Problem Set #4 Questions #7-#12

7. Dynamics! I immediately remember that the sum of the forces = ma, and I start to draw the forces in. I realize that it is accelerating down the incline, so this will be my “x” direction and I will decompose gravity into these two directions. If gravity is $m_o g$, then the x component must be about 0.9 mg, and the y component must be about 0.4 mg as judged from the drawing. The “y” component is compensated by the normal force, and the x component accelerates the block downward.

a) If the block is frictionless, a =
b) If 1 m/s^2, μ_k=
c) Does mass matter?

$\Sigma F_x = ma_x$

$F_{gx} = ma_x$

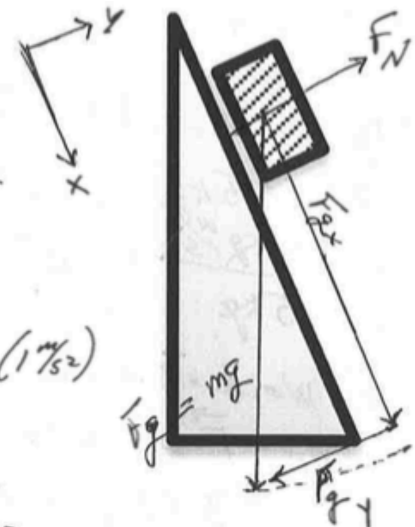
$\sim 0.9\cancel{m}g = \cancel{m}a_x$

$a_x \sim 9 \text{ m/s}^2$

$\Sigma F_y = ma_y = 0$

$F_N - F_{gY} = 0$

$F_N = F_{gY} \approx .4 m_o g$



b) if $a = 1 \text{ m/s}^2$, Then $\Sigma F_x = ma = m_o(1 \text{ m/s}^2)$

now we have F_f in $-\hat{x}$.

$F_{gx} + \bar{F}_f = ma_x \quad \therefore F_f = F_{gx} - ma_x$

$= (9 \text{ m/s}^2 - 1 \text{ m/s}^2) m_o$

$F_f = 8 \text{ m/s}^2 \, m_o$

$F_f = \mu F_N \approx \mu \, 0.4 m_o g$

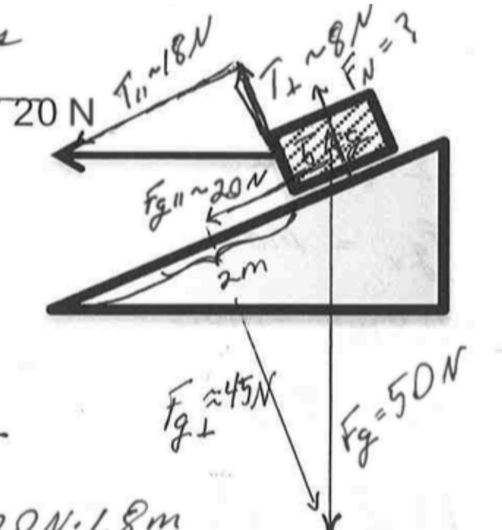
$8 \text{ m/s}^2 \, \cancel{m_o} \approx \mu \cdot 0.4 \cancel{m_o} g$, $g = 10 \text{ m/s}^2$

$\boxed{2 \approx \mu}$ very high coefficient of friction

c) We see from above that mass cancels. If you double mass, you also double F_g, F_N, F_f so the acceleration stays the same.

