N) cause acceleration. Write $\sum\vec{F} = m\vec{a}$, identify the body (my body), the forces, and the direction of acceleration in a FBD.*
   a) You test the scale in the lobby by standing on it. What does it read? Why do you know it reads that? Show that if a = 0, then the forces are = and opposite.
   b) You ***hear*** that the acceleration on the elevator is 15 m/s². If this is the case, what should the scale read as the elevator begins its ascent? Again, please follow the full protocol and show that I feel very "heavy" as the scale provides 1250 N of normal force upward on my body.
   c) Near the end of the ascent (just before you come to your destination) you find yourself standing on the ceiling of the elevator, upside down, on your scale (scale against the ceiling), which now reads 300 N. Never mind how I got into this position, what must be my present acceleration? Again, please follow the full protocol and show that the full force on my body is 800 N downward, corresponding to an acceleration of 16 m/s².

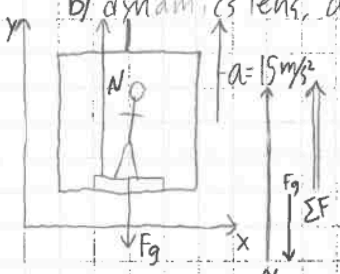
① a) dynamics lens, a=0, standing still on flat ground $\Sigma\vec{F}=m\vec{a}$ m=50kg



$\Sigma\vec{F}=m\vec{a}$ ($\vec{a}\to 0$)
$\Sigma F = N + F_g$ ($\Sigma F \to 0$)
$N = -F_g$
$= -(mg)$
$= -(50kg \cdot -10 m/s^2)$
$N = \boxed{500N}$ ✓

We know this because the scale reflects what would be the Normal force (N), and because you are in equilibrium N is equal and opposite to Fg.

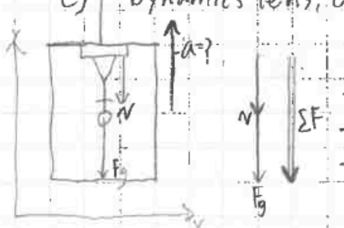
b) dynamics lens, a=15, and forces cause acceleration m=50kg a=15m/s²



$\Sigma\vec{F}=m\vec{a}$
$\Sigma F = N + F_g$
$N = \Sigma F + F_g$
$= ma + mg$
$= 50kg \cdot 15 m/s^2 + 50kg \cdot 10 m/s^2$ ✓
$= \boxed{1250N}$

Great!

c) Dynamics lens, a causes forces $\Rightarrow \Sigma\vec{F}=m\vec{a}$ m=50kg a=? N=300N



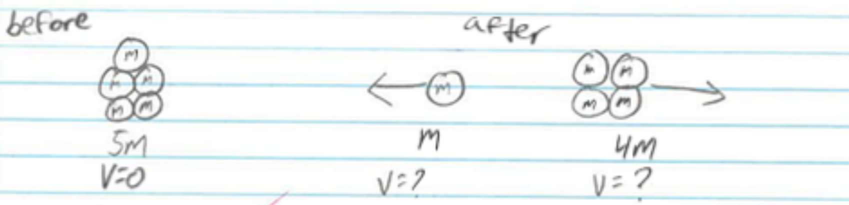
$\Sigma\vec{F}=m\vec{a}$
$N + F_g = ma$
$-N + -F_g = ma$
$-300N - 500N = 50kg \cdot a$
$-800N = 50kg \cdot a$
$a = -16 m/s^2$

$\boxed{a = -16 m/s^2}$ ✓

2. *From an old midterm. Even if you've never heard of fusion, you have the basic skills to draw a picture and analyze this problem.* Fusion is the process that powers the sun and hydrogen bombs: small nuclei are fused into larger nuclei. One fusion process involves a triton (two neutrons and a proton – recall that neutrons and protons have about the same mass) and a deuteron (one neutron and a proton) fusing to form a supercharged 5-nucleon nucleus, which gives off its energy by blasting apart into a single neutron and a helium nucleus (or alpha particle) at high speeds. I want to know which of the particles gets more of the energy. Let's simplify the problem to just the explosive breakup: Protons and neutrons have the same mass, so we can think of this process as <u>**a 5-ball cluster (in space, at rest) breaking up into one ball and a 4-ball cluster. Do the two pieces equally share the kinetic energy or does one get all or more kinetic energy?**</u> You will be graded not on your answer, but on your reasons, drawings, and lens descriptions.

