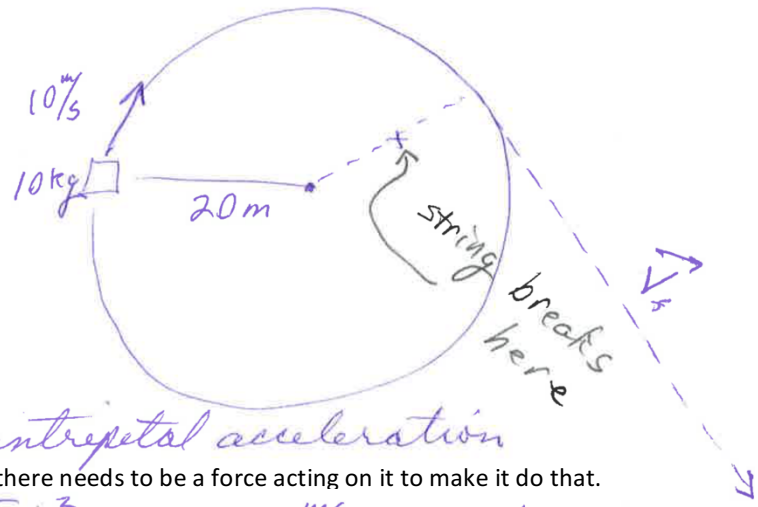
Problem Set #5 due beginning of class, Tuesday, Nov. 1. Remember to start each question with a description of what concept is central to your strategy and *why*. Don't forget your 4 lenses

#1 This is in a video for Tuesday of Week 7. See it early if you like.

#2



a) The $\vec{v}$ is changing direction. This is centripetal acceleration

Things don't accelerate in a circle by themselves, there needs to be a force acting on it to make it do that.

b) $a = \frac{v^2}{r} = \omega^2 r = \left(\frac{0.5}{s}\right)^2 20m = \underline{\underline{5\ m/s^2}}$ or $\frac{1}{2}g$

c) $F = ma = 10kg \cdot 5\ m/s^2 = 50N$ Direction: Inward

d) say it.... "I don't know, but I know there must be *some* force to cause the centripetal acceleration"

e) $T = F = ma = 50N$

f) when string ~~pull~~ breaks, $\vec{F} = 0$, $\vec{a} = 0$, $\vec{v} = const$ straight line like this

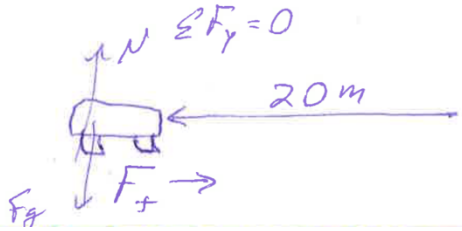
g) This must be gravity! $F = 50N = \frac{m_1 m_2}{r^2} G$ center planet

$m_2 = \frac{50N\, r^2}{m_1 G} = \frac{50N (20m)^2}{10kg \cdot 6.67\times10^{-11} \frac{Nm^2}{kg^2}} \approx 3\times10^{13} kg$

$V_{sphere} = \frac{4}{3}\pi r^3 \approx 4 \cdot 8000 m^3 \approx 32{,}000 m^3 = 3.2\times10^4 m^3$

$\rho = \frac{m}{V} \approx \frac{3\times10^{13} kg}{3\times10^4 m^3} \approx 10^9 \frac{kg}{m^3}$ or 10^6 times the density of water.

