Problem set #6

1) You will not solve this problem! You are graded on your narrative, setting up the problem and reflecting on what you think from your experiences. I stand at the end of an 8 m diving board with 2 m between pylons A and B. I have a mass of 70 kg and the board is a uniform plank of mass 30 kg. Find the force provided by Pylon A and Pylon B (including direction) while explaining your reasoning with a drawing *of your own*. Please find the forces on pylons A and B. Indicate if these forces are up or down.

CM
A-
(9/9)

$\leftarrow 4m \rightarrow$
$\leftarrow 2m \rightarrow$

$m = 70 kg$

$\leftarrow 8m \rightarrow$

Dynamics lens, b/c there are forces, torques, and accelerations.
$\vec{a} = 0$, so $\Sigma \vec{F} = m\vec{a} = 0$
also
$\alpha = 0$, so $\Sigma \tau = I\alpha = 0$.

F_1 must be downward and F_2 must be upward, b/c otherwise $\Sigma \vec{F} \neq 0$
or $\Sigma \tau \neq 0$.

Since the system is static, we can set the point of rotation anywhere. I'll set it at Pylon A.

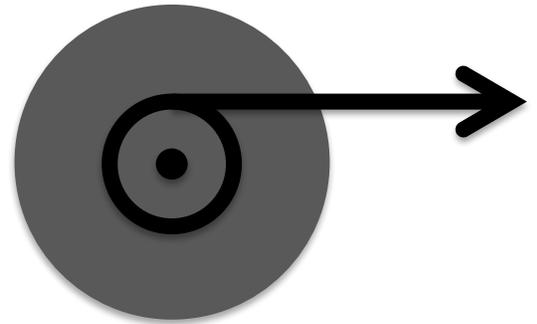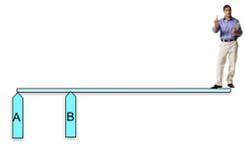$F_2 - F_1 - F_3 - F_4 = 0$; $F_2 r_2 - F_3 r_3 - F_4 r_4 = 0$

$F_1 = F_2 - F_3 - F_4$; $F_2 = \frac{F_3 r_3 + F_4 r_4}{r_2}$

$F_1 = 3400N - 300N - 700N$ (center of mass)

$\boxed{F_1 = 2400N}$ downward

$F_2 = \frac{(300N)(4m) + (700N)(8m)}{2m}$

$\boxed{F_2 = 3400 N}$ upward

4) The forces on Pylon A + B will be equal in magnitude to the forces they provide, but opposite in direction, b/c of Newton's Third Law.

2) A concrete flywheel of uniform thickness has a mass of 50 kg and a radius of 40 cm. If I pull on 2 m of string with a force of 100 N that is wound around a pulley of radius 16 cm.

Bt 2)

40 cm
0.4 m

100 N →

50 kg

16 cm
0.16 m

a) I use an angular dynamics lens since I see torques causing angular acceleration. I know that $\tau = I\vec{\alpha} = \vec{r}_\perp F$ and that $I_{disk} = \frac{1}{2}mr^2$:

$\tau = (0.16m)(100N) = \frac{1}{2}(50 kg)(0.4m)^2 \vec{\alpha} \Rightarrow$

$\vec{\alpha} = \frac{(0.16m)(100N)}{\frac{1}{2}(50 kg)(0.4m)^2} = 4 \, rad/s^2$

So, after 10 s: $\vec{\omega} = (4 \, rad/s^2)(10s) = \boxed{40 \, rad/s = \vec{\omega}}$

b) I would use a rotational kinetic energy lens since I see energy transformations as force is applied to the flywheel. I know that $W = F \cdot dx$ & $E_k = \frac{1}{2}I\omega^2$:

$W = (100N)(2m)$
$= 200 J$

great Start!

$200 J = E_k = \frac{1}{2}I\omega^2$

$\vec{\omega} = \sqrt{\frac{200J}{\frac{1}{2}I}} = \sqrt{\frac{200 J}{\frac{1}{2}(\frac{1}{2})(50 kg)(0.4m)^2}} = 10 \, rad/s$

Using a straight forward kinematics lens, we see ω_{ave} can be $\frac{1}{2}\omega_f$ because we know the wheel started from rest and there was constant torque, so constant angular acceleration. $\omega_{ave} = 5/s$. Total angular displacement, $\theta = \frac{l}{r} = \frac{2\,m}{0.16\,m} = 12.5$ radians, which should take 2.5 s. Knowing the final kinetic energy of the wheel, we should be able to use an energy lens and find average power $= dE/dt$, and get 80 W. Let me know if I'm wrong here.