Looking at this through an energy lens, we see that some kind of nuclear energy is transformed into kinetic energy. However, we don't know how much nuclear energy we started with, nor how the two pieces share that energy. But we know that there's no external forces, and we know it's at rest to begin with, so we can use a momentum lens and see that the final momentum = initial momentum = 0. Hence the two clusters must have equal and opposite momenta in order to have a sum momenta of zero. From there, please show that the single neutron must have 4 times the speed as the 4-ball cluster, in comparing the kinetic energies, the neutron should take 80% of the energy of the explosion, or 4 times that of the larger cluster.

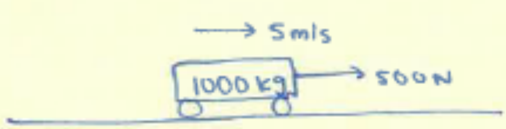
Problem 2



There appears to be a collision of sorts, so let's use the momentum lens for this problem. The two masses move in opposite directions, and their starting momentum was zero, so their momentums must cancel out. If the momentum of the 4m mass was $p = 4mv$, the smaller ball with mass m must have a momentum of $p = -4mv$ to cancel out. The velocity of the single particle m must be $-4v$. So the single particle gets more kinetic energy than the other 4 particles. by a factor of 4

3. Exercise 6 in 2.4, Vectors I would use an energy lens because my work turns the kinetic energy of the car to heat… or I am doing negative work on the car to be subtracted from its kinetic energy. I do -500J of work on the car, so the final kinetic energy of the car is 7,500 J, corresponding to a speed of square root of 15 m/s or just shy of 4 m/s… ~ 3.9 m/s

3. Ex 6 in 2.4



$V_f = ?$

Lens: Energy Lens because work is being done on the car

$W = F \cdot \Delta x = \Delta E_{k\ car}$

$E_{k\ car} = W + E_{k_0}$

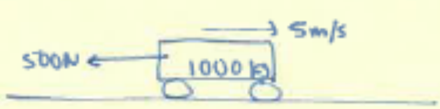
$\frac{1}{2} m v_f^2 = (F \cdot \Delta x) + (\frac{1}{2} m v_0^2)$