8. At right, you see that I pull a 5 kg mass down a 2 m long incline with a 20 N, horizontal force. With good communication and without a calculator or angle measuring, please calculate:

a) For a frictionless surface, please calculate the acceleration of the block, and the normal force of the surface on the block.
b) For a frictionless surface, please calculate the total work I do, and the block's final speed.
c) If the coefficient of friction is 0.25, please calculate the acceleration of the block.
d) If the coefficient of friction is 0.25, please calculate the amount of heat produced (in Joules), and please calculate the final speed of the block.



a) Dynamics because we have forces and want $\vec{a}$

$\Sigma F_{\parallel} = m a_{\parallel}$

$T_{\parallel} + F_{g\parallel} = m a_{\parallel}$

$18N + 20N = 5kg\ a_{\parallel}$

$a_{\parallel} \approx \frac{38\,kg\,m/s^2}{5kg} = 7.6\,m/s^2$

$\Sigma F_{\perp} = m a_{\perp} = 0$

$F_N + T_{\perp} + F_{g\perp} = 0$

$F_N = F_{g\perp} - T_{\perp}$

$\approx 45N - 8N$

$\approx 37N$

b) Work + Energy

$W_p = \vec{F} \cdot \vec{dx} \approx 18N \cdot 2m$, or $\approx 20N \cdot 1.8m$

$= 36J$, but we are also losing PE_g $mg\Delta h$

$PE_g \approx 5kg \cdot 10\,m/s^2 (\sim 0.8\,m)$

$\Delta PE \sim 40J$

Conserving Energy: $E_0 = E_f$

$PE_0 + \cancel{KE_0} + W_{Pete} = KE_f$

$\sim 76J = \frac{1}{2} m v^2$ $\quad V_f \approx 5.5\,m/s$

c) now we must consider $F_f = \mu F_N \approx 0.25 \cdot 37N \approx 9N$

$\oplus\ T_{\parallel} \overset{\oplus}{+} F_{g\parallel} \overset{\ominus}{+} F_f = 18N + 20N - 9N = 29N = ma \quad a \sim 6\,m/s^2$

d) W_f Work/Energy lens. The work of friction = heat.

$W_f = \vec{F_f} \cdot \vec{\Delta x} \approx 9N \cdot 2m = 18J$ $\quad F_f \parallel \Delta x$ so just multiply

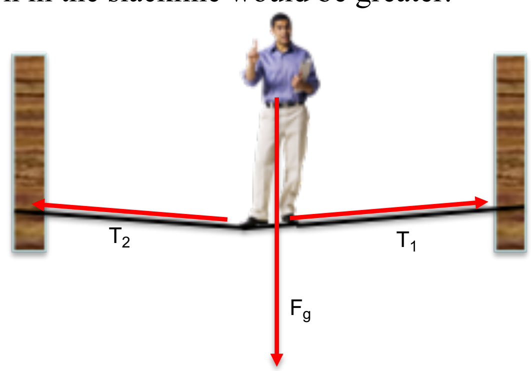
Now the total energy at the end is only 58J because 18J ⇒ heat. $\frac{1}{2} m v_f^2 \approx 58J$ $\quad \boxed{V_f \approx 5\,m/s}$

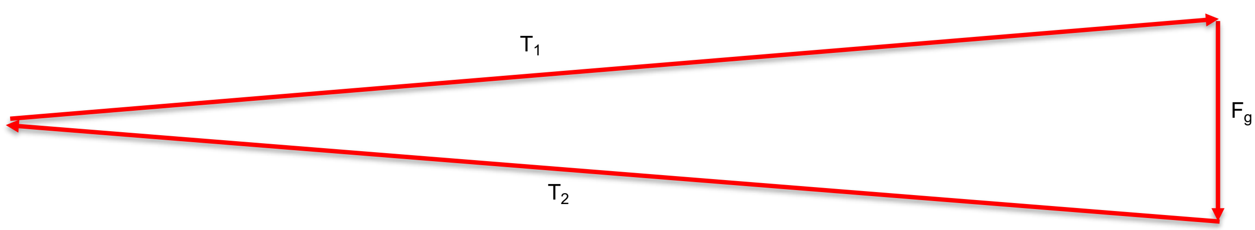
9. At right, you see two pictures of me at 70 kg, slack lining.

