Problem Set #5 due beginning of class, Monday, Feb. 12.

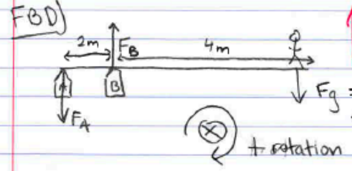
1) Section 4.6 Exercise 2, Diving Board Problem.

1) 4.6 #2 Diving board Lens: Rotational Dynamics, Statics since $\Sigma\vec{\tau}=0$ ✓

$\Sigma\vec{F}=m\vec{a}$

$\vec{\tau}=rF$

FBD)



fulcrum = pt. B

$\tau_A+\tau_g+F_B=0$

$rF_A+rF_g+rF_B=0$

$(2m)(F_A)+(4m)(900N)+0(F_B)=0$

$2mF_A+(-3600N\cdot m)=0$

$2m\,F_A=3600N\cdot m$

$\boxed{F_A=-1800N}$

$(0m)F_A+2m(F_B)+6m(900N)=0$

$2m(F_B)-5400\,N\cdot m=0$

$F_B=\frac{5400\,N\cdot m}{2}$

B	2700 N
A	−1800 N
PERSON	−900 N

✓

$\Sigma\vec{F}=F_A+F_B+F_G=0$

$=\boxed{-1800\,N+2700\,N-900\,N=0}$

$\Sigma\vec{\tau}=\vec{\tau}_A+\vec{\tau}_B+\vec{\tau}_G=0$

$=F_A(2m)+F_B(0m)+F_G(4m)=0$

$-1800N(2m)+0+900N\cdot 4m=0$

$\boxed{-3600\,N\cdot m+3600\,N\cdot m=0}$

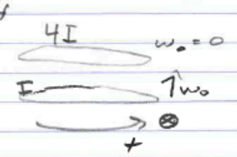
2) Section 4.7, Exercise 1. Dropping larger disk on rotating disk

Note that 4/5 of the original kinetic energy is "lost" to thermal energy.

2) 4.7 Ex 1

a) Angular Momentum: $\vec{l}$ is conserved since no outside torques interact. ✓

$\Sigma\vec{l}_i=\Sigma\vec{l}_f$



$\vec{l}=I\omega$

$I_1\omega_1+I_2\omega_2=I_f\omega_f$

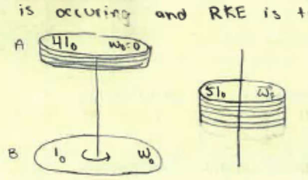
$4I(0)+I\omega_0=I_f\omega_f$

$I\omega_0=5I\omega_f$

$\boxed{\omega_f=\frac{1}{5}\omega_0}$ ✓

b) Energy: similar to a linear inelastic collision, mechanical energy is not conserved, some being lost to E_{th}. Yet energy of the system is conserved.

2) Angular momentum lens and Rotational Energy lens because an inelastic collision between 2 bodies is occuring and RKE is transformed



$l_i=l_f$

$l_A+l_B=0$

Assuming that this a inelastic collision between 2 rotating bodies, RKE is not conserved b/c Energy is lost as heat due to friction

$4I_0\omega_0+I_0\omega_0=5I_f\omega_f$

$1\cdot I_0\omega_0=5I_f\omega_f$

$\Rightarrow\boxed{\omega_f=\frac{1}{5}\omega_0}$ ✓

$\frac{1}{2}I_0\omega_0^2\Rightarrow\frac{1}{2}(5I_0)(\frac{1}{5}\omega_0)^2$

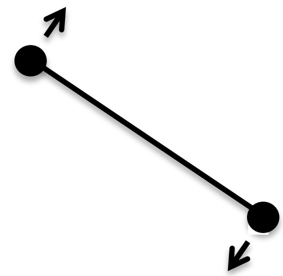
$\frac{5}{2}I_0\frac{1}{25}\omega_0^2$

$\frac{1}{2}I_0\omega_0^2\Rightarrow\frac{1}{10}I_0\omega_0^2$

c) the ΔL would be $+\frac{4}{5}L$ for the stationary disks and $-\frac{4}{5}L$ moving disks. ✓

