Problem Set #5 due beginning of class, Monday, Feb. 5.

1) Section 4.0 Exercise 1, collision of rotating bodies – what is the linear analogue for this problem?

A 2) (4.0, #1) Two identical disks ("A" and "B") are spinning in opp. directions in space, $\omega_A = 3\omega_B$.

I would use an angular momentum lens since I'm comparing their angular momentums. $\vec{l}_A = I\vec{\omega}$. For disk A, it would be $l_A = I_A \omega_A$, and for B it's, $\vec{l}_B = I_B \vec{\omega}_B = I_B(\frac{1}{3}\omega_A)$. Since this disks are identical, $I_A = I_B$. So, $\vec{l}_B = \frac{1}{3}\vec{l}_A$ ... $\boxed{\vec{l}_A = 3\vec{l}_B}$

For their kinetic energies, I would use a rotational kinetic energy lens since I'm dealing with their transfers of energy. $E_R = \frac{1}{2}I\vec{\omega}^2$. $I_A = I_B = I$

$E_{R(A)} = \frac{1}{2}I_A\vec{\omega}_A^2 = \frac{1}{2}I(3\vec{\omega}_B)^2$ $\quad E_{R(B)} = \frac{1}{2}I_B\vec{\omega}_B^2 = \frac{1}{2}I\vec{\omega}_B^2$

$E_{R(A)} = 9(\frac{1}{2}I\vec{\omega}_B)$ $\quad E_{R(B)} = \frac{1}{2}I\vec{\omega}_B^2$

$\boxed{E_{R(A)} = 9E_{R(B)}}$

- The two bodies collide and stick together.
  - The linear analogue to those disks colliding would be two carts or blocks colliding.
  - I would use an angular momentum lens here since this collision is in an isolated system and therefore momentum is conserved. (No outside forces)

$\vec{l}$ is a vector

$\vec{\omega}_A = -3\vec{\omega}_B$

$\vec{l}_i = \vec{l}_f \rightarrow I_A\vec{\omega}_A + I_B\vec{\omega}_B = (I_A + I_B)\vec{\omega}_f \rightarrow I_A = I_B = I$

$\omega_f = \frac{I_A\vec{\omega}_A + I_B\vec{\omega}_B}{I_A + I_B} = \frac{I(3\omega_B) + I\omega_B}{2I} = \frac{\cancel{4}^{1}\cancel{I}\omega_B}{\cancel{2I}} = \cancel{2}\omega_B = \vec{\omega}_B$

$\omega_B = \frac{1}{3}\omega_A \Rightarrow \omega_f = \cancel{2}\omega_B = \cancel{2}(\frac{1}{3}\omega_A) = \boxed{\frac{1}{3}\omega_A = \omega_f}$

- I would use an energy lens here since I see that there's a transformation of energy in this collision.

$\sum\vec{E}_{Ri} = \sum\vec{E}_{R(A)} + \sum E_{R(B)}$
$= 9E_{R(B)} + E_{R(B)}$
$= 10E_{RB}$
$= 10(\frac{1}{2}I\vec{\omega}_B^2)$
$= 5I(\vec{\omega}_B)^2 = 10(\frac{1}{2}I\vec{\omega}_B^2)$
$= 10\vec{E}_{R(B)} = \sum\vec{E}_{Ri}$

$\sum\vec{E}_{Rf} = \frac{1}{2}I_{A+B}\vec{\omega}_f^2$
$= \frac{1}{2}(I_A + I_B)(\frac{1}{3}\omega_A)^2$
$= \frac{1}{2}(2I)(\frac{1}{9}\omega_A^2)$
$= \frac{1}{9}I(3\omega_B)^2$
$= \cancel{1}I\omega_B^2 = \cancel{2}(\frac{1}{2}I\omega_B^2)$
$= \cancel{2}E_{R(B)} = \sum\vec{E}_{Rf}$

80% of the original $E_K \Rightarrow E_{th}$

Is this the same as in an inelastic collision?

$\boxed{m \rightarrow v_0 \quad 3v_0 \leftarrow m}$

and that $E_{RA} = 9E_{RB}$

Using E_{RB}, I found the initial and final energies of the system. $\sum E_{Ri} = 10E_{RB}$ while $\sum E_{Rf} = 2E_{RB}$. Therefore, we see that an energy equal to $8E_{RB}$ was "lost" to heat due to the collision.

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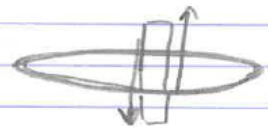
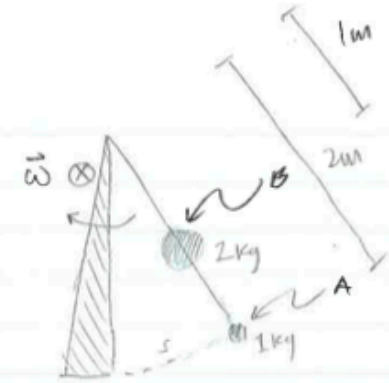
2) Section 4.1 Example 1, Rotation Direction

4.1 Ex 1 (rotational acceleration)

$\boxed{C.}$ Shows torque in the leftward $-\hat{x}$ direction.