3) Chapter 5.0, Exercise 1

③ • Yes, from dynamics lens, $\vec{a}_c \neq 0$, so there must be a force acting on the rock.

• Dynamics lens → there are forces + accelerations

$\Sigma \vec{F} = m\vec{a}_c$; $\vec{a}_c = \dfrac{v^2}{r} = \dfrac{(10^m/s)^2}{20m} = \boxed{5\,^m/s^2}$

• $\Sigma \vec{F} = m\vec{a}_c$, $F = (10kg)(5^m/s^2) = \boxed{50N}$

• I have no idea b/c I can't see what is going on.

• Tension Force

$F_T = m\vec{a}_c = \boxed{50N}$

• The velocity of the rock is always tangential to its circular path. If the string breaks, the rock no longer has a centripetal acceleration; no force keeping its motion circular. So, the rock will continue in a straight line.
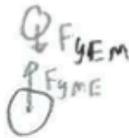
$v = 10^m/s$, F_c, $r = 20m$
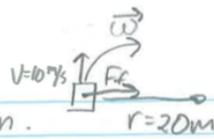
• Must be friction.

$\Sigma \vec{F} = m\vec{a}_c$; $F_f = m\vec{a}_c$; $\mu F_N = m\vec{a}_c$

$\mu = \dfrac{m\vec{a}_c}{F_N} = \dfrac{m\vec{a}_c}{mg} = \dfrac{\vec{a}_c}{g} = \dfrac{5^m/s^2}{10^m/s^2} = \boxed{\frac{1}{2}}$

− This is static friction b/c the tires are no sliding, they are rolling.

4) Chapter 5.1, Exercise 1

F_{yEM}

F_{yME}

④ − Dynamics lens, involves forces and acceleration

$\dfrac{385,000\ km}{6400\ km} = 60_r$ Radius from Earth to moon is 60 times greater than Earth's center to surface. Inverse square law says that gravity will by equal to $10^m/s^2 \cdot \dfrac{1}{r^2}$

So $10^m/s^2 \cdot \dfrac{1}{60^2} = \boxed{.002\ m/s^2}$. − This is acceleration is caused by gravity

− Moon's acceleration is the same as that. Moon just has tangential velocity, which is why it doesn't crash into Earth. There is just one force acting on the moon. This doesn't change.

− a_{grav} is directed toward center, so it is equivalent so centripetal acceleration.

$a_{grav} = a_c$

$.002\ m/s^2 = \dfrac{v^2}{385,000,000\ m}$ → $\boxed{v = 880\ m/s}$

Circumference is distance

− Period = time of orbit. Kinematics lens, involves motion and time.

$\Delta x = vt$ → $\dfrac{\Delta x}{v} = t$ → $\dfrac{2\pi(385,000,000\ m)}{880\ m/s} = \boxed{2748843\,s} = \boxed{31\ days}$

5) Chapter 5.2, Exercise 2

(5) Dynamics lens - There are forces + — I made
accelerations
At sea level, $R_S = 6,400$ km
At LEO, $R_L = 6,560$ km
So, $R_L = (1.025) R_S$
So $F_L = (1.025^2) F_S = (1.051) F_S$
So $\boxed{a_L = 1.051\, a_S}$, so a factor of 1.051 is roughly the same

• Kinematics lens b/c motion of sat. is an explicit function of time.
$\vec{a}_c = \dfrac{v^2}{r}$, $10 \text{ m/s}^2 = \dfrac{v^2}{6560 \text{ km}}$
$\boxed{v = \sqrt{(6560000 \text{ m})(10 \text{ m/s}^2)}\,,\ v = 8100 \text{ m/s}}$
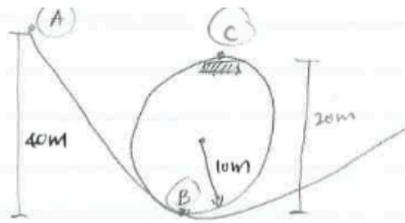
$T = \Delta t = \dfrac{\Delta d}{\bar{v}} = \dfrac{2\pi r}{8100 \text{ m/s}} = \boxed{1.5 \text{ hours}}$

⊚ 24 hours in a day,
going around Earth 1.5 hours at a time
So (16 times)

6) Chapter 5.3, Do exercise 1 and 2, but don't hand it in. – Solutions are in text.
7) Chapter 5.3, Exercise 4

7) (5.3, #14) You go on a R=10m loop ride, cart is
let go on low friction track, pulled downhill
by gravity. I have mass = 70 kg.
a) Start from vertical height h = 40 m. I use