h) $F_f = N\mu = 50N$ $\boxed{\mu = 0.5}$



3. In 1930, it was discovered that a beta decay:

$n \Rightarrow p^+ + e^-$

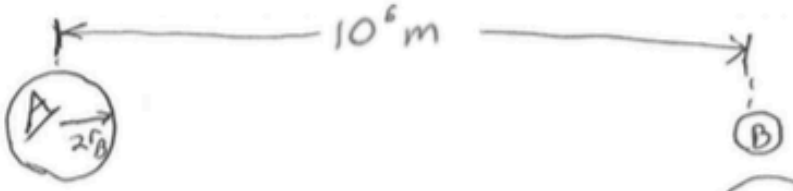
didn't conserve energy, momentum or *angular momentum*. Wolfgang Pauli postulated the creation of a new particle, the *neutrino.* We now estimate that 65 billion neutrinos from the sun pass through each square centimeter on earth, *per second*. How fast does the sun produce neutrinos? We just need to find the number of square centimeters… that these neutrinos are spread out over… so construct a sphere around the sun of radius equal to the earth's orbital radius… ~150 million km, or 1.5×10^{11} m = 1.5×10^{13} cm. The area is $4\pi r^2$ wow… we're looking at about 2×10^{38}/s. How many pass through you during this class? What if my surface area were half a square meter… or 5000 square centimeters. Class is about 3500 s long, then I'd have about 10^{14}. That's a lot. Good thing they don't hurt. How about if you were on Venus? The ratio of the distances of the two planets is in a ratio of about 1:1.4 or square root of two. So the area of the sphere that the neutrinos have to pierce at Venus's distance is half that of the sphere at the earth's distance, so Venus would have about twice the intensity of neutrinos as here at earth.

4. Exercise 1: The distance between the centers of the earth and moon is about 385,000 km and where the radius of the earth is about 6,400 km. Please draw a good picture!

a) If the moon were to stop moving, what about what would be the moon's acceleration toward the earth? I just use a math lens… and inverse square relationship lens. we know that the mass of the earth causes an acceleration of 10 m/s^2 one earth radius from the center. Twice as far would be ¼ that acceleration… the moon is much further than that, or about 60 times as far, so the acceleration should be about 1/3600 as much or about 1/360 m/s^2.
b) What would cause that acceleration? Gravity between the earth and moon.
c) But the moon *is* moving! In this case, should its acceleration be the same as in "a" above? Why or why not? Dynamics, yes. It's the same force of gravity, it'll cause the same acceleration.
d) From your answers above, estimate the speed of the moon in its orbit around the earth. By setting the acceleration = centripetal acceleration, we find about 1.0 km/s.
e) From your answer above, estimate the period of the moon… is it close to a month? Just kinematics, I get about 27 days.
f) Please calculate the mass of the earth. Dynamics because we know the acceleration on the earth's surface, and this is caused by gravitational force. I get 6×10^{24} kg, pretty close.

5. There are two planets with centers 10^6 m apart: Planet A, and Planet B. The radius of Planet A is twice that of planet B, or $r_A=2r_B$. Both planets are made of the same rocks, and therefore have the same density. There are no other objects, so we are only looking at the force of gravity acting between the two planets. *Provide reasons for your answers before showing the work, before showing the answer.*

5



9

a) $m \propto$ Volume $\quad m = V\rho \quad V = \frac{4}{3}\pi r^3$

so as $r = 2r \quad V \Rightarrow 2^3 V_0$

or $V \propto m \quad m \Rightarrow 8 m_0$

Dynamics - Forces + acceleration.

b) this is a <u>single</u> gravitational interaction

$\vec{F}_{AB} = \vec{F}_{BA}$ Forces are =, opposite

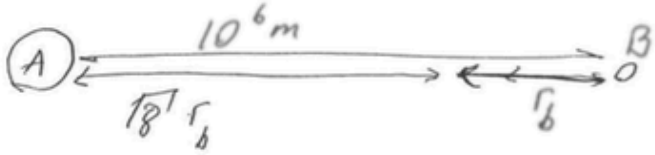
dynamics - Forces and acceleration

c) $a = F/m$ so the larger mass will accelerate <u>less</u>: $a_A = \frac{1}{8} a_B$

d) $F_g = \frac{m_{Planet}\, m_{me}}{(Planet - me)^2} G$ in order to be pulled equally hard by both planets, I must be closer to the smaller one. If the numerator (mass) of a is 8 × the mass of b, then $r_a^2 = 8 r_b^2$ or $r_a = \sqrt{8}\, r_b$

e)

(b) where is $F_g = 0$?