3) Section 4.2 Exercise 1, Rotation and kinetic energy of two masses



) (4.2, # 1) Rigid rod, driven by motor, rotates about pivot at top of triangular support at $\vec{\omega} = 10\ \text{rad/s}$. The 2m long rod is very light, w/ 1kg mass @ end and 2 kg mass in the middle of the rod.

a) I would use a rotational kinematics lens for this since I'm analyzing the rod's rotational motion as an explicit function of time.

$$\vec{\omega} = 10\ \frac{\text{rad}}{\text{s}} \times \frac{1\ \text{revolution}}{2\pi\ \text{rad}} \times \frac{60\ \text{s}}{1\ \text{min.}} = \boxed{95.493\ \text{rpm}}$$

without a calculator

b) I would use a rotational kinematics lens for the same reason. I know that the rod rotates at $\vec{\omega} = 10\ \text{rad/s}$, therefore each mass must also rotate at $\vec{\omega} = 10\ \text{rad/s}$, like any point on the rod, since they all cover the same $\Delta\theta$ in the same Δt. However, mass A must move faster translationally than mass B since mass A covers a larger arc length "s" in the same Δt that mass B covers a smaller s. Since $s = r\Delta\theta$,

$$\vec{V} = \frac{ds}{dt} = \frac{r\,d\theta}{dt} = r\vec{\omega} \Rightarrow \begin{array}{l} \vec{V}_A = (2\text{m})(10\ \text{rad/s}) = 20\ \text{m/s} = \vec{V}_A \\ \vec{V}_B = (1\text{m})(10\ \text{rad/s}) = 10\ \text{m/s} = \vec{V}_B \end{array}$$

c) I would use a rotational kinetic energy lens for this since I'm find the E_K of each mass $\Rightarrow$ there's energy transformations. We know $E_{RK} = \frac{1}{2} I \omega^2$ and $I = mr^2$, But you could also just use $E_K = \frac{1}{2}mv^2$

$$E_{RK(A)} = \frac{1}{2} I_A (\omega_A)^2 = \frac{1}{2} m_A (r_A)^2 (\omega_A)^2 = \frac{1}{2}(1\text{kg})(2\text{m})^2(10\ \text{rad/s})^2 = 200\ \text{kg}\ \text{m}^2/\text{s}^2 = \boxed{200\ \text{J}}$$

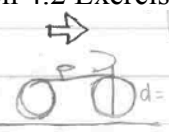
$$E_{RK(B)} = \frac{1}{2} I_B (\omega_B)^2 = \frac{1}{2} m_B (r_B)^2 (\omega_B)^2 = \frac{1}{2}(2\text{kg})(1\text{m})^2(10\ \text{rad/s})^2 = 100\ \text{kg}\cdot\text{m}^2/\text{s}^2 = \boxed{100\ \text{J}}$$

d) I use a rotational kinetic energy lens here for the same reason.

$$\sum E_K = \sum E_{RK} = E_{RK(A)} + E_{RK(B)} = 200\ \text{J} + 100\ \text{J} = \boxed{300\ \text{J}}$$

4) Section 4.2 Exercise 2, Rotation and linear speed, bicycle problem

(4)



$d = 0.700\,m$ $\quad \Delta t = 5\,s$ $\quad v = 10\,m/s$

a.) kinematics lens

$\vec{a} = \frac{\Delta v}{\Delta t} = \frac{10\,m/s}{5\,s} = 2\,m/s^2$ ← possible on level ground

b.) rotational kinematics lens

$\vec{\omega} = \frac{\vec{v}}{r} = \frac{10\,m/s}{\frac{1}{2}(0.7\,m)} = \boxed{28.57\,rad/s}$

right hand rule ⇒ ⊗

c.) $\vec{\alpha} = \frac{\Delta \omega}{\Delta t} = \frac{28.57\,rad/s}{5\,s} = \boxed{5.71\,rad/s^2}$

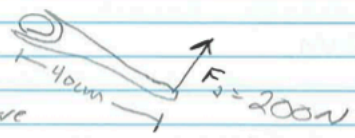
d.) $\vec{\omega} = \frac{\Delta \vec{\theta}}{\Delta t} \Rightarrow \Delta \vec{\theta} = \vec{\omega}_{average}\,\Delta t$

$= \frac{(28.57\,rad/s)(5\,s)}{2}$

$= \boxed{71.45\,rad}$

5) Section 4.3 Exercise 2, Turning a wrench

(6) 4.3 Ex) 2



a) First, dynamics lens b/c there are τ's and α's

$\tau = F \cdot r = 200N \cdot .4m = 80\,Nm$

Now, energy lens, b/c work is done on the nut as it turns.

