

a) It's an energy problem because you are analyzing velocity through the gravitational potential energy, since it can calculate the escape velocity, which will be final velocity in space because of zero forces.

$$r = 6,371 \text{ km}$$

$$G = 6.67 \times 10^{-11}$$

$$PE = -\frac{m_1 m_2}{r} G = -\frac{(6 \times 10^{24} \text{ kg})(100 \text{ kg})}{6,371,000} (6.67 \times 10^{-11} \text{ m}^3/\text{kg s}^2) = -6.28 \times 10^9 \text{ J}$$

$$KE = \frac{1}{2} mv^2 = \frac{1}{2} (100 \text{ kg})(12,000 \text{ m/s})^2 = 7.2 \times 10^9 \text{ J}$$

$$E_T = KE + PE = 7.2 \times 10^9 - 6.28 \times 10^9 = 9.2 \times 10^8 \text{ J}$$

$$KE = \frac{1}{2} mv^2$$

$$9.2 \times 10^8 \text{ J} = \frac{1}{2} (100 \text{ kg}) v^2$$

$$v^2 = 18,400,000 \text{ m}^2/\text{s}^2$$

$$v = 4289.5 \text{ m/s}$$

Too many S.F.

(-1)

b) $r_1 = 6,371,000 \text{ m}$

$$r_2 = 385,000 \text{ km}$$

$$E_T = 9.2 \times 10^8 \text{ J}$$

$$E_T = -PE + KE = -\frac{(6 \times 10^{24} \text{ kg})(100 \text{ kg})}{3.85 \times 10^8 \text{ m}} (6.67 \times 10^{-11} \text{ m}^3/\text{kg s}^2) + \frac{1}{2} (100 \text{ kg}) v^2$$

$$9.2 \times 10^8 \text{ J} = -1.04 \times 10^8 \text{ J} + 50 \text{ kg } v^2$$

$$1.02 \times 10^9 \text{ J} = 50 \text{ kg } v^2$$

$$2.05 \times 10^7 \text{ m}^2/\text{s}^2 = v^2$$

$$v = 4525.4 \text{ m/s}$$

c) $F_g = -\frac{m_1 m_2}{r^2} G =$

$$= -\frac{(6 \times 10^{24} \text{ kg})(100 \text{ kg})(6.67 \times 10^{-11} \text{ m}^3/\text{kg s}^2)}{(385,000,000 \text{ m})^2}$$

$$100 a = -2.7$$

$$a = -.0027 \text{ m/s}^2$$

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$$d) \frac{1}{2} kx^2 = \frac{1}{2} mv^2$$

$$\frac{1}{2} k(10m)^2 = \frac{1}{2} (100kg)(12000m/s)^2$$

$$k50m = 7.2 \times 10^7 \text{ Kg m}^2/s^2$$

$$k = 144,000,000 \text{ Kg m}^2/s^2$$

$$e) F = kx = ma$$

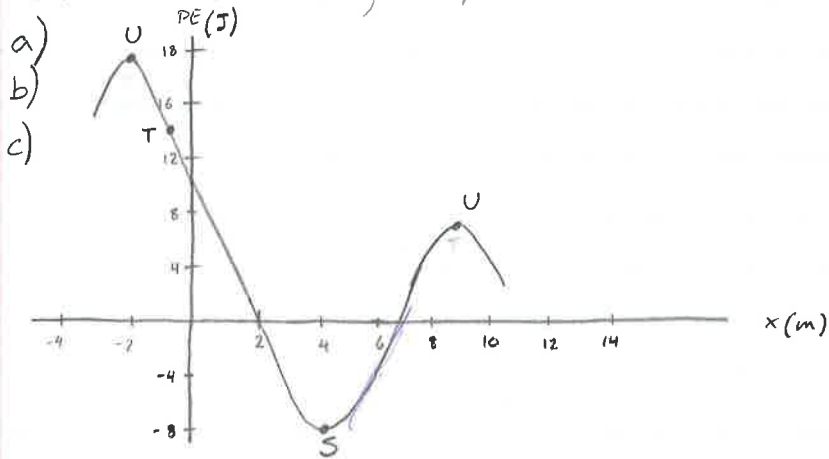
$$\frac{(1.44 \times 10^8 N)(10m)}{100kg} = \frac{(100kg)a}{100kg}$$

$$14,400,000 m/s^2 = a$$

$$a = 1.44 \times 10^7 m/s^2 \quad \text{This acceleration is too fast for a person}$$

$a \Rightarrow$ 1 million gravities! it would ultra centrifuge your body into layers of water, protein, lipids, different pulverized bone fragments, etc....