$500 v_f^2 = (500)(20m) + 500(25)$

$v_f = \sqrt{20 + 25}$

$v_f = 3\sqrt{5}$ m/s

$v_f = 6.71$ m/s $\approx \boxed{7 \text{ m/s}}$



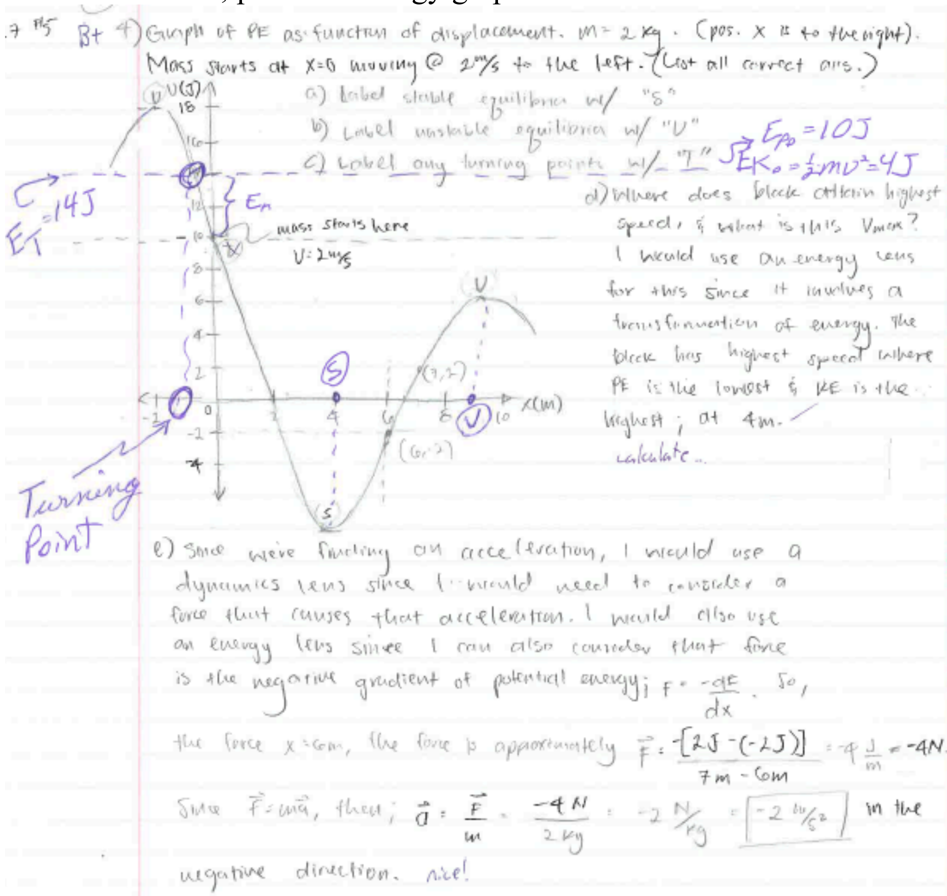
$\frac{1}{2} m v_f^2 = (F \cdot \Delta x) + \frac{1}{2} m v_0^2$

$500 v_f^2 = (500)(-10m) + 500(25)$

$v_f = \sqrt{25 - 10}$

$\boxed{v_f = 3.87 \text{ m/s} \approx 4 \text{ m/s}}$

4. Exercise 5 in 2.7, potential energy graph.


2.7 #5 B+ 4) Graph of PE as function of displacement. m = 2 kg. (pos. x is to the right). Mass starts at x = 0 moving @ 2 m/s to the left. (List all correct ans.)

a) label stable equilibria w/ "S"

b) Label unstable equilibria w/ "U"

c) Label any turning points w/ "T"

$E_p = 10J$, $E_{K_0} = \frac{1}{2}mv^2 = 4J$

$E_T = 14J$

mass starts here, v = 2 m/s

Turning Point

d) Where does block attain highest speed, & what is this V_{max}? I would use an energy lens for this since it involves a transformation of energy. The block has highest speed where PE is the lowest & KE is the highest; at 4m. calculate..

e) Since we're finding an acceleration, I would use a dynamics lens since I would need to consider a force that causes that acceleration. I would also use an energy lens since I can also consider that force is the negative gradient of potential energy; $F = \frac{-dE}{dx}$. So, the force x = 6m, the force is approximately $\vec{F} = \frac{-[2J - (-2J)]}{7m - 6m} = -4\frac{J}{m} = -4N$.

Since $\vec{F} = m\vec{a}$, then; $\vec{a} = \frac{\vec{F}}{m} = \frac{-4N}{2kg} = -2\frac{N}{kg} = \boxed{-2 m/s^2}$ in the negative direction. nice!