$W = \tau \cdot \Delta\theta = 80Nm \cdot 2\pi \approx 500\,J$

b) This energy was converted to heat through friction.

c) $P = \frac{W}{\Delta t} = \frac{500J}{2s} = 250\,W$

great!

6) Section 4.3 Exercise 4, Pedaling a bicycle

$F = 200\,N$ $\quad \vec{\omega} = 90\,rpm$

$r = 175\,mm$

a.) rotational dynamics lens

$\tau = r\vec{F}_\perp$

$= (0.175\,m)(200\,N)$

$= \boxed{35\,Nm}$

$\frac{90\,r}{1\,min} \cdot \frac{2\pi\,rad}{1\,r} \cdot \frac{1\,min}{60\,sec}$

$\vec{\omega} = 9.42\,rad/sec$

$P = \frac{\Delta E}{\Delta t} = \frac{W}{\Delta t} = \tau \Delta\vec{\omega} = (35\,Nm)(9.42\,rad/sec)$

$\approx \boxed{330\,W}$

b.) rotational kinematics lens

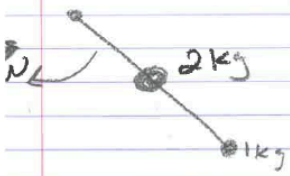
$\vec{v} = \frac{\Delta s}{\Delta t} = \frac{r\Delta s}{\Delta t} = r\vec{\omega} = (0.175\,m)(9.42\,rad/s) = \boxed{1.64\,m/s}$

$P = \frac{\Delta E}{\Delta t} = \frac{F\Delta x}{\Delta t} = F \cdot \vec{v} = (200\,N)(1.64\,m/s) \approx \boxed{330\,W}$

7) Section 4.4 Exercise 2, Kinetic energy of two masses

7 4.4 Ex 2

$\omega = 10/s$



Lens: Energy. Finding inertia as how difficult to rot. move and KE.

(e) $I = m_1 r_1^2 + m_2 r_2^2$

$= 2kg(1m)^2 + 1kg(2m)^2$

$\boxed{I = 6\ kgm^2}$ ✓

(f) $E_R = \frac{1}{2} I \omega^2$

$= \frac{1}{2}(6kgm^2)(10/s)^2$ ✓

$= 30\ kg\frac{m^2}{s^2} = \boxed{300J}$

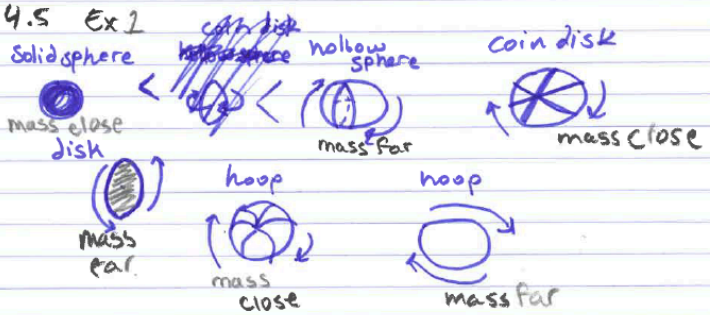
$= E_R$

(g) My answer agrees!

(h) Finding KE w/ rotational equations is better because there's one less step without having to find V_{Tan}.

8) Section 4.5 Exercise 1, Ranking Several Objects

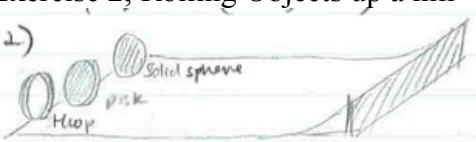
4.5 Ex 1



Solid sphere < hollow sphere < coin disk < standing hoop ✓ < disk < hoop.

9) Section 4.5 Exercise 2, Rolling Objects up a hill

(c) (4.5, #2)



$I_{solid\ sphere} = \frac{2}{5}mr^2$

$I_{disk} = \frac{1}{2}mr^2$

$I_{Hoop} = mr^2$

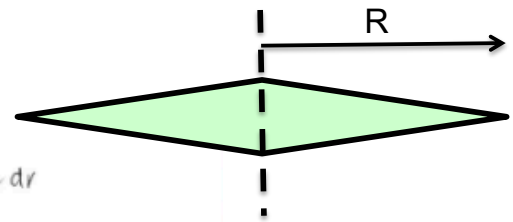
I use a rotational dynamics lens since I see that torques are causing rotational acceleration. We know that moment of inertia is a measure of how hard it is to rotationally accelerate a body; the more I a body has, the harder it is to apply an angular acceleration on it. Because the hoop has the highest I, it will go further up the hill than the others because it's harder for the force of gravity, and therefore friction to apply an angular acceleration (negative) to slow the disk down. great! I would use Energy lens to get the same answer! Can you try that too?