Con energy (ers since I see E_p turning into E_k as the cart moves
down the loop. $\sum E = E_p + E_k$, $E_p = mg\Delta h$, $E_k = \tfrac{1}{2} m v^2$

$E_{P(A)} = E_{K(C)} + E_{P(C)}$ $E_{P(A)} = E_{K(B)}$
$mgh_A = \tfrac{1}{2} m v_c^2 + mgh_c$ $mgh_A = \tfrac{1}{2} m v_B^2$
$\boxed{v_c = \sqrt{2g(h_A - h_c)}}$ $\boxed{v_B = \sqrt{2gh_A}}$
$v_c = \sqrt{2(10 \text{ m/s}^2)(40 \text{ m} - 20 \text{ m})}$ $v_B = \sqrt{2(10 \text{ m/s}^2)(40 \text{ m})}$
$v_c = 20 \text{ m/s}$ $v_B = 28.28 \text{ m/s}$

Then I use a dynamics lens since I see something
moving in a circle, I know $a_c = \dfrac{v^2}{r}$, so :   →

what does the scale read...
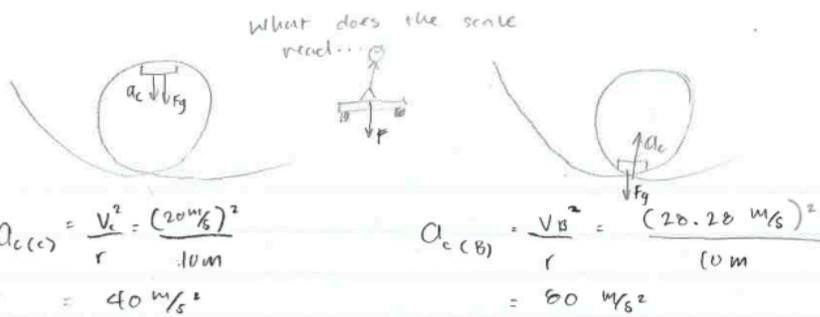
$$a_{c(C)} = \frac{V_c^2}{r} = \frac{(20\,m/s)^2}{10\,m}$$

$$= 40\,m/s^2$$

$$F_{c(C)} = m\,a_{c(C)}$$
$$= (70\,kg)(40\,m/s^2)$$
$$= 2800\,N$$

$$\Sigma F = F_{c(C)} - F_g$$
$$= 2800 - (70\,kg)(10\,m/s^2)$$
$$= \boxed{2100\,N \text{ at the top}}$$

$$a_{c(B)} = \frac{V_B^2}{r} = \frac{(28.28\,m/s)^2}{10\,m}$$

$$= 80\,m/s^2$$

$$F_{c(B)} = m\,a_{c(B)}$$
$$= (70\,kg)(80\,m/s^2)$$
$$= 5600\,N$$

$$\Sigma F = F_{c(B)} + F_g$$
$$= 5600\,N + (70\,kg)(10\,m/s^2)$$
$$= \boxed{6300\,N \text{ at the bottom}}$$

At the bottom, you feel a ~~lot more force than~~ normal pushing up that is *>> mg more than you are used to*, to the point where it's dangerous, especially for a pregnant woman.

b) I use an energy lens for the same reason as in (a). If I start the cart at the same height as the top of the loop, my E_K at $h = 20\,m$ would be zero — it's all E_p. therefore, I would have zero E_K at the top of the loop, therefore 0 velocity, and I would not be able to complete the loop.

(*) c) I use an energy lens for the same reason.

$$\Sigma E = E_p + E_K$$
$$mgh_A = mgh_c + \tfrac{1}{2}m V_{c(c)}^2$$

$$h_A = \frac{(10\,m/s^2)(20\,m) + \tfrac{1}{2}(10\,m/s)^2}{(10\,m/s^2)}$$

$$\boxed{h_A = 25\,m}$$

$$a_c > g$$
$$\frac{V_c^2}{r} > g$$
$$V_c > \sqrt{rg}$$
$$V_c > \sqrt{(10\,m)(10\,m/s^2)} = 10\,m/s$$

$h_A = ?$

$N > 0 \Rightarrow a_c > 0$

$N \geq 0$
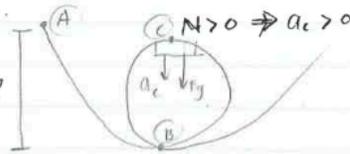
8) Chapter 5.3, Exercise 6

8) (5.3, # 6)   N — what scale reads

a)