$r_b + \sqrt{8}\, r_b = 10^6 m$

$(1+\sqrt{8})\, r_b = 10^6 m$

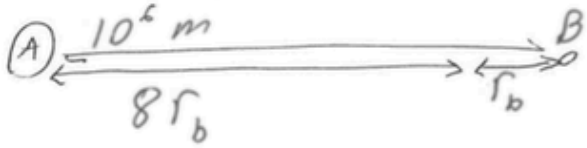
$r_b = \dfrac{10^6 m}{1+\sqrt{8}}$

almost 3

$r_b \lesssim \frac{1}{4}(10^6 m) \approx 2.5 \times 10^5 m$

$r_a = 10^6 m - r_a$, $r_a \sim 7.5 \times 10^5 m$

Where are the potentials = ?



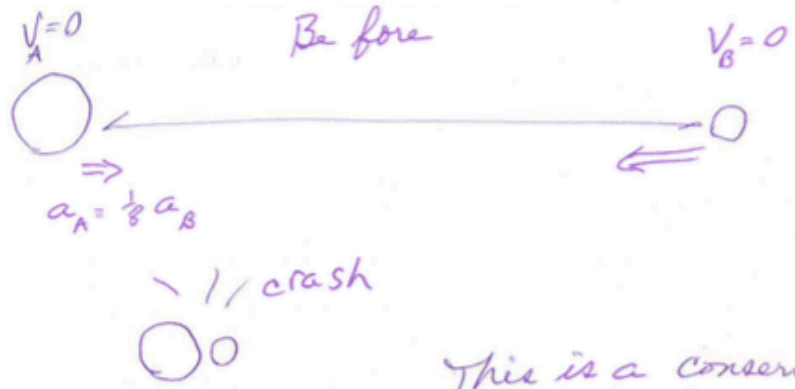
$9 r_b = 10^6 m$

$r_b = \frac{1}{9} \cdot 10^6 m$

$\approx \underline{1.1 \times 10^5 m}$

$r_a = 10^6 m - r_b \approx \underline{8.9 \times 10^5 m}$

f)



This is a conservation of momentum Problem!

$\vec{P}_0 = \vec{P}_A + \vec{P}_B = 0$ there's a collision, so $\Sigma \Delta \vec{P} = 0$

right before collision $\vec{P}_A = -\vec{P}_B$!

$$m_A V_A = -m_B V_B$$

$m_A = 8 m_B$, so $V_A = \frac{1}{8} V_B$!

at the end, $\vec{V}_A = \vec{V}_B = \vec{P}_A = \vec{P}_B = \Sigma \vec{P} = 0$ again!

g) If $V_B = 8*V_A$, then the distance to A will be 8 times the distance to B: $r_B = 8*r_A$ and the sum of the two is a million meters!. Find that the distance to the larger planet A is 111,111 m. This position is the center of mass of the two planets! It's the point that the system would balance on if it were a solid system. We'll learn more about the center of mass later.

6)

Here we have neglected to identify the lenses. We can see this is a combination of energy and dynamics: Energy because we are finding velocity by recognizing that the height lost will translate to PE => KE. Dynamics because in a circle we know that we undergo centripetal acceleration and recognize the sum of the forces (in this case gravity and normal force) = ma. Also notice that this student chose to calculate the energy in Joules. This is not necessary. You could skip this step by setting PE=KE and just solving for V^2, which is what you need for centripetal acceleration. In any case, we see that a "9g" force at the bottom is more than a pregnant woman (and many other people) should experience.

a) $PE_I = (70\,kg)(10\,m/s^2)(40\,m) = 28000\,J$

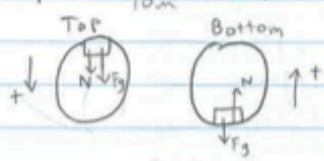
$PE_{Top\ of\ Loop} = (70\,kg)(10\,m/s)(20\,m) = 14000$

$28000\,J = \frac{1}{2}mv^2,\ V_{bottom} = 28.28\,m/s$

$14000\,J = \frac{1}{2}mv^2,\ V_{top} = 20\,m/s$

$a_{c\,bottom} = \frac{(28.28\,m/s)^2}{10\,m} = 80\,m/s^2$

$a_{c\,top} = \frac{(20\,m/s)^2}{10\,m} = 40\,m/s^2$



$\Sigma F_c = ma_c$

$N + F_g = ma_c$

$N = ma_c - F_g$

$N = (70\,kg)(40\,m/s^2) - (70\,kg)(10\,m/s^2) = \boxed{2100\,N}$

$\Sigma F_c = ma_c$

$N - F_g = ma_c$

$N = ma_c + F_g$

$N = (70\,kg)(80\,m/s^2) + (70\,kg)(10\,m/s^2) = \boxed{6300\,N}$

→ As you round the bottom of the loop, you feel more force pushing down on you. This is not a good ride for a pregnant woman!