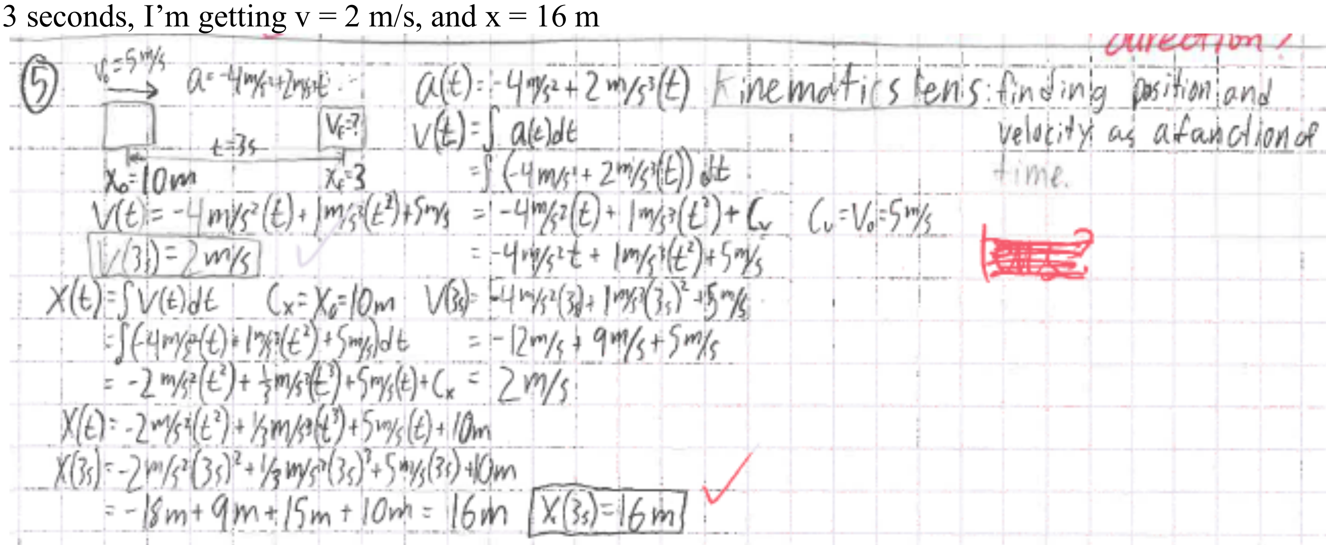
5. An object starts at 10 m with a speed of 5 m/s and has an acceleration of – 4 m/s^2 + 2 m/s^3(t). Find the velocity and position after 3 seconds.

This is a straight kinematics lens because we're given and need to find motion as an explicit function of time. We recognize that a = dv/dt and v = dx/dt, so we have to integrate acceleration to get velocity and integrate velocity to get displacement:

Δv = – 4 m/s^{2}t + m/s^3(t^2)… but at t = 0, the speed is 5 m/s so we have to add 5 m/s as the "integration constant" v(t) = 5 m/s – 4 m/s^{2}t + m/s^3(t^2)…

Δx = 5 m/s(t) – 2 m/s^{2}t^2 + (1/3) m/s^3(t^3)… but at t = 0, the position is 10 m so we have to add 10 m as the "integration constant": x = 10 m + 5 m/s(t) – 2 m/s^{2}t^2 + (1/3) m/s^3(t^3)…

At 3 seconds, I'm getting v = 2 m/s, and x = 16 m


5) $V_0 = 5 m/s$, $a = -4 m/s^2 + 2 m/s^3 t$, t = 3s, $V_f = ?$, $X_0 = 10m$, $X_f = ?$

$a(t) = -4 m/s^2 + 2 m/s^3(t)$ Kinematics lens: finding position and velocity as a function of time.