2. 2 kg mass
starts at $x=0$ moving 2 m/s left



$$\begin{aligned}
 KE &= \frac{1}{2}mv^2 \\
 &= \frac{1}{2}(2\text{kg})(2\text{m/s})^2 \\
 &= 4\text{J} \\
 \text{start at } 10\text{J} \\
 10\text{J} + 4\text{J} &= 14\text{J} = \text{max}
 \end{aligned}$$

d) $PE_i + KE = 14\text{J}$

$$-2\text{J} + KE_i = 14\text{J}$$

$$KE_i = 16\text{J}$$

$$\frac{1}{2}mv^2 = 16\text{J}$$

$$mv^2 = 32\text{J}$$

$$2\text{kg}v^2 = 32\text{J}$$

$$\sqrt{v^2} = \sqrt{16\text{m}^2/\text{s}^2}$$

$$v = 4\text{m/s}$$

at $x=6$, $v = 4\text{m/s}$

e) $(5.5, -4), (6.5, 0)$

$$a = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - (-4)}{6.5 - 5.5} = 4\text{J/m} = \frac{4\text{kg}\cdot\text{m}^2/\text{s}^2}{2\text{kg}} = 2\text{m/s}^2$$

$a = 2\text{m/s}^2$

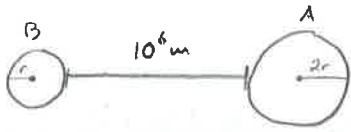
~~$F = -dE$~~

$$F_x = -\frac{d(PE)}{dx} = -\left(\frac{4\text{J}}{\text{m}}\right) = m\vec{a}$$

$$a = -2\text{m/s}^2$$

2kg

3.



volume

$m \propto V \propto r^3$

a) $d = \frac{m_A}{V_A}$ $d = \frac{m_B}{V_B}$
 $m_A = dV_A$ $m_B = dV_B$
 $m_A = d\left(\frac{4}{3}\pi r^3\right)$ $m_B = d\left(\frac{4}{3}\pi 8r^3\right)$
 $m_A = \frac{4\pi r^3}{3} d$ $m_B = 8\left(\frac{4\pi r^3}{3}\right) d$

$m_A = 8m_B$

b) $F_A = F_B$

c) $F = m_A a_A$

$a_A = \frac{F}{8m_B}$ $a_B = \frac{F}{m_B}$

$a_A = a_B$

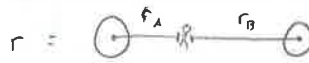
$a_A = \frac{F}{8m_B} = \frac{F}{m_B} = a_B$

$\frac{a_A}{\frac{F}{8m_B}} = \frac{a_B}{\frac{F}{m_B}} \Rightarrow \frac{8m_B a_A}{F} = \frac{m_B a_B}{F} \left(\frac{F}{8m_B}\right)$
 $a_A = \frac{1}{8} a_B$

$a_A = \frac{1}{8} a_B$

d) $F_A = G \frac{m_A m_A}{r^2}$

$F_B = G \frac{m_B m_B}{r^2}$



$F_A = F_B$

~~$G \frac{m_A m_A}{r^2} = G \frac{m_B m_B}{r^2}$~~

$\frac{8m_A}{r_A^2} = \frac{m_B}{r_B^2}$

$\frac{m_A r_A^2}{m_A} = \frac{8m_B r_B^2}{m_B}$

$\sqrt{r_A^2} = \sqrt{8r_B^2}$

$r_A = \sqrt{8} r_B$

$r_A = \sqrt{8} r_B$

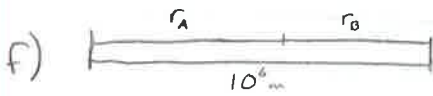
e) ~~$mg_A h_A = mg_B h_B$~~ → only true when $F_g = mg = \text{const.}$