a) In which drawing is the line tighter? This is a dynamics problem… in particular a statics problem because there is no acceleration. We look at the forces on the man at rest and we add them like vectors to sum to zero. The x components of the tension cancel each other, and the y components add to be equal and opposite to gravity (because we're in equilibrium). On the left the ratio of the horizontal and vertical components is about 5:1, on the right, it's more like 10:1. So, if the force of gravity on the man is 700N, then the tension on the *right* would be about 3,500 N, and on the left would be about half that.

b) Using your force drawing, please estimate the tension on the slack line at right. *See below.*

c) In a classic physics problem, a car is stuck in the mud, so you tie a rope to a tree on the other side of the road as tight as you can and then push the rope – do you pull it along the rope, or push it perpendicular? Would it be a good idea to slack line on it? If you were slack lining on it, would it be a good idea to jump on it? *We just showed* that if you push perpendicular to the rope, the tension will be greater than the perpendicular force you apply…. As long as the rope is inelastic enough to not deflect to high angles… That is, for the slackliner example, please show that if the slack line made an angle of 30 degrees with the horizon, that the tension would be the same as the force of gravity on the man. If you bounced on the slackline, then at the bottom of your oscillations, you would be accelerating upward, and thus the tension in the slackline would be greater.





10. You need to build a massive slingshot that propels a 100 kg object (you in a capsule) at 13 km/s so you can go into space (infinity)! For ***each question***, start with a statement of which of the 4 mechanics concepts is central to this problem and why.

10 All these questions are about energy because Each variable can be expressed in terms of an energy and no energy is lost to heat or anything - it can all be accounted for:

Speed ⇒ Kinetic energy, $\frac{1}{2}mv^2$

Distance to earth's center ⇒ Gravitational Potential Energy, $-\frac{m_e m_2}{r}G$

Compression of spring ⇒ Spring Potential Energy: $\frac{1}{2}kx^2$

a) Deep Space is when $r \to \infty$

E_o ← earth surface $=$ E_f ← deep space

$$\frac{1}{2}\cancel{m}V_o^2 + -\frac{m_e \cancel{m_2}}{r_e}G = \frac{1}{2}\cancel{m}V_f^2 + \underbrace{-\frac{m_e \cancel{m_2}}{r_{ds}}G}_{=0 \text{ at } r=\infty}$$

we find that mass cancels, so if you define the speed of something leaving the earth, you can find its speed anywhere

$$V_f^2 = V_o^2 - \frac{2m_e}{r_e}G$$

$m_e = 6.0 \times 10^{24}$ kg

$r_e = 6.4 \times 10^6$ m

$r_{e\text{-}m} = 3.8 \times 10^8$ m $\approx 60\, r_e$

$G = 6.67 \times 10^{-11} \frac{Nm^2}{kg^2}$

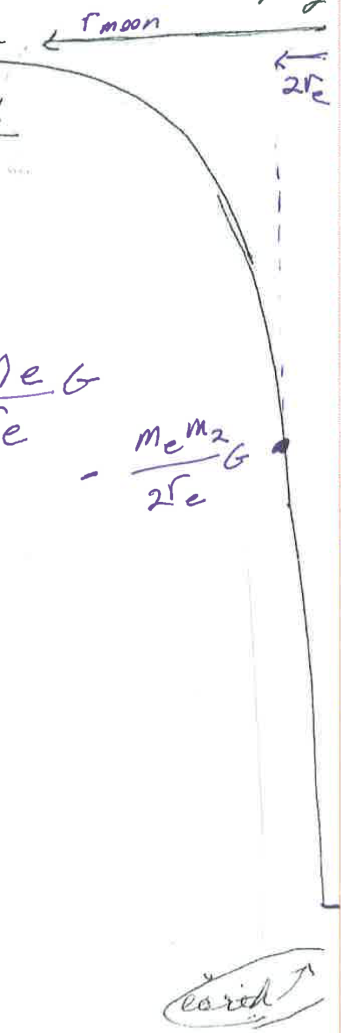
$$V_f^2 = (13 \times 10^3 \, m/s)^2 - \frac{2 \cdot 6.0 \times 10^{24} \, kg}{6.4 \times 10^6 \, m} \, 6.67 \times 10^{-11} \, \frac{N \, m^2}{kg^2}$$

