Problem Set #6 due beginning of class, Tuesday, May 15.

1) Chapter 5.0, Exercise 1, You see something moving in a circle.

(3) • Yes, from dynamics lens, $\vec{a}_c \neq 0$, so there must be a force acting on the rock.

• Dynamics lens → there are forces + accelerations

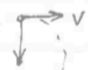
$\Sigma\vec{F} = m\vec{a}_c$; $\vec{a}_c = \frac{v^2}{r} = \frac{(10\,m/s)^2}{20m} = \boxed{5\,m/s^2}$

• $\Sigma\vec{F} = m\vec{a}_c$, $F = (10kg)(5\,m/s^2) = (50N)$

• I have no idea b/c I can't see what is going on.

• Tension Force

$F_T = m\vec{a}_c = (50N)$



• The velocity of the rock is always tangential to its circular path. If the string breaks, the rock no longer has a centripetal acceleration; no force keeping its motion circular. So, the rock will continue in a straight line.

• Must be friction. r=20m

$\Sigma\vec{F} = m\vec{a}_c$; $F_f = m\vec{a}_c$; $\mu F_N = m\vec{a}_c$

$\mu = \frac{m\vec{a}_c}{F_N} = \frac{m\vec{a}_c}{mg} = \frac{\vec{a}_c}{g} = \frac{5\,m/s^2}{10\,m/s^2} = \boxed{\frac{1}{2}}$

- This is static friction b/c the tires are no sliding, they are rolling.

2) Chapter 5.1 Exercise 1, Lunar period!

(4) - Dynamics lens, involves forces and acceleration

$\frac{385{,}000\,km}{6400\,km} = 60$, Radius from Earth to moon is 60 times greater than Earth's center to surface. Inverse square law says that gravity will be equal to $10\,m/s^2 \cdot \frac{1}{r^2}$

So $10\,m/s^2 \cdot \frac{1}{60^2} = \boxed{.002\,m/s^2}$. - This acceleration is caused by gravity

- Moon's acceleration is the same as that. Moon just has tangential velocity, which is why it doesn't crash into Earth. There is just one force acting on the moon. This doesn't change.

- a_{grav} is directed toward center, so it is equivalent so centripetal acceleration.

$a_{grav} = a_c$

$.002\,m/s^2 = \frac{v^2}{385{,}000{,}000\,m}$ → $\boxed{V = 880\,m/s}$

Circumference is distance

- Period = time of orbit, Kinematics lens, involves motion and time.

$\Delta x = vt$ → $\frac{\Delta x}{v} = t$ → $\frac{2\pi(385{,}000{,}000\,m)}{880\,m/s} = \boxed{2748893\,s} = \boxed{31\,days}$

3) Chapter 5.2, Exercise 2, Lower Earth Orbit The key to this one is that the moon is 60 times further from the earth's center as the earth's radius… and the force of gravity scales like r^{-2}.

(3) Dynamics lens - There are forces & accelerations I made

At sea level, $R_S = 6{,}400\,km$

At LEO, $R_L = 6{,}560\,km$

So, $R_L = (1.025) R_S$

So $F_L = (1.025^2) F_S = (1.051) F_S$

So $\boxed{\vec{a}_L = 1.051\, \vec{a}_S}$, so a factor of 1.051 is roughly the same

• Kinematics lens b/c motion of sat. is an explicit function of time.

$\vec{a}_c = \frac{v^2}{r}$, $10\,m/s^2 = \frac{v^2}{6560\,km}$,

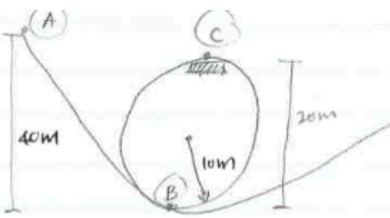
$V = \sqrt{(6560000\,m)(10\,m/s^2)}$, $\boxed{V = 8100\,m/s}$

$T = \Delta t = \frac{\Delta d}{\bar{v}} = \frac{2\pi r}{8100\,m/s} = 1.5$ hours

24 hours in a day, going around Earth 1.5 hours at a time so 16 times

4) Chapter 5.3, Do exercise 1 and 2, but don't hand it in. Does it stay in the bucket? Answers at end of chapter.

5) Chapter 5.3, Exercise 4, Loop the loop.