$v(t) = \int a(t)dt = \int(-4 m/s^2 + 2 m/s^3(t))dt = -4 m/s^2(t) + 1 m/s^3(t^2) + C_v$ $C_v = V_0 = 5 m/s$

$= -4 m/s^2 t + 1 m/s^3(t^2) + 5 m/s$

$V(t) = -4 m/s^2(t) + 1 m/s^3(t^2) + 5 m/s$

$\boxed{V(3) = 2 m/s}$

$V(3s) = -4 m/s^2(3s) + 1 m/s^3(3s)^2 + 5 m/s = -12 m/s + 9 m/s + 5 m/s = 2 m/s$

$X(t) = \int V(t)dt$ $C_x = X_0 = 10m$

$= \int(-4 m/s^2(t) + 1 m/s^3(t^2) + 5 m/s)dt$

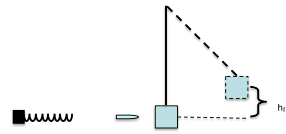
$= -2 m/s^2(t^2) + \frac{1}{3} m/s^3(t^3) + 5 m/s(t) + C_x$

$X(t) = -2 m/s^2(t^2) + \frac{1}{3} m/s^3(t^3) + 5 m/s(t) + 10m$

$X(3s) = -2 m/s^2(3s)^2 + \frac{1}{3} m/s^3(3s)^3 + 5 m/s(3s) + 10m$

$= -18m + 9m + 15m + 10m = 16m$ $\boxed{X(3s) = 16m}$

6. A loaded gun is cocked by compressing a spring of k = 10^4 N/m. and then releasing it behind a 20 g bullet. The bullet strikes and sticks inside of a 0.5 kg ballistics pendulum and swings upward to a final height of 50 cm. Presume the spring is massless and there is no friction in the system. Please find:



a) The bullet's speed.
b) how far the spring was compressed.
c) Does the bullet have constant acceleration in the gun, or does the acceleration change over time? Please explain your answer… identify a lens.
d) Please find the maximum acceleration of the bullet in the gun.
e) Did you identify the lenses at the very beginning, or one at a time for each question? Which do you think would be a better approach?

As soon as we see this, we are tempted to use an energy lens equating the initial spring potential energy to the final gravitational potential energy. However, the great majority of the bullet's kinetic energy is converted to thermal energy in the inelastic collision. Thus, we can find the kinetic energy of the bullet/block immediately after collision using an energy lens. However, to find the bullet's speed, we need to use the momentum lens because there is negligible outside forces so the momentum is conserved in the collision. The bullet's kinetic energy does come from the spring potential energy.

For letter "c" and "d", constant acceleration would be the result of a constant force. However, the force of the spring is proportional to the spring's compression. This the maximum acceleration would be when the spring is maximally compressed. This acceleration comes out to be 6000 times the acceleration of gravity, but so is the life a bullet! In fact, this acceleration is small compared to the acceleration the bullet experiences when it hits the target!

6). Lens: Energy Lens because energy is being converted from potential to kinetic and momentum

$k = 10^4$ N
$m_b = .02$ kg
$m_{box} = .5$ kg
$h_f = .5$ m

$E_{spring} \longrightarrow KE_{Bullet} \longrightarrow E_{therm} + KE_{Block} \longrightarrow PE_g$

$\frac{1}{2} kx^2 \longrightarrow \frac{1}{2} mv^2 \longrightarrow \qquad \longrightarrow mgh$

$\frac{1}{2}(10000 \frac{N}{m})x^2 \xrightarrow{\text{units}} \frac{1}{2}(.02 \text{ kg})v^2 \qquad \longrightarrow (.5 \text{ kg})(.5 \text{ m})(10 \text{ m/s}^2)$

$\frac{1}{2}mv^2 \longrightarrow 2.6$ J

inelastic collision of bullet / box

lens? $\vec{p}$ because $F_{outside} = 0$

$m_b v_b = m_{box} v_{box}$

$\frac{1}{2}(.52 \text{ kg})v^2 = 2.6$ J

$v = \sqrt{\frac{5.2 \text{ J}}{.52 \text{ kg}}}$

$(.02 \text{ kg})v_b = (.52 \text{ kg})(3.16 \text{ m/s})$

$v_{box} = 3.16$ m/s

$v_{bullet} = \frac{(.52 \text{ kg})(3.16 \text{ m/s})}{(.02 \text{ kg})} = \boxed{82.16 \text{ m/s}}$ a

