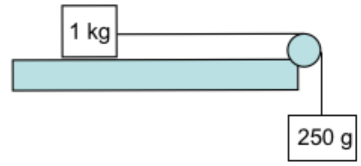
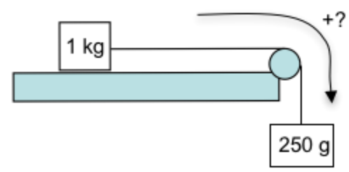
PS#6, some answers



1. Consider the system of masses above. Here are some important questions:
We can be sure that the system will move to the right because this will lower the potential energy, gaining kinetic energy as it falls. There will be tension in the string in order to accelerate the 1 kg. Thus the acceleration of the 250 g mass must be less than gravity because the tension provides an upward force. However, because the 250 g mass is accelerating downward, the tension must be less than the force of gravity on the 250 g mass. Only the 250 g mass loses (2.5 J) potential energy, but the kinetic energy is gained by both the masses, which might also explain why the acceleration is less than gravity. Thus by conserving energy, I think we found that the final speed is 2 m/s, the average speed is 1 m/s, so the time to travel 1 m is 1 s, and the acceleration of the system is 2 m/s^2. Now we look with a dynamic lens at either mass. The force on the 1 kg mass must be 2 N in order to accelerate it at 2 m/s^2. The force necessary to accelerate the 250 g mass must be 0.5 N (downward). However, gravity provides 2.5 N downward, so the upward tension on the mass must be 2N. It's good that the tension is the same at both ends, because it has to be!



2. Consider the system above through a dynamics lens of forces on the whole system. Make a good Free Body Diagram.

Here we can see that there is only one force acting on the system, the force of gravity on the hanging mass, 2.5 N. This force has to accelerate 1.25 kg. Thus we can see that a=F/m = 2 m/s^2. As in the problem above, we can use dynamics to solve for the tension in the string.

3. What if there is a coefficient of friction $(\mu_d = 0.1)$ on the 1 kg mass as is slides across the horizontal surface?

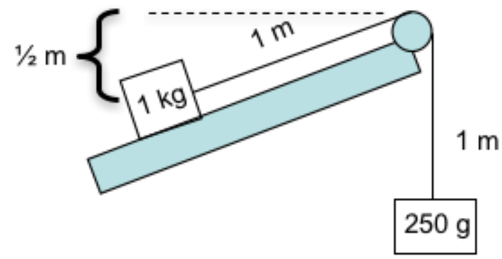
a) How would this change the energy considerations in Example 1? Find the new speed of the system as it hits the ground 1 m below. We now have to consider that some kinetic energy is turned to thermal energy. What is the