b) $\boxed{E_{th}=\frac{4}{10}I_0\omega_0^2}$ ✓

3) This is a variation of Section 4.7, Exercise 2



(14) 4.7 Ex) 2

a) - Momentum is conserved b/c there are no outside forces.
- Energy is always conserved, but KE would also be conserved here b/c there is no loss to heat. maybe!
- Kinematics, $\vec{v}$ stays the same, so $\vec{\omega}$ would increase since the radius decreases.
- There are no outside force. The only force is that of the motor pulling the mass's inward.

b) $I = mr^2$, $r = \frac{1}{3} r_0$, so $I_F = \frac{1}{9} I_0$

c) $L_0 = L_F$; Momentum lens

$I_0 \vec{\omega}_0 = I_F \vec{\omega}_F$; $I_F = \frac{1}{9} I_0$

$\cancel{I_0} \vec{\omega}_0 = \frac{1}{9} \cancel{I_0} \vec{\omega}_F$

$\vec{\omega}_0 = \frac{1}{9} \vec{\omega}_F$, $\vec{\omega}_F = 9 \vec{\omega}_0$

d) Energy lens, there is KE_{rot}

$KE_0 = \frac{1}{2} I_0 \vec{\omega}_0^2$ $\quad \frac{1}{2} I_0 \vec{\omega}_0^2$; $\frac{1}{2}(\frac{1}{9} I_0)(9\vec{\omega}_0)^2$

$KE_F = \frac{1}{2} I_F \vec{\omega}_F^2$ $\quad \frac{1}{2} I_0 \vec{\omega}_0^2$; $\frac{1}{2}(9) I_0 \omega_0^2$

$KE_F = 9 KE_0$ $\quad$ I guess KE is not conserved

yes!

e) Energy lens

$\frac{1}{2} I_0 \vec{\omega}_0^2 = \frac{1}{2}(\frac{1}{9} I_0)(\vec{\omega}_F)^2$

$\frac{1}{2} I_0 \vec{\omega}_0^2 = \frac{1}{18}(I_0)(\vec{\omega}_F)^2$ so $\vec{\omega}_F = \sqrt{2}\, \vec{\omega}_0$

f) Yes, momentum lens

$L_F = \frac{\sqrt{2}}{9} L_0$

g) I think that L is conserved and KE_{rot} is not, because work must be done in order to move the masses inward. yes!

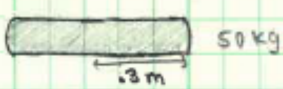
4) You have an ax to grind, and you decide to grind it on the outer rim of a round 5 kg stone grinding wheel of uniform thickness and radius 30 cm. The coefficient of friction between steel and stone is 0.3. You spin the wheel up to 1000 rpm with a 100 W motor.

a) What is the angular velocity of 1000 rpm?

b) How long does it take to spin the wheel up to 1000 rpm? What lens do you use?

c) Then I push the ax against the wheel with a force of 100 N and the sparks fly! But as soon as you start, the electricity goes out and the wheel is spinning freely without power. What is the angular acceleration of the wheel as you push against it with the ax?

4.



$\mu = 0.3$ $\quad$ spun @ 1000 rpm w/ 100 W motor

Lens: Rotational Dynamics because a force is causing acceleration

$I = \frac{1}{2}(50\text{ kg})(.3\text{m})^2 = 25 \times .09 = 2.25$

$1\text{ rpm} = \frac{2\pi}{60}\text{ rad/s}$

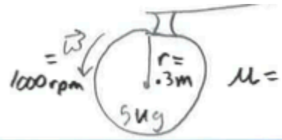
$\omega = \frac{d\theta}{dt}$

a) $1000\text{ rpm} \cdot \frac{2\pi/60\text{ s}}{1\text{ rpm}} = 200\cancel{0}\pi/6\cancel{0}\text{ s}$

$\boxed{33.33\ \pi/\text{s}}$

Because pi is about 3, ω ~ 100/s or 100 radians/s.