Energy lens, because $E_s \Rightarrow E_K$

$\frac{1}{2}(10000 \frac{N}{m})x^2 = \frac{1}{2}(.02 \text{ kg})(82.16 \text{ m/s})^2$

$x = \sqrt{\frac{(.02 \text{ kg})(82.16 \text{ m/s})^2}{10000 \text{ N/m}}} = \boxed{0.12 \text{ m}}$ b

$\left(\frac{\text{kg} \frac{m^2}{s^2}}{N/m}\right)^{\frac{1}{2}} = (m^2)^{\frac{1}{2}} = m$

c) The force of the bullet comes from the force of the spring ($F = kx$) To have a constant acceleration, there must be a constant force. The force of the spring is not constant because as the ~~displacement of compression change~~ spring reaches equilibrium, the F_{spring} decreases. Thus the bullet does not have constant a.

Please include units.

Nice!

d) $F_s = F_B = ma$

$\cancel{\frac{1}{2}(10000)(.12)^2 = (.02)a}$ $\cancel{\boxed{a = 58000 \text{ m/s}^2}}$

E_p

$F = kx = 10000 \frac{N}{m} \cdot 0.12 m = 1200 N!$

e) Lenses are the way to go!

Yes!

$a = \frac{F}{m} = \frac{1200 N}{0.02 kg} = 60000 \frac{kg \, m/s^2}{kg}$

$= 6 \times 10^4 \, m/s^2 = 6000$ gravities!

7. Using an energy lens, please show that if you drop a 5 kg box from 60 m, it hits the ground at ~35 m/s. But then, you *throw* the box *downward* from 60 meters height with an initial speed of 35 m/s.

a) Find the speed that it has when it hits the ground.
b) What if I throw it *upwards* at 35 m/s, what is the speed when it hits the ground?
c) What if I throw it straight off the cliff at 35 m/s horizontally, what speed does it have when it hits the ground now?
d) <u>*Can*</u> I throw a 5 kg box at 35 m/s? Please back up your answer.

I show in a video that if I double the energy, then the speed increases by root 2 or about 49 m/s. Also, conserving energy, it doesn't matter what angle I throw the box, the final kinetic energy (and speed) will be the same. We also see, using an energy lens, it is very unlikely I could throw 5 kg at that speed, requiring power and force from my arm that is really more than one would expect from me.

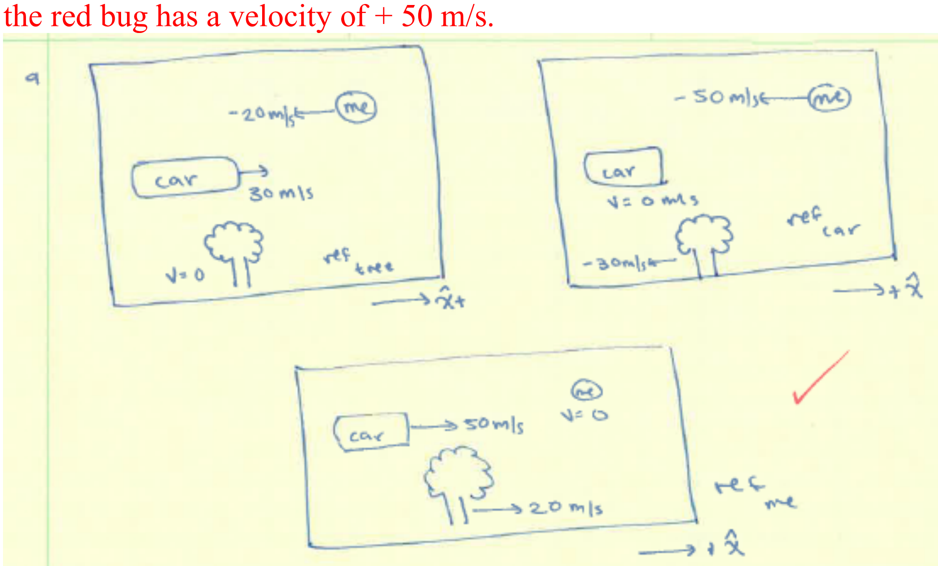
8. According to the hydrodynamic flow equations you'll learn in PHYS-132, the speed of water coming from a 200 PSI fire house is about 45 m/s (~100 mph!). Wikipedia claims these hoses are 25 mm in diameter. Imagine if you were hit with water by one of these hoses, like if you were protesting the Dakota Access Pipeline, and the fire department was called to clear the area (please see some drama: https://www.youtube.com/watch?v=K3Iv9okL4QU). I'd like to know the force that this water puts on someone's body. Let's model the water as a moving column that hits you and disperses all directions perpendicular to its original direction of travel, as in the figure of the demonstrator at right.