7) (5.3, #14) You go on a R=10m loop ride, cart is let go on low friction track, pulled downhill by gravity. (I have mass: 70 kg.)

a) Start from vertical height h: 40m. I use an energy lens since I see E_p turning into E_K as the cart moves down the loop. $\sum E = E_p + E_K$, $E_p = mg\Delta h$, $E_K = \frac{1}{2}mv^2$

$E_{p(A)} = E_{K(C)} + E_{p(C)}$ $\qquad$ $E_{p(A)} = E_{K(B)}$

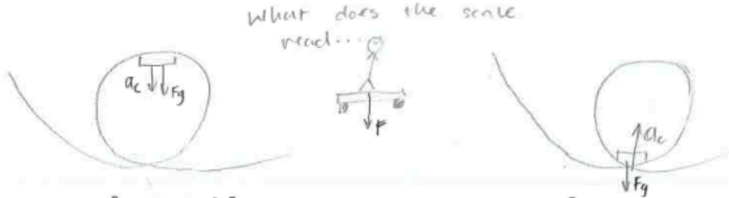
$mgh_A = \frac{1}{2}mv_C^2 + mgh_C$ $\qquad$ $mgh_A = \frac{1}{2}mv_B^2$

$V_C = \sqrt{2g(h_A - h_C)}$ $\qquad$ $V_B = \sqrt{2gh_A}$

$V_C = \sqrt{2(10\,m/s^2)(40m - 20m)}$ $\qquad$ $V_B = \sqrt{2(10\,m/s^2)(40\,m)}$

$V_C = 20\,m/s$ $\qquad$ $V_B = 28.28\,m/s$

Then I use a dynamics lens since I see something moving in a circle, I know $a_c = \frac{v^2}{r}$. so: →



$a_{c(c)} = \frac{V_c^2}{r} = \frac{(20\,m/s)^2}{10\,m}$

$= 40\,m/s^2$

$F_{c(c)} = m\,a_{c(c)}$

$= (70\,kg)(40\,m/s^2)$

$= 2800\ N$

$\sum F = F_{c(c)} - F_g$

$= 2800 - (70\,kg)(10\,m/s^2)$

$= \boxed{2100\ N \text{ at the top}}$

$a_{c(B)} = \frac{V_B^2}{r} = \frac{(28.28\,m/s)^2}{10\,m}$

$= 80\,m/s^2$

$F_{c(B)} = m\,a_{c(B)}$

$= (70\,kg)(80\,m/s^2)$

$= 5600\ N$

$\sum F = F_{c(B)} + F_g$

$= 5600\,N + (70\,kg)(10\,m/s^2)$

$= \boxed{6300\ N \text{ at the bottom}}$

At the bottom, you feel a ~~lot more force than~~ normal pushing up that is $\gg mg$ more than you are used to ~~down on you~~, to the point where it's dangerous, especially for a pregnant woman.

b) I use an energy lens for the same reason as in (a). If I start the cart at the same height as the top of the loop, my E_K at $h = 20\,m$ would be zero — it's all E_p. therefore, I would have zero E_K at the top of the loop, therefore 0 velocity, and I would not be able to complete the loop.

(*) c) I use an energy lens for the same reason.

$\sum E = E_p + E_K$

$mgh_A = mgh_c + \frac{1}{2}mV_{c(c)}^2$

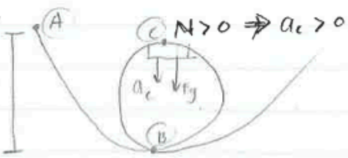
$a_c > g$

$\frac{V_c^2}{r} > g$

$V_c > \sqrt{rg}$

$V_c > \sqrt{(10\,m)(10\,m/s^2)} = 10\,m/s$

$h_A = ?$



$h_A = \frac{(10\,m/s^2)(20\,m) + \frac{1}{2}(10\,m/s)^2}{(10\,m/s^2)}$

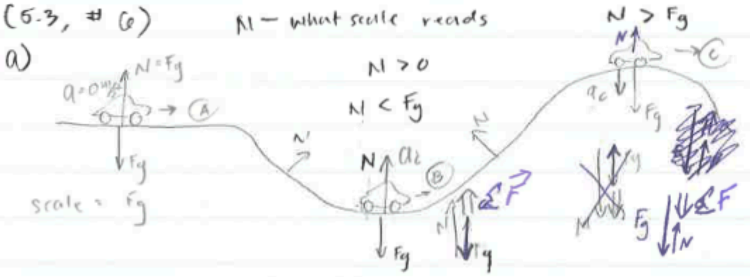
$\boxed{h_A = 25\ m}$

$N > 0$

6) Chapter 5.3, Exercise 6, Driving up and down hills in a car

6) (5.3, # 6) N — what scale reads

a)