Noting that $I = \frac{1}{2} mr^2$ for a solid disk of uniform thickness,



$\mu = .3$

a) Energy lens, b/c the work done by the motor is converted to the rotational kinetic energy of the wheel

$KE_{rot} = \frac{1}{2} I \omega^2$; $I = \frac{1}{2} m r^2$ (disk)

$\vec{\omega} = \frac{1000 \text{ rot}}{\text{min}} = \frac{(2000\pi)}{60s} \approx 100/s$

$KE_{rot} = \frac{1}{2}(\frac{1}{2} 5kg (.3m)^2)(100/s)^2$

$KE_{rot} = 1{,}125 J$

$P_{motor} = 100W = \frac{100J}{s}$; $1125J \cdot \frac{s}{100J} \approx \boxed{11.25s}$

b) Dynamics lens, b/c there are forces and accelerations.

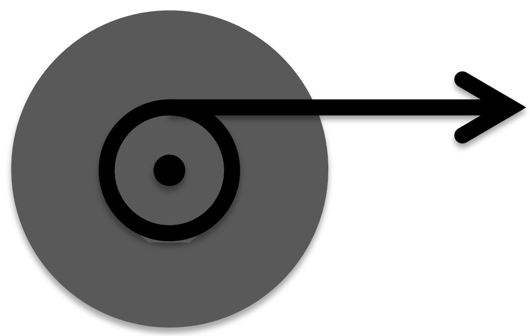
$\Sigma \vec{\tau} = I \vec{\alpha}$ (For Fixed I)

$\tau = F \cdot r = F_F \cdot r = \mu F_N \cdot r$

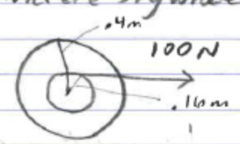
$\tau = (.3)(100N) \cdot (.3m) = 9Nm$

$\vec{\alpha} = \frac{9Nm}{\frac{1}{2}(5kg)(.3m^2)} \approx \boxed{40/s^2}$

5) A concrete flywheel of uniform thickness has a mass of 50 kg and a radius of 40 cm. If I pull on the string with a force of 100 N that is wound around a pulley of radius 16 cm.



Concrete flywheel



$m = 50 kg$

Rotational dynamics: Forces acting and torque, to cause α.

$\Sigma \vec{F} = m\vec{a}$ $\quad \vec{\tau} = I\vec{\alpha}$

$I = \frac{1}{2} m r^2$

Kinematics: function of time

a) $\vec{F}(r) = [mr^2 \frac{1}{2}]\alpha$

$100N(.16m) = [(50kg)(.4m)^2 \frac{1}{2}]\alpha$

$16 N\cdot m = \frac{1}{2} 50kg \cdot .16m \, \alpha$

$\alpha = 4 \, 1/s^2$

$\omega = \int \alpha \, dt$

$= 4 s^{-2}(10s)$

$\boxed{\omega = 40 \, 1/s}$

b) Lens: Energy. Work done over a certain distance causing change in E.

$E_i = 0 \quad W = F\,dx = 100N \cdot (2m)$

$\boxed{W = 200J} = \Delta E$

$E_{rot} = \frac{1}{2} I \omega^2$

$I = \frac{1}{2} m r^2 = 50kg(.4m)^2(\frac{1}{2})$

$200J = \frac{1}{2}(4 kg\,m^2)\omega^2$

$100 \, 1/s^2 = \omega^2$ $\quad \boxed{\omega = 10 \, 1/s}$

c) $\omega_f = 7.07 \, 1/s$ $\quad$ Ave $\omega = \frac{7.07 \, 1/s}{2} = \boxed{3.51 \, 1/s}$

$\alpha = \frac{\tau}{I}$ $\quad \alpha = \frac{d\omega}{dt}$ $\quad 4 \, 1/s^2 = \frac{10 \, 1/s}{dt} = \boxed{2.5 s}$

$= \frac{.16m \cdot 100N}{\frac{1}{2} \cdot 50kg(.4)^2} = \frac{16 N\cdot m}{\frac{1}{2} 8 kg\,m^2} = 4 \, 1/s^2$

Ave Power $= \frac{\Delta E}{\Delta t} = \frac{200J}{2.5s} = \boxed{80W}$

6) Chapter 5.0, Exercise 1, You see something moving in a circle.