a) Clearly map out why this problem should be solved with conservation of momentum.
b) What is the volume, mass and momentum of a 1-meter column of water *before* it hits your body?
c) What is the momentum of water *after* it hits your body?
d) How long did it take the water to change momentum?
e) Find the force that this water puts on your body. Could it knock you over?

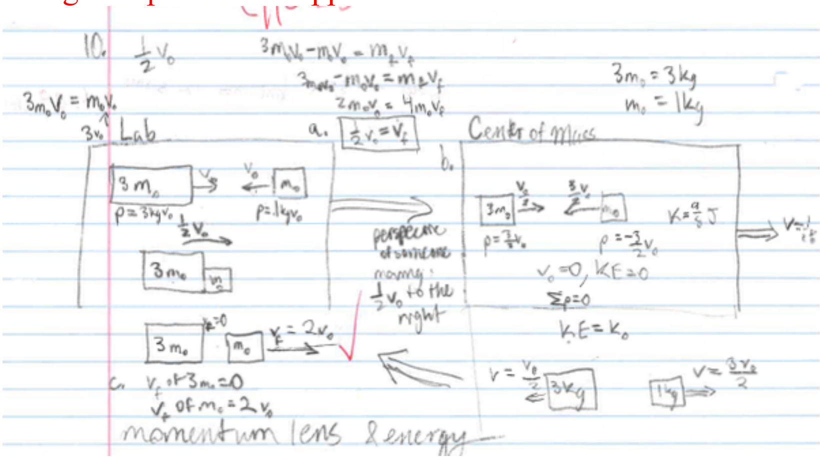
Please see fire hose video from Week 3.

9. Exercise 1 in 3.0

This is a simple kinematics lens because we are just looking at relative velocity. Each object sees itself at rest, but still sees the same relative velocity. For instance, in order to see itself moving at 0 m/s, the blue cart must add + 20 m/s to the velocity of each cart. Thus the tree has a velocity of + 20 m/s and the red bug has a velocity of + 50 m/s.



10. Exercise 2, in 3.1, What are the final velocities in this elastic collision? We did this in class. We find that the velocity of the center of mass is $v_o/2$. We remember that we want to be in this reference frame because this is where one would see the system as having zero momentum. Thus the final momentum must also be zero. We should find that the larger mass is at rest after the collision and the smaller mass has twice its original speed in the opposite direction.