$N = F_g$

scale = F_g

$N > 0$
$N < F_g$

scale > 0, $> F_g$
$$F_N = mg + \frac{mv^2}{r}$$

$N \geq 0$
$N > F_g$

$N > 0$
scale > 0, $< F_g$
$$F_N = mg - \frac{mv^2}{r} \quad \text{great!}$$
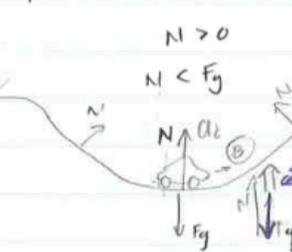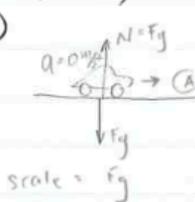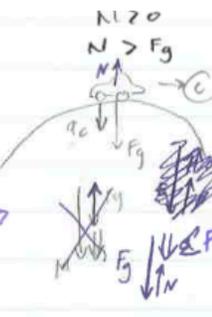
I use a dynamics lens here since I see things moving in a circle (somewhat) and forces that cause acceleration.

b) From a dynamics lens ($\vec{F}$'s and $\vec{a}$'s), we can see that $\Sigma \vec{F} = 0$ in case 1, so you feel completely normal. In case 2, $\vec{a}$ is upward, so you feel "heavier". In case 3, $\vec{a}$ = downward, so you feel "lighter". Does this match experience?

9) Chapter 5.3, Exercise 7

B+ 9) (5.3, #7) Do you weigh more at the North Pole? At the equator, you weigh...
you feel lighter.

I use a dynamics lens since I see things moving in a circle.

North Pole   Equator   At the equator, I'm traveling in a circle, so therefore
I have a_c towards the center of the Earth. Therefore
the (normal force) N that causes a_c is less than F_g, otherwise I wouldn't accelerate in a circular path. If $N = F_g$, I
would continue in a straight line. Therefore, I weigh less at the equator
since I am not in equilibrium, as opposed to the North Pole, where $N = F_g$. $\vec{a} = 0$

$-F_N) = \Sigma F$ causes the a_c here

B- 10) (5.4, #3) Calculate the escape speed at Earth's surface.

$r_e = 6.4 \times 10^6$ m $M = m_e = 5.97 \times 10^{24}$ $\Sigma E_i = 0J$

6.67×10^{-11}

$0 = \Sigma E_f = E_p + E_k$ $\Sigma E_f = 0J$

$0 = -\dfrac{mM}{r_e} G + \dfrac{1}{2}mv^2$ m → some object
                                              M → earth

$\dfrac{mMG}{r_e} = \dfrac{1}{2}mv^2$ $v = \sqrt{\dfrac{2MG}{r_e}} \cdot \sqrt{\dfrac{2(5.97 \times 10^{24} kg)(6.67 \times 10^{-11})}{6.4 \times 10^6 m}}$ $\boxed{11.195 \text{ km/s}}$

b) If I threw a rock at 14 km/s, it would still have
kinetic energy when it gets very far from earth.
As the distance from the object and earth approaches
infinity, its speed approaches but never reaches zero.
Energy lens! Find energy rock has and how much it takes to escape, then
see what you have left.

10) Chapter 5.4, Exercise 3

acceleration!!

$r = R_E$ ⟹ $r = \infty$

Energy lens, You would need enough KE so that PE = 0.
KE = PE_g;
$\dfrac{1}{2}mv^2 = \dfrac{mm_e}{r} G \cdot \dfrac{5}{r}$; $\dfrac{1}{2}mv^2 = \dfrac{mm_e}{r^2} G \cdot r$; $\dfrac{1}{2}mv^2 = mgr$ ⟹

$\dfrac{1}{2}v^2 = gr$

$v = \sqrt{2gr} = \sqrt{2 \cdot (10 \text{m/s}^2) \cdot (6,400,000 \text{ m})}$

$\boxed{v = 11,300 \text{ m/s}}$

• Energy lens still
$PE = -\dfrac{mm_e}{r} G$; $KE_F = KE_0 + PE_0$

$\dfrac{1}{2}mv_F^2 = \dfrac{1}{2}mv_0^2 - \dfrac{mm_e}{r} G$; $\dfrac{1}{2}v_F^2 = \dfrac{1}{2}v_0^2 - \dfrac{m_e}{r} G$

$v_F^2 = v_0^2 - \dfrac{2m_e}{r} G$;

$v_F = \sqrt{(14,000 \text{ m/s})^2 - \dfrac{2(5.972 \times 10^{24} kg)}{(6,400,000 \text{ m})}(6.67 \times 10^{-11} m^3 kg^{-1} s^{-2})}$

$v_c = 8,460 \text{ m/s}$