b) If you start at the height as the top of the loop, you won't have enough velocity to make it around the top of the loop, because if energy is conserved, then KE will be 0 as you approach the top of the loop

c) $PE_I = PE_T + KE_T$

$mgh = mg(20\,m) + \frac{1}{2}mv^2$

$gh = g(20\,m) + \frac{1}{2}v^2$

$\Downarrow$

$(10\,m/s^2)h = (10\,m/s^2)(20\,m) + \frac{1}{2}(10.01\,m/s)^2$

$\boxed{h > 25\,m}$

$\Sigma F = ma_c$

$F_g < ma_c$

$mg < ma_c$

$a_c > g$

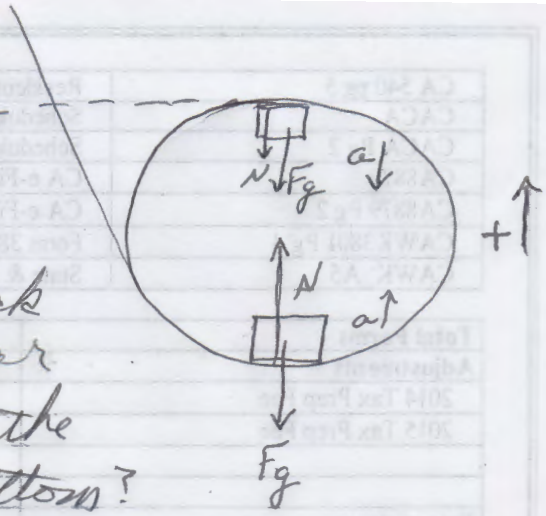
$\frac{v^2}{r} > g$

$v > \sqrt{gr}$

$v > 10\,m/s$

Wait, here is a different, but similar question that may be more helpful to look at

So, how high above the top of the circle would we need to start a frictionless cart from rest to stay on the track at the top of the circular loop and what would the Normal force be at the bottom?



Looking through a dynamics lens, FBD we see:

Top

$\Sigma \vec{F} = m\vec{a}_{Top}$

$-N_T - mg = -ma_T$

or $m\frac{V_T^2}{r} = mg + N$

The slowest you can go is when $N_T = 0$, so this is when $V_T^2 = gr$ (oops! m's cancel)

"T" is for "top"

Bottom

$\Sigma \vec{F} = ma_{Bottom}$

$N_B - mg = ma_{Bottom}$

$N_B = mg + \frac{mV_B^2}{r}$

$= mg + \frac{m5gr}{r} = 6mg$

we can find Δh by using an energy lens because

$\Delta E_p \Rightarrow E_{K_{Top}}$, or $mg\Delta h = \frac{1}{2}mV_T^2 = \frac{1}{2}mgr$

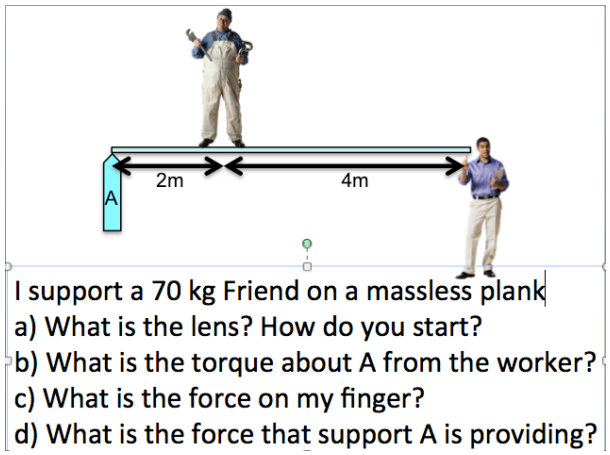
so $\boxed{\Delta h = \frac{1}{2}r}$ regardless of size or mass

we use the energy lens again to find V^2_{Bottom} because

$\Delta E_p \Rightarrow E_{K_{Bottom}}$, or $mg\Delta h = \underbrace{mg\left(2\tfrac{1}{2}r\right) = \tfrac{1}{2}mV_B^2}_{V_B^2 = 5gr}$

$N_B = 6mg$; or 6 times your weight!