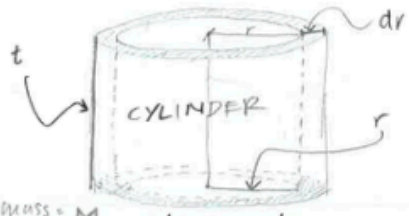
10) Read Section 4.5, exercise 3 and do the following:

11) A solid object of mass M, has a thickness of t_o at the axis (at $r=0$) that tapers evenly to a sharp edge at $r=R$, or $t = t_o(1-r/R)$.

a) Estimate the moment of inertia.

b) Calculate the moment of inertia.





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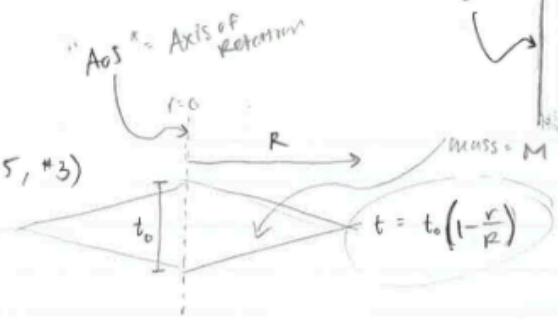
11) (4.5, #3)



$V = 2\pi r t$

$dV = 2\pi r dr\,(t)$

$dV = 2\pi r dr\,(t_o)\left(1-\frac{r}{R}\right)$

(a) To estimate the moment of inertia I of the discus, I must first analyze how the mass is distributed. Compared to a disk, it definitely has less I since a lot of the discus mass seems to be concentrated near the AoS. It's also less than a solid/hollow sphere since mass is more spread out in spheres. I would estimate $I_{discus} < I_{solid\ sphere} = \frac{2}{5}mr^2$.

(b) Exact moment of inertia. $I = mr^2$

To find volume, integrate using cylinders

$dV = 2\pi r\,dr\,(t)$

$$\text{Volume} = \int_{r=0}^{r=R} dV = \int_0^R 2\pi r\,dr\,(t_o)\left(1-\frac{r}{R}\right) = 2\pi t_o \int_0^R r\left(1-\frac{r}{R}\right)dr$$

$$= 2\pi t_o \int_0^R \left(r\,dr - \frac{r^2}{R}dr\right) = 2\pi t_o\left(\frac{r^2}{2} - \frac{r^3}{3R}\Big]_0^R\right)$$

$$= 2\pi t_o\left[\left(\frac{R^2}{2} - \frac{R^3}{3R}\right) - \left(\frac{0^2}{2} - \frac{0^3}{3R}\right)\right] = 2\pi t_o\left(\frac{R^2}{2} - \frac{R^2}{3}\right)$$

$$= 2\pi t_o\left(\frac{R^2}{6}\right) = \frac{1}{3}\pi t_o R^2 = V_{discus}$$

$$\rho = \frac{M}{V} = \frac{M}{\frac{1}{3}\pi t_o R^2} = \frac{3M}{\pi t_o R^2}$$

constant

$M = \rho V$

$dM = \rho\, dV$

$(t_o)\left(1-\frac{r}{R}\right)$

integrate over mass

$$I = mR^2, \quad dI = dm\,R^2 = \rho\, dV r^2 = \rho(2\pi r dr(t))\,r^2 = \rho 2\pi r^3 dr\,(t)$$

$$I = \int_0^R dI = \int_0^R \rho 2\pi r^3 dr\,(t_o)\left(1-\frac{r}{R}\right) = 2\pi\rho t_o \int_0^R \left(1-\frac{r}{R}\right) r^3 dr$$

$$= 2\pi\rho t_o \int_0^R \left(r^3 dr - \frac{r^4}{R}dr\right) = 2\pi\rho t_o\left[\left(\frac{r^4}{4} - \frac{r^5}{5R}\right)\Big]_0^R\right]$$

$$= 2\pi\rho t_o\left(\frac{R^4}{4} - \frac{R^5}{5R} - \frac{0^4}{4} + \frac{0^5}{5R}\right) = 2\pi\rho t_o\left(\frac{R^4}{20}\right) = \frac{1}{10}\pi\rho t_o R^4$$

$$= \frac{1}{10}\pi\left(\frac{3M}{\pi t_o R^2}\right) t_o R^2 = \boxed{\frac{3}{10}MR^2 = I_{discus}}$$

You can also see my solution at

http://sharedcurriculum.wikispaces.com/file/view/PS7_SUS_W15_Q5%20solution.jpg/542965166/PS7_SUS_W15_Q5%20solution.jpg