$$V_f^2 \approx 1.69 \times 10^8 \, \frac{m^2}{s^2} - 1.2 \times 10^8 \, \frac{m^2}{s^2} \approx 5 \times 10^7 \, \frac{m^2}{s^2}$$

$$V_f \approx \left(50 \times 10^6 \, \frac{m^2}{s^2}\right)^{\frac{1}{2}} \approx 7 \times 10^3 \, \frac{m}{s} = \boxed{7 \, km/s}$$

b) now, we need to consider that we haven't climbed all the way out of the gravitational PE_g Potential well that the earth is in.

lunar distance ≈ 50 r_e ~~using that first~~



$$\frac{1}{2} m V_0^2 - \frac{m_e m_2}{r_e} G = \frac{1}{2} m V_f^2 - \frac{m_e m_2}{2r_e} G$$

$$V_f^2 = V_0^2 - \frac{2 m_e}{r_e} G + \frac{2 m_e}{2 r_e} G = V_0^2 - \frac{m_e}{r_e} G$$

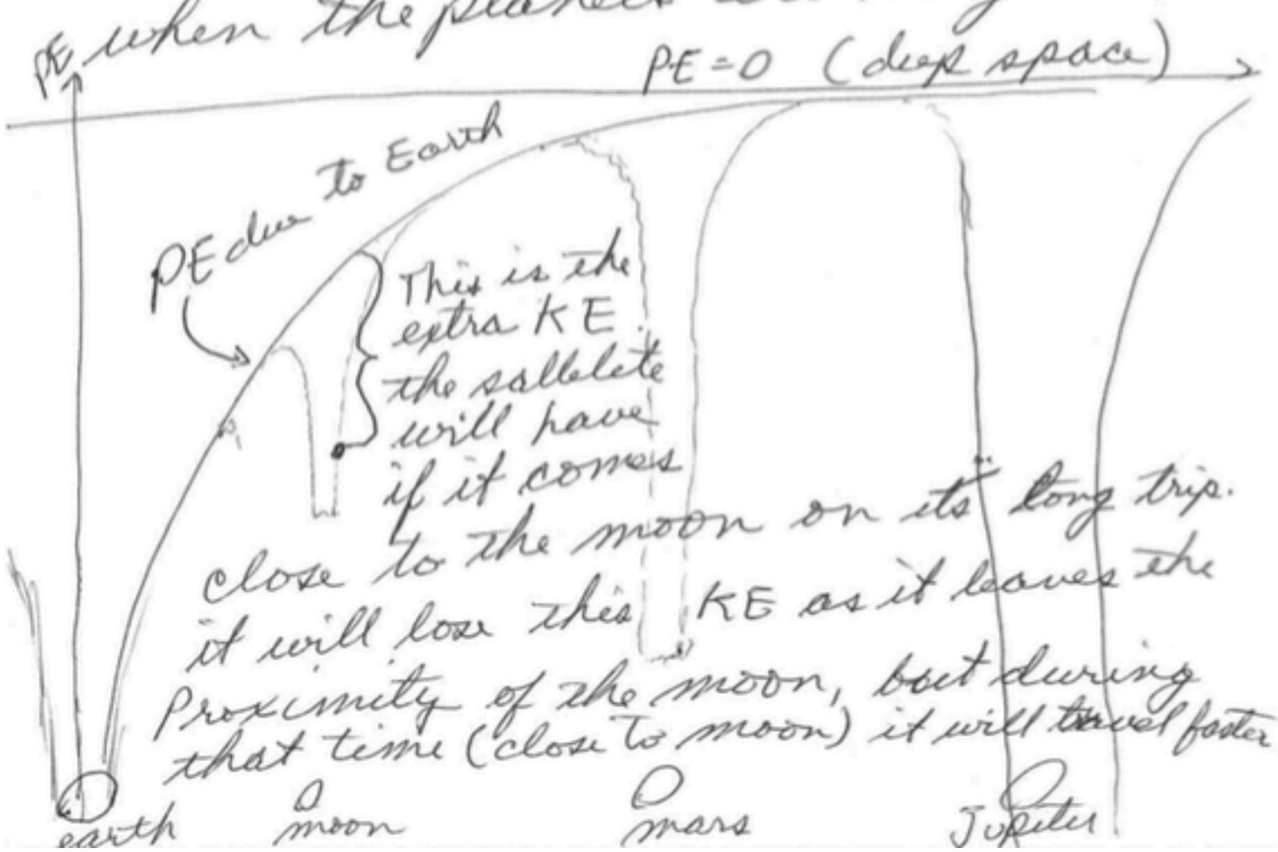
$$V_{f(r=2r_e)} = \left(V_0^2 - \frac{m_e G}{r_e}\right)^{\frac{1}{2}}$$

