Problem Set #5 due beginning of class, Tuesday, May 8.

1) Section 4.6 Exercise 2, Diving Board Problem. I almost don't want to post this solution because we did such a good job with it in class. Below is a very good protocol for solving the statics problem. However, in embracing this method, don't lose sight of the good conceptual and visual way we looked at it in class Tuesday.

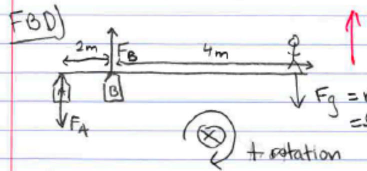
1) 4.6 #2 Diving board

Lens: Rotational Dynamics, Statics since $\Sigma \vec{\tau} = 0$

$\Sigma \vec{F} = m\vec{a}$

$\vec{\tau} = rF$

FBD)



fulcrum = pt. B

$\tau_A + \tau_g + F_B = 0$

$rF_A + rF_g + rF_B = 0$

$(2m)(F_A) + (4m)(900N) + 0(F_B) = 0$

$2mF_A + (-3600N\cdot m) = 0$

$2m\,F_A = 3600N\cdot m$

$\boxed{F_A = -1800N}$

$(0m)F_A + 2m(F_B) + 6m(900N) = 0$

$2m(F_B) - 5400\,N\cdot m = 0$

$F_B = \frac{5400\,N\cdot m}{2}$

B	2700 N
A	−1800 N
PERSON	−900 N

$\Sigma \vec{F} = F_A + F_B + F_G = 0$

$= \boxed{-1800\,N + 2700\,N - 900\,N = 0}$

$\Sigma \vec{\tau} = \vec{\tau}_A + \vec{\tau}_B + \vec{\tau}_G = 0$

$= F_A(2m) + F_B(0\,m) + F_G(4m) = 0$

$-1800N(2m) + 0 + 900N\cdot 4m = 0$

$\boxed{-3600\,N\cdot m + 3600\,N\cdot m = 0}$

2) Section 4.7, Exercise 1. Dropping larger disk on rotating disk

Note that 4/5 of the original kinetic energy is "lost" to thermal energy.

2) 4.7 Ex 1

a) Angular Momentum: $\vec{l}$ is conserved since no outside torques interact.

$\Sigma l_i = \Sigma l_f$

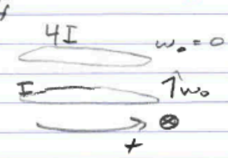
$\vec{l} = I\omega$

$I_1\omega_1 + I_2\omega_2 = I_f\omega_f$

$4I(0) + I\omega_0 = I_f\omega_f$

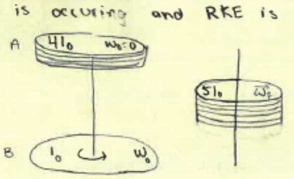
$I\omega_0 = 5I\omega_f$

$\boxed{\omega_f = \frac{1}{5}\omega_0}$



b) Energy: similar to a linear inelastic collision, mechanical energy is not conserved, some being lost to E_{th}. Yet energy of the system is conserved.

2) Angular momentum lens and Rotational Energy lens because an inelastic collision between 2 bodies is occuring and RKE is transformed



$l_i = l_f$

$l_A + l_B = 0$

Assuming that this a inelastic collision between 2 rotating bodies, RKE is not conserved b/c Energy is lost as heat due to friction

$4I_0\omega_0 + I_0\omega_0 = 5I_f\omega_f$

$1\cdot I_0\omega_0 = 5I_f\omega_f$

$\Rightarrow \boxed{\omega_f = \frac{1}{5}\omega_0}$

$\frac{1}{2}I_0\omega_0^2 \Rightarrow \frac{1}{2}(5I_0)(\frac{1}{5}\omega_0)^2$

$\frac{5}{2}I_0\frac{1}{25}\omega_0^2$

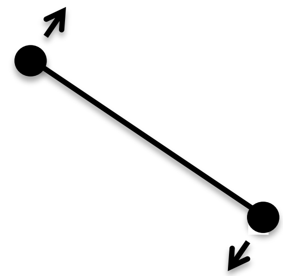
$\frac{1}{2}I_0\omega_0^2 \Rightarrow \frac{1}{10}I_0\omega_0^2$

b) $\boxed{E_{th} = \frac{4}{10}I_0\omega_0^2}$

c) the ΔL would be $+\frac{4}{5}L$ for the stationary disks and $-\frac{4}{5}L$ moving disks.