7) a) Know how to start a statics problem. We know that acceleration and angular acceleration are both zero, so the sum of the forces and sum of the torques are both zero. THEN we make a good and clear FBD – I can't force you to do this, but you won't get credit for the problem if you don't. Label the forces acting on the body of interest… what is that? It's important that in this scenario, we define the *board* as the body of interest because all the forces (and torques) are acting on it.



b) 1400 Nm into the paper

c) Your finger must provide the opposite torque, with a distance of 6 m from the pivot, or a force of 1400Nm/6m = 233N

d) now we need to balance forces so they add to zero, and see that A must support ~ 467 N

Problem #8 is the same as #1. Sorry for the redundancy.

9. A child's carousel has a mass of 100 kg and a diameter of 3 meters, and is spinning clockwise as viewed from above at 1.5 revolutions per second. Assume that the mass is uniformly distributed over the circular area. Two kids, 30 kg point masses, each are dropped from rest simultaneously on opposite sides of the carousel, 1 meter from the center.
   a) Find the moment of inertia of the carousel and the moment of inertia of the two children.
   b) Find the initial angular velocity, ω_o, please include direction using the right hand rule.
   c) What happens to the rotation rate of the carousel after the kids are dropped onto the surface? Why is this? Please identify the appropriate physics concept in your answer.
   d) Please find the final angular velocity, ω_f.
   e) Please find the initial and final kinetic energy of the carousel + children system before and after the stationary kids were dropped onto the carousel's surface. Was kinetic energy conserved? If not where did it go, and how?

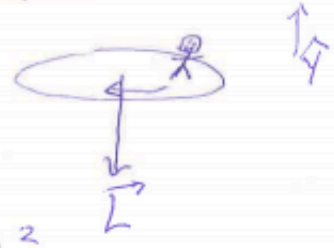
This is a "rotational collision". There are no external torques, so $\Delta\vec{L} = \vec{\tau}\,dt = 0$ conserve $\vec{L}$. KE is lost to heat as the kids slide onto the moving surface.

$\vec{L}_0 = I_0 \vec{\omega}_0 = I_f \omega_f = \vec{L}_0$ (2.25)50

1.5
1.5
1.5
75

a) $I_{disk} = \left(\frac{1}{2}\right) M R^2 = \frac{1}{2}(100\,kg)(1.5\,m)^2 = \underline{112.5\ kg\,m^2}$ 2.25

b) $\omega_0 = \frac{1.5\ \cancel{rev}}{s} \frac{\overbrace{2\pi}^{\sim 6.3}\ radians}{\cancel{rev}} \approx 9.5/s\ (-\hat{y})$



c) $\omega \Downarrow$ $L = I\omega$ ← must decrease ← increase

d) $I_f = I_{disk} + I_{kids}$ $I_{kids} = 2(30\,kg)(1\,m)^2$ (2 kids)

$= 60\ kg\,m^2$

$= 112.5\ kg\,m^2 + 60\ kg\,m^2$

$\approx 172.5\ kg\,m^2$

$L = I_0\omega_0 = I_f\omega_f$

$\omega_f = \frac{I_0\omega_0}{I_f} \approx \frac{112.5\ \cancel{kg\,m^2}\ 9.5/s}{172.5\ \cancel{kg\,m^2}}$

$\approx 6.4/s\ (-\hat{y})$

angular momentum lens

e) $\vec{\tau} = \frac{d\vec{L}}{dt} \doteq \frac{I\,\Delta\omega}{\Delta t} = \frac{112.5\ kg\,m^2 (3.1/s)\hat{y}}{.05\ s}$

$\approx 7000\ Nm\ \hat{y}$

f) at top see above statement

g) closer to the middle would have reduced I_{kids}, reducing $\Delta\omega$