11. Dragsters have a mass of about 1000 kg and the best dragsters get to 44 m/s in about 0.8 s.
    a) What's the acceleration?
        This is straight kinematics because we have explicit descriptions about motion. The acceleration is 55 m/s^2, outrageously large… 5.5 gravities!!
    b) Estimate the coefficient of friction necessary to make this happen if you were in a regular car on flat ground.
        This is a forces (dynamics) problem because we have a force (friction) causing acceleration. The acceleration is outrageous, so the friction coefficient must be as well. First use a dynamics analysis in the y direction with a nice drawing where the acceleration is zero to find that the normal force = the force of gravity. You need a frictional coefficient of 5.5… impossible? Maybe. We'll see below that it really doesn't have to be that large.
    c) What's the average power output during this 0.8 s?
        This is an energy lens because we are looking at how the energy changes as a function of time, and the energy conversion is mechanical work (from the engine) to kinetic energy in the motion of the dragster. This is about 1.2 MW, or about 1600 HP… and outrageous amount of horsepower….. like 10 times as much as an average car. But again, dragsters aren't average. It was brought to my attention that this wasn't an adequate estimation: We calculated that this is the power the car received from the engine. However, the mechanical output of the engine was also turned into heat released from the spinning tires on the ground. We didn't include that. So, the engine must certainly be putting significantly more power than the 1600 HP we calculated. It's worth noting that if you don't spin your tires, there is little kinetic energy converted to heat, so you don't need to include this consideration.
    d) Dragsters have their exhaust pipes pointed *upwards*, which ejects a huge amount of exhaust straight up into the air at very high velocity. What effect does this thrust have on the ability of the car to accelerate? *Why? Please start with clarification of reasons, drawings, lenses.*
        We use a dynamics lens looking at the forces in the y direction. Force is the rate of change of momentum of the heated exhaust upward, there is an equal downward force on the dragster because the force is between the dragster and the air. We can then examine the forces in the y direction on the dragster and realize that now the normal force must be equal to the force of gravity *and* this down force combined.

    According to my calculations, the engines kick out about 18 kg of exhaust every second at about 230 m/s. This corresponds to a momentum change of 4400 kg m/s every second, exerting a force of 4400 N.
    e) What is the momentum of this amount of gas?
    f) How much force should this put on the vehicle? In which direction?

g) With this extra "downforce", what coefficient of friction is necessary in order to accelerate the dragster? Now, the normal force must be 14100 N, requiring a friction force of only 3.9, which is still very large, but more attainable.

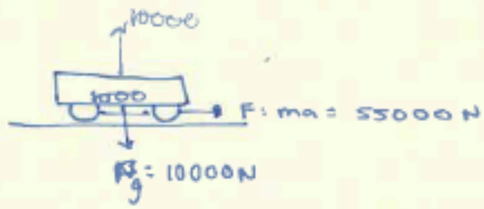
11. m = 1000 kg

best dragsters get to 44 m/s in .8 s

$F_f = \mu N$

a) $a = \Delta v / \Delta t = 44\ m/s / .8 s = \boxed{55\ m/s^2}$

Dynamics Lens because $\Sigma \vec{F} = m\vec{a}$



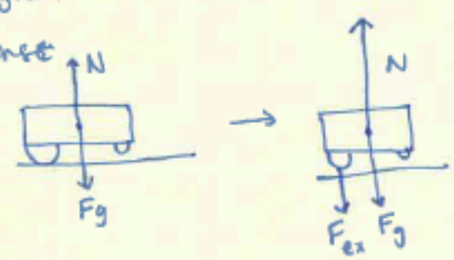
55000 = (μ)10000

b) $\boxed{\mu = 5.5}$

c) ~~P = ΔW/Δt = 55000 .8~~ $P = \frac{1/2 (1000)(44)^2}{.8 s} = \boxed{1210\ kW}$

d) When the exhaust ~~ejects~~ ~~exerts~~ exerts force downwards on the wheel, the normal force increases significantly, consequently increasing the force of friction which allows for greater acceleration

Dynamics Lens



e) $p = (18)(230) = \boxed{4140\ kg\ m/s}$

f) $F = ma = (18)(230\ m/s^2) = \boxed{4140\ N}$ $\boxed{\text{downwards}}$

g) $55000 = (14140)\mu \Rightarrow \mu = \underline{3.89}$