$$\approx 10.3 \, \frac{km}{s}$$

C) From the perspective of an <u>energy lens</u>[3]
If you get close to the moon, your PE ⇓ because of the negative gravitational Potential between you and the moon: $\frac{-m_{you}\, m_{moon}}{r_{you-moon}}\, G$.

So, KE and speed increase

NASA makes use of this opportunity in launching deep space sattelites so they get close to the planets <u>especially</u> when the planets are aligned.



d) Dynamics → ~~we~~ calculate $\vec{F}$ to find $\vec{a}$

$$a = \frac{F}{m} = \frac{\frac{M_e \cancel{M_m}}{r_{em}^2} G}{\cancel{m_m}} = \frac{m_e}{r_{em}^2} G$$

$$= \frac{6.0 \times 10^{24} kg \cdot 6.67 \times 10^{-11} N \frac{m^2}{kg^2}}{(3.8 \times 10^8 m)^2}$$

$$\approx \cancel{2.8\times}\ 2.8 \times 10^{-3} m/s^2$$

.... wait, this is about $\frac{10 m/s^2}{2.8 \cdot 10^{-3} m/s^2} \approx 4000$

This is about $\frac{1}{4000}$ the acceleration of gravity on the earth's surface - does this make sense? ~~test~~ let's see!

$F_g \propto \frac{1}{r^2}$ what is the ratio of

$$\frac{r_m}{r_e} \approx \frac{3.8 \times 10^8 m}{6.4 \times 10^6 m} \approx 60$$

so $60^2 = 3600 \approx \underline{\underline{4000}}$

so yes! if you increase your distance to the earth's surface by 60, expect gravitational force on you to decrease by ~ 4000

e) Energy! - I'm going to turn

$PE_{spring} \Rightarrow KE \Rightarrow PE_{gravity} + KE$ very little

$\frac{1}{2} k x^2 = \frac{1}{2} m v^2$

$$k = \frac{m v^2}{x^2} = \frac{(100\,kg)(12 \times 10^3\,m/s)^2}{(10\,m)^2}$$

$$= \frac{\cancel{10^2}\,kg}{\cancel{10^2}\,\cancel{m^2}}\,169 \times 10^6\,\frac{\cancel{m^2}}{s^2}$$

$$= 1.69 \times 10^8\,\frac{kg}{s^2} \qquad \frac{kg}{s^2}\left(\frac{m}{m}\right) = N/m$$

f) This is a dynamics problem $F = ma$

$F = kx = 1.69 \times 10^8\,\frac{N}{\cancel{m}} \cdot 10\,\cancel{m} = 1.69 \times 10^9\,N$

This is the force of gravity on 10^8 kg or 10^5 Tons

$$a = F/m = \frac{1.69 \times 10^9\,N}{100\,kg} \approx 10^7\,m/s^2 \text{ or } 10^6\,g\text{s}$$