$\frac{m_A h_A}{r_A^2} = \frac{m_B h_B}{r_B^2}$

$\frac{m_A h_A}{r_A^2} = \frac{8m_B h_B}{r_B^2}$

$\frac{m_A}{r_A} = \frac{m_B}{8r_B}$ $\frac{1}{r_A} = \frac{1}{8r_B}$

$r_A = 8r_B$



$$r_A = \sqrt{8} r_B$$

$$10^6 \text{ m} = r_A + r_B$$

$$10^6 \text{ m} = \sqrt{8} r_B + r_B$$

$$\frac{10^6 \text{ m}}{(1+\sqrt{8})} = \frac{(1+\sqrt{8}) r_B}{(1+\sqrt{8})}$$

$$r_B = 261,203.9 \text{ m}$$

$$r_A = \sqrt{8}(261,203.9 \text{ m}) = \boxed{738,796.1 \text{ m}}$$

$$r_A = 8r_B$$

$$10^6 \text{ m} = r_A + r_B$$

$$= r_A + \frac{r_A}{8}$$

$$= \frac{9}{8} r$$

$$r = \boxed{1,125,000 \text{ m}}$$

g) use momentum, because they have the same momentum at the beginning

$$m_A v_A = m_B v_B$$

$$m_A v_A = 8 m_B v_B$$

$$m_A v_A = \frac{m_A}{8} v_B$$

$$v_A = \frac{1}{8} v_B$$

h) 

$$t = 5.52 \text{ s}$$

$$\boxed{v_A = \frac{1}{8} v_B}$$

$$\vec{p}_{\text{system}} = 0$$

$$\vec{p}_A = -\vec{p}_B$$



$$r_A = 2r_B$$

$$r_B = 10^4 \text{ m}$$

$$r_A = 2 \times 10^4 \text{ m}$$

$\forall \rho r^3$, so

$$m_A = 8m_B$$

$$r_e \sim 6.4 \times 10^6 \text{ m}$$

$$m_e \sim 6.0 \times 10^{24} \text{ kg}$$

$$r_e = 6.4 \times 10^2 r_B$$

so, if all planets (A, B, Earth have same density):

$$m_e = (6.4 \times 10^2)^3 m_B \approx 260 \times 10^6 m_B \approx 3 \times 10^8 m_B$$

$$m_B \approx 0.25 \times 10^8 m_e \approx 0.75 \times 10^{16} \text{ kg}$$

$$m_a = 8m_B = 6 \times 10^{16} \text{ kg}$$

We know $v_0 = 0$, so the kinetic energy they have at the end is equal to the ΔPE

$$PE = -\frac{m_A m_B}{r} G \quad r_0 = 10^6 \text{ m} \quad r_f = r_A + r_B = 3 \times 10^4 \text{ m}$$

$$\Delta PE = -\frac{m_A m_B}{r_f} G - \left(-\frac{m_A m_B}{r_0} G \right) = m_A m_B G \left(\frac{1}{r_0} - \frac{1}{r_f} \right)$$

$$= 6 \times 10^{16} \text{ kg} \cdot 0.75 \times 10^{16} \text{ kg} \cdot 6.7 \times 10^{-11} \frac{\text{Nm}^2}{\text{kg}^2} \left(\frac{1}{10^6 \text{ m}} - \frac{1}{3 \times 10^4 \text{ m}} \right)$$

$$\approx -10 \times 10^{17} \text{ J} = -10^{18} \text{ J}$$

so the KE gained $\sim 10^{18} \text{ J} = KE_A + KE_B$

$$\text{So } 10^{18} \text{ J} = \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2 \quad 2 \text{ unknowns}$$

but to conserve momentum, we know from (2)

$$v_A = \frac{1}{8} v_B, \text{ or we could do this:}$$

knowing $p_A = p_B = p$, we can say

$$10^{18} \text{ J} = KE_A + KE_B = \frac{p_A^2}{2m_A} + \frac{p_B^2}{2m_B} = p^2 \left(\frac{1}{2m_A} + \frac{1}{2(8)m_A} \right)$$