I use a dynamics lens here since I see things moving in a circle (somewhat) and forces that cause acceleration.

scale > 0, > F_g

$F_N = mg + \frac{mv^2}{r}$

scale > 0, < F_g

$F_N = mg - \frac{mv^2}{r}$ great!

F_g

b) From a dynamics lens ($\vec{F}$'s and $\vec{a}$'s), we can see that $\Sigma\vec{F} = 0$ in case 1, so you feel completely normal. In case 2, $\vec{a}$ is upward, so you feel "heavier". In case 3, $\vec{a}$ = downward, so you feel "lighter". Does this match experience?

Chapter 5.3, Exercise 7, does the scale read more at the equator? Clearly a dynamics problem because the forces (normal force and gravity) cause acceleration… But the acceleration on the pole is zero (and so you are in equilibrium, and $F_N = F_g$) because I have no velocity there. On the equator, the centripetal acceleration is inward because of the uniform circular motion. You will need a good FBD here to show that at the equator, $F_N < F_g$, where at the pole the forces are equal and opposite. Also, do a good vector addition diagram to show that if the acceleration is inward, the net force is inward, and thus $F_N < F_g$. Correct answer: b)

7) Infamous Tow Truck Problem: A 2-ton Tow Truck pulls a 1-ton car on a smooth level road, with a rope that has a tension of 3000 N on it. If the wheels of the car are free to roll, what coefficient of friction is necessary between the Tow Truck's wheels and the ground? This is a multi-step problem that will require some thought and some drawing. *This is a dynamics problem through and through because the force of friction on the wheels is accelerating the entire 3-ton system, and the tension in the rope is accelerating the 1-ton car, and the tension and friction are accelerating the 2-ton tow truck. It's up to you to make a good FBD and know which of the systems to look at for each one. It's important to recognize that the tension is an internal force and thus provides no force to the system. Examining the 1-ton car alone, we find that the acceleration must be 3 m/s^2. Examining the 3-ton system, we see that the force of friction accelerating the system must be 9 kN. The necessary coefficient of friction must be 0.45.*

8) Chapter 6.0, carefully consider Examples 1, 2, and 3 – solutions are provided in the text. Then do the following: *The system at right is dropped through 1 m. Even if the question doesn't give you a distance, you can always assume a certain distance.*
   a) How would this change the energy considerations in Example 1? Find the new speed of the system as it hits the ground 1 m below. Then find the time to fall and the acceleration.

Using the energy lens, we realize that now some of the potential energy is changed to thermal energy as well as kinetic energy, reducing the final speed and the resulting acceleration:
$E_{gp} = E_k + E_{th}$. Using a dynamics lens, we look at the 1 kg mass, knowing it is in equilibrium in the y direction, so the normal force = mg = 10N. Thus, the frictional force is 1 N and 1 J of thermal energy is produced. The 0.25 kg block loses 2.5 J of E_{gp} so now only 1.5 J of kinetic energy is shared between the two blocks (corresponding to a final speed of about 1.5 m/s, and average speed of 0.75 m/s) as opposed to the 2.5 J without friction (corresponding to a final speed of 2 m/s, or average speed of 1 m/s). Corresponding times and accelerations of 1.3 s, and 1.2 m/s^2, and 1 s and 2 m/s^2 without friction.

   b) How would this change the dynamics considerations of the system? Find the new acceleration directly (is it the same as you found above?) and tension in the string.

Using a dynamics lens we can write that $\boxed{\sum \vec{F}_{system} = m_{system}\vec{a}_{system}}$*. Please make a good FBD with all relevant forces on the system (note that some forces don't matter like the tension in the string because it pulls in both directions, so it exerts no force on the system), label the + direction as to the right and down, so there are now two forces, + 2.5N and – 1N, so the total force is 1.5 N, resulting in an acceleration of 1.2 m/s^2 as opposed to 2.0 m/s^2 with no friction.*