This acceleration is 1 million gravities. This would ultracentrifuge your biomatter into neat layers of lipids, water, hemoglobin, etc. It wouldn't be good for you.

g) $m_m = 7.4\times10^{22}\,kg \qquad r_m \approx 1.7\times10^{6}\,m$

we use an energy law

$$E_{surface} = E_{Deep\ space}$$

$$PE + KE = PE + KE$$

$$-\frac{m_m m_{pste}}{r_m} G + \frac{1}{2} m_{pste} v_0^2 = \cancelto{0}{\frac{m_m m}{\infty} G} + \frac{1}{2} m_{pste} \cancelto{0}{(0\,m/s)^2}$$

$$\frac{1}{2} \cancel{m_{pste}} v_0^2 = \frac{m_m \cancel{m_{pste}}}{r_m} G$$

$$v_0 = \left[\frac{2 m_m}{r_m} G\right]^{\frac{1}{2}}$$

$$= \left(\frac{2 \cdot 7.4\times10^{22}\,\cancel{kg}\; 6.67\times10^{-11}\,N\frac{m^2}{kg^2}}{1.7\times10^{6}\,m}\right)^{\frac{1}{2}}$$

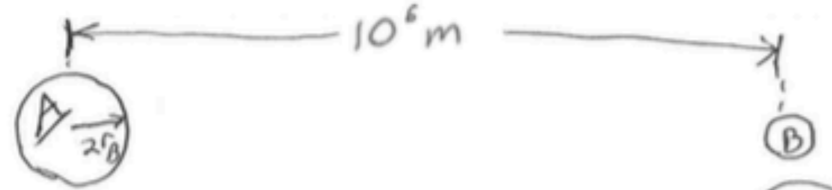
$$\approx \underline{\underline{2.4\ km/s}}$$

$$\frac{Nm}{kg} = \frac{m^2}{s^2}$$

11. In 1930, it was discovered that a beta decay:
$n \Rightarrow p^{+} + e^{-}$
didn't conserve energy, momentum or *angular momentum*. Wolfgang Pauli postulated the creation of a new particle, the *neutrino.* We now estimate that 65 billion neutrinos from the sun pass through each square centimeter on earth, *per second*. How fast does the sun produce neutrinos? We just need to find the number of square centimeters… that these neutrinos are spread out over… so construct a sphere around the sun of radius equal to the earth's orbital radius… ~150 million km, or 1.5×10^{11} m = 1.5×10^{13} cm. The area is $4\pi r^2$ wow… we're looking at about 2×10^{38}/s. How many pass through you during this class? What if my surface area were half a square meter… or 5000 square centimeters. Class is about 3500 s long, then I'd have about 10^{14}. That's a lot. Good thing they don't hurt. How about if you were on Venus? The ratio of the distances of the two planets is in a ratio of about 1:1.4 or square root of two. So the area of the sphere that the neutrinos have to pierce at Venus's distance is half that of the sphere at the earth's distance, so Venus would have about twice the intensity of neutrinos as here at earth.

12. There are two planets with centers 10^6 m apart: Planet A, and Planet B. The radius of Planet A is twice that of planet B, or $r_A=2r_B$. Both planets are made of the same rocks, and therefore have the same density. There are no other objects, so we are only looking at the force of gravity acting between the two planets. *Provide reasons for your answers before showing the work, before showing the answer.*

#3) 9



a) $m \propto$ Volume $\quad m = V\rho \quad V = \frac{4}{3}\pi r^3$

so as $r = 2r \quad V \Rightarrow 2^3 V_0$

or $V \propto m \quad m \Rightarrow 8 m_0$

b) Dynamics – Forces + acceleration.

this is a <u>single</u> gravitational interaction

$\vec{F}_{AB} = -\vec{F}_{BA}$ Forces are =, opposite

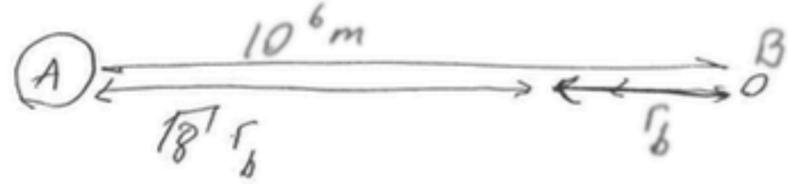
c) $a = F/m$ so the larger mass will accelerate <u>less</u>: $a_A = \frac{1}{8} a_B$

dynamics – Forces and acceleration

d) $F_g = \frac{m_{Planet}\, m_{me}}{r^2_{(Planet-me)}} G$ in order to be pulled equally hard by both planets, I must be closer to the smaller one. If the (mass) numerator of a is 8× the mass of b, then $r_a^2 = 8 r_b^2$ or $r_a = \sqrt{8}\, r_b$

e) using the same reasoning as d) above, but for PE, we don't square the denominator, so $r_a = 8\, r_b$

f) ⓑ where is $F_g = 0$?



$r_b + \sqrt{8}\, r_b = 10^6 m$

$(1+\sqrt{8})\, r_b = 10^6 m$

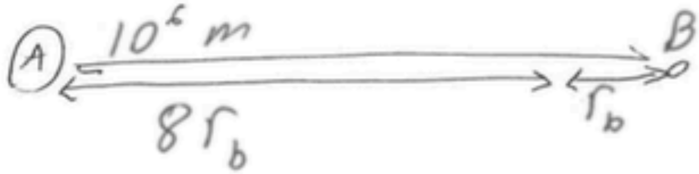
$r_b = \frac{10^6 m}{1+\sqrt{8}}$

almost 3

$r_b \lesssim \frac{1}{4}(10^6 m) \approx 2.5 \times 10^5 m$

$r_a = 10^6 m - r_a$, $r_a \sim 7.5 \times 10^5 m$

Where are the potentials = ?



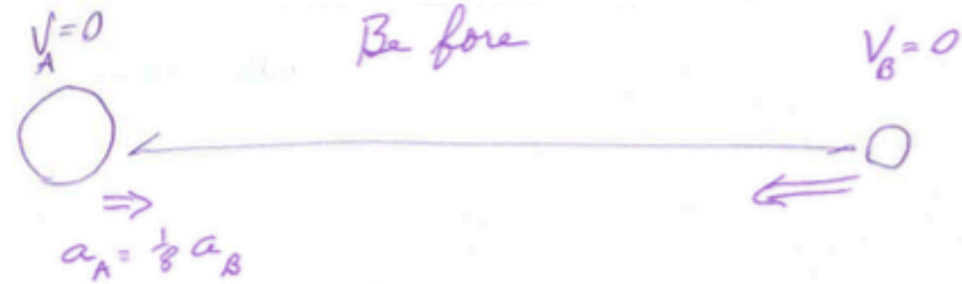
$9\, r_b = 10^6 m$

$r_b = \frac{1}{9} \cdot 10^6 m$

$\approx \underline{1.1 \times 10^5 m}$

$r_a = 10^6 m - r_b \approx \underline{8.9 \times 10^5 m}$

9)



This is a conservation of momentum Problem!

$\vec{P}_0 = \vec{P}_A + \vec{P}_B = 0$ there's a collison, so $\Sigma \Delta \vec{P} = 0$

right before collision $\vec{P}_A = -\vec{P}_B$!

$m_A V_A = -m_B V_B$

$M_A = 8 m_B$, so $\underline{\underline{V_A = \frac{1}{8} V_B}}$!

at the end, $\vec{V}_A = \vec{V}_B = \vec{P}_A = \vec{P}_B = \Sigma \vec{P} = \underline{0}$

again!