$$= p^2 \left(\frac{8}{16m_A} + \frac{1}{16m_A} \right)$$

$$= \frac{9}{16m_A} p^2$$

$$p^2 = (KE_{\text{total}}) \frac{16}{9} m_A = 10^{18} \text{ J} \frac{16}{9} \cdot \frac{1}{8} \times 10^{16} \text{ kg J}$$

$$= \frac{32}{9} \cdot 10^{34} \text{ kg} \frac{\text{kg m}^2}{\text{s}^2}$$

$$p^2 \approx 10^{35} \left(\frac{\text{kg m}}{\text{s}} \right)^2$$

$$p \approx 3 \times 10^{17} \text{ kg m/s} = m_B v_B$$

$$v_B \approx \frac{3 \times 10^{17} \text{ kg m/s}}{0.75 \times 10^{16} \text{ kg}} \approx 40 \text{ m/s}$$

$$v_A = \frac{1}{8} v_B \approx 5 \text{ m/s}$$

4. a. person $x = 7 \cdot t$

$v = 7 \text{ m/s}$

• bus $x = 20 + \frac{1}{2}(1)t^2$

$v = 1 \cdot t$

so $7t = 20 + \frac{1}{2}t^2$

$\frac{1}{2}t^2 - 7t + 20 = 0$

$\frac{1}{2}(t^2 - 14t + 40) = 0$

$\frac{1}{2}(t-10)(t-4) = 0$

$t = 4 \text{ or } 10$

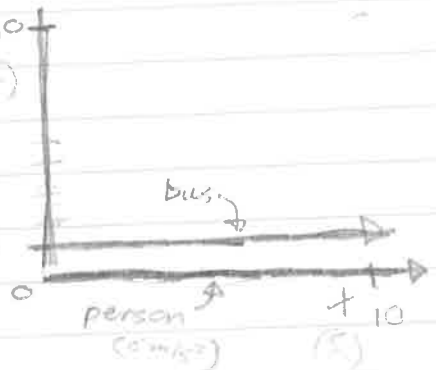
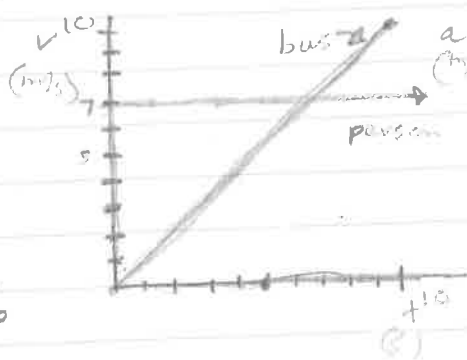
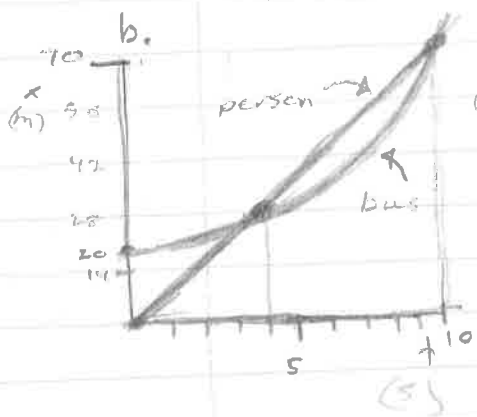
person: $v(4) = 7 \text{ m/s}$

$v(10) = 7 \text{ m/s}$

bus $v(4) = 4 \text{ m/s}$

$v(10) = 10 \text{ m/s}$

you will catch the bus at $t=4$ seconds,
if you were to continue, the bus would
pass you at $t=10$ s.



c. a. person $x = 7 \text{ m/s} \cdot t$

bus $x = 30 \text{ m} + 0 \text{ m/s} \cdot t + \frac{1}{2} (1 \text{ m/s}^2) t^2$

so $7 \text{ m/s} \cdot t = 30 \text{ m} + \frac{1}{2} (1 \text{ m/s}^2) t^2$

$\frac{1}{2} t^2 - 7t + 30 = 0$

$\frac{1}{2} (t^2 - 14t + 60) = 0$

the function has no zero's, therefore the position of the person will never be the same as the position of the bus driver - you will not catch the bus.

b.