(3) • Yes, from dynamics lens, $\vec{a}_c \neq 0$, so there must be a force acting on the rock.

• Dynamics lens → there are forces + accelerations

$\Sigma \vec{F} = m\vec{a}_c$; $\vec{a}_c = \frac{v^2}{r} = \frac{(10 m/s)^2}{20m} = \boxed{5 m/s^2}$

• $\Sigma \vec{F} = m\vec{a}_c$, $F = (10kg)(5 m/s^2) = (50N)$

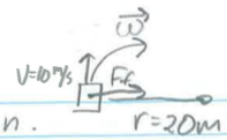
• I have no idea b/c I can't see what is going on.

• Tension Force

$F_T = m\vec{a}_c = (50N)$

• 

The velocity of the rock is always tangential to its circular path. If the string breaks, the rock no longer has a centripetal acceleration; no force keeping its motion circular. So, the rock will continue in a straight line.



• Must be friction.

$\Sigma \vec{F} = m\vec{a}_c$; $F_f = m\vec{a}_c$; $\mu F_N = m\vec{a}_c$

$\mu = \frac{m\vec{a}_c}{F_N} = \frac{m\vec{a}_c}{mg} = \frac{\vec{a}_c}{g} = \frac{5 m/s^2}{10 m/s^2} = \boxed{\frac{1}{2}}$

- This is static friction b/c the tires are no sliding, they are rolling.

7) Chapter 5.1 Exercise 1, Lunar period!

(4) - Dynamics lens, involves forces and acceleration

$\frac{385{,}000 km}{6400 km} = 60$. Radius from Earth to moon is 60 times greater than Earth's center to surface. Inverse square law says that gravity will be equal to $10 m/s^2 \cdot \frac{1}{r^2}$

So $10 m/s^2 \cdot \frac{1}{60^2} = \boxed{.002 m/s^2}$. - This acceleration is caused by gravity

- Moon's acceleration is the same as that. Moon just has tangential velocity, which is why it doesn't crash into Earth. There is just one force acting on the moon. This doesn't change.

- a_{grav} is directed toward center, so it is equivalent so centripetal acceleration.

$a_{grav} = a_c$

$.002 m/s^2 = \frac{v^2}{385{,}000{,}000 m}$ → $\boxed{V = 880 m/s}$

Circumference is distance

- Period = time of orbit, Kinematics lens, involves motion and time.

$\Delta x = vt$ → $\frac{\Delta x}{v} = t$ → $\frac{2\pi(385{,}000{,}000 m)}{880 m/s} = \boxed{2748893 s} = \boxed{31 days}$

8) Chapter 5.2, Exercise 2, Lower Earth Orbit The key to this one is that the moon is 60 times further from the earth's center as the earth's radius… and the force of gravity scales like r^{-2}.

(5) Dynamics lens - There are forces + accelerations

I made

At sea level, $R_S = 6{,}400\ km$

At LEO, $R_L = 6{,}560\ km$

So, $R_L = (1.025) R_S$

So $F_L = (1.025^2) F_S = (1.051) F_S$

So $\boxed{\vec{a}_L = 1.051\ \vec{a}_S}$, so a factor of 1.051 is roughly the same

- Kinematics lens b/c motion of sat. is an explicit function of time.

$\vec{a}_c = \frac{v^2}{r}$, $10\ m/s^2 = \frac{v^2}{6560\ km}$,

$V = \sqrt{(6560000\ m)(10\ m/s^2)}$, $\boxed{V = 8100\ m/s}$

$T = \Delta t = \frac{\Delta d}{\bar{v}} = \frac{2\pi r}{8100\ m/s} = $ 1.5 hours

24 hours in a day, going around Earth 1.5 hours at a time

So 16 times