3) This is a variation of Section 4.7, Exercise 2

Two identical bodies are tied together with a string, are spinning in space about the center at angular speed, ω_i, when a motor at the center pulls them both inward such that the final diameter of their paths is 1/3 the original diameter, or, $d \Rightarrow$ <u>1/3 d_i</u>. If this is in outer space, we can be sure there are no outside forces. We might consider conserving energy and/or angular momentum.



a) Is it possible for the angular momentum to change? If so how?

b) Is it possible for energy to change? if so, where did the energy go or come from?

Is it possible to conserve both angular momentum ***<u>and</u>*** energy? Let's find out!

c) What happens to the moment of inertia with this change? $I \Rightarrow$ ___ I_i

d) If we conserve angular momentum what should be the new angular velocity? $\omega \Rightarrow$ ___ ω_i,

e) Would this change the kinetic energy? If so, by what factor: $KE \Rightarrow$___ KE_i

f) Can we conserve angular momentum ***<u>and</u>*** energy? If not, which one must have changed, and where did that change come from?

14 4.7 Ex) 2

a) - Momentum is conserved b/c there are no outside forces.

- Energy is always conserved, but KE would also be conserved here b/c there is no loss to heat ... maybe!

- Kinematics, $\vec{v}$ stays the same, so $\vec{\omega}$ would increase since the radius decreases.

- There are no outside force. The only force is that of the motor pulling the mass's inward.

b) $I = mr^2$, $r = \frac{1}{3} r_0$, so $\boxed{I_F = \frac{1}{9} I_0}$

c) $L_0 = L_F$; Momentum cons

$I_0 \vec{\omega}_0 = I_F \vec{\omega}_F$; $I_F = \frac{1}{9} I_0$

$\cancel{I_0} \vec{\omega}_0 = \frac{1}{9} \cancel{I_0} \vec{\omega}_F$

$\vec{\omega}_0 = \frac{1}{9} \vec{\omega}_F$, $\boxed{\vec{\omega}_F = 9\vec{\omega}_0}$

d) Energy cons, there is KE_{rot}

$KE_0 = \frac{1}{2} I_0 \vec{\omega}_0^2$ $\quad \frac{1}{2} I_0 \vec{\omega}_0^2$; $\frac{1}{2}(\frac{1}{9} I_0)(9\vec{\omega}_0)^2$

$KE_F = \frac{1}{2} I_F \vec{\omega}_F^2$ $\quad \frac{1}{2} I_0 \vec{\omega}_0^2$; $\frac{1}{2}(9) I_0 \omega_0^2$

$\boxed{KE_F = 9 KE_0}$ I guess KE is not conserved

yes!

e) Energy cons

$\frac{1}{2} I_0 \vec{\omega}_0^2 = \frac{1}{2}(\frac{1}{9} I_0)(\vec{\omega}_F)^2$

$\frac{1}{2} I_0 \vec{\omega}_0^2 = \frac{1}{18}(I_0)(\vec{\omega}_F)^2$ so $\boxed{\vec{\omega}_F = \sqrt{2}\, \vec{\omega}_0}$

f) Yes, momentum cons

$\boxed{L_F = \frac{\sqrt{2}}{9} L_0}$

g) I think that L is conserved and KE_{rot} is not, because work must be done in order to move the masses inward. yes!

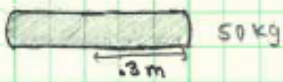
4) You have an ax to grind, and you decide to grind it on the outer rim of a round 5 kg stone grinding wheel of uniform thickness and radius 30 cm. The coefficient of friction between steel and stone is 0.3. You spin the wheel up to 1000 rpm with a 100 W motor.

a) What is the angular velocity of 1000 rpm?

b) How long does it take to spin the wheel up to 1000 rpm? What lens do you use?

c) Then I push the ax against the wheel with a force of 100 N and the sparks fly! But as soon as you start, the electricity goes out and the wheel is spinning freely without power. What is the angular acceleration of the wheel as you push against it with the ax?

4.



$\mu = 0.3$ spun @ 1000 rpm w/ 100 W motor

Lens: Rotational Dynamics because a force is causing acceleration

$I = \frac{1}{2}(50\,kg)(.3m)^2 = 25 \times .09 = 2.25$

$1\ rpm = \frac{2\pi}{60}\ rad/s$

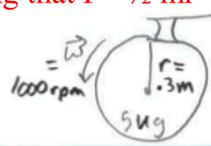
$\omega = \frac{d\theta}{dt}$

a) $1000\ rpm \cdot \frac{2\pi/60s}{1\ rpm} = 200\cancel{0}\pi/6\cancel{0}s$

$\boxed{33.33\ \pi/s}$

Because pi is about 3, ω ~ 100/s or 100 radians/s.

Noting that $I = \frac{1}{2} mr^2$ for a solid disk of uniform thickness,



$\mu = .3$

a) Energy lens, b/c the work done by the motor is converted to the rotational kinetic energy of the wheel

$KE_{rot} = \frac{1}{2} I \omega^2$; $I = \frac{1}{2} mr^2$ (disk)

$\vec{\omega} = \frac{1000\ rot}{min} = \frac{2000\pi}{60s} \approx 100/s$

b) $KE_{rot} = \frac{1}{2}\left(\frac{1}{2} 5kg (.3m)^2\right)(100/s)^2$

$KE_{rot} = 1,125 J$

$P_{motor} = 100W = \frac{100J}{s}$; $1125J \cdot \frac{s}{100J} \approx 11.25 s$

b) Dynamics lens, b/c there are forces and accelerations.

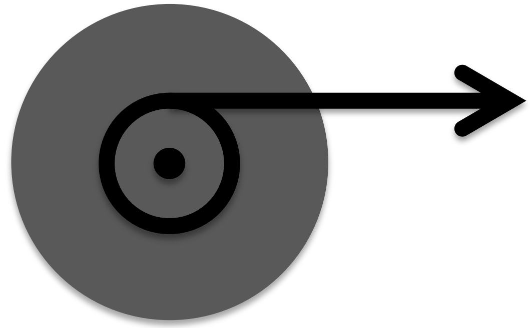
$\Sigma \vec{\tau} = I \vec{\alpha}$ (For Fixed I)

$\tau = F \cdot r = F_F \cdot r = \mu F_N \cdot r$

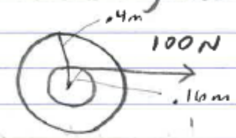
$\tau = (.3)(100N) \cdot (.3m) = 9Nm$

$\vec{\alpha} = \frac{9Nm}{\frac{1}{2}(5kg)(.3m^2)} \approx \boxed{40/s^2}$

5) A concrete flywheel of uniform thickness has a mass of 50 kg and a radius of 40 cm. If I pull on the string with a force of 100 N that is wound around a pulley of radius 16 cm.



Concrete flywheel



$m = 50$ kg

Rotational dynamics: Forces and torque, acting to cause α.

$\sum \vec{F} = m\vec{a}$ $\quad \vec{\tau} = I\vec{\alpha}$

$I = \frac{1}{2}mr^2$

Kinematics: function of time

(a) $\vec{F}(r) = [mr^2 \tfrac{1}{2}]\alpha$

$100\,N(.16m) = [(50kg)(.4m)^2 \tfrac{1}{2}]\alpha$

$16\,N\cdot m = \tfrac{1}{2} 50\,kg \cdot .16\,m\ \alpha$

$\alpha = 4\ ^1/_{s^2}$

$\omega = \int d\alpha\, dt$

$= 4\,s^{-2}(10\,s)$

$\boxed{\omega = 40\ ^1/_s}$

(b) Lens: Energy. Work done over a certain distance causing change in E.

$E_i = 0$ $\quad W = F\,dx = 100N \cdot (2m)$

$\boxed{W = 200J} = \Delta E$

$E_{rot} = \frac{1}{2}I\omega^2$ $\quad I = \frac{1}{2}mr^2 = 50kg(.4m)^2(\tfrac{1}{2})$

$200\,J = \tfrac{1}{2}(4\,kg\,m^2)\omega^2$

$100\ ^1/_{s^2} = \omega^2$ $\quad \boxed{\omega = 10\ ^1/_s}$

(c) $\omega_f = 7.07\ ^1/_s$ $\quad$ Ave $\omega = \frac{7.07\ ^1/_s}{2} = \boxed{3.54\ ^1/_s}$

$\alpha = \frac{\vec{\tau}}{I}$ $\quad \alpha = \frac{d\omega}{dt}$ $\quad 4\ ^1/_{s^2} = \frac{10\ ^1/_s}{dt} = \boxed{2.5\ s}$

$= \frac{.16\,m \cdot 100\,N}{\frac{1}{2} \cdot 50\,kg(.4)^2} = \frac{16\,N\cdot m}{\frac{1}{2} 8\,kg\,m^2} = 4\ ^1/_{s^2}$ $\quad$ Ave Power $= \frac{\Delta E}{\Delta t} = \frac{200\,J}{2.5\,s} = \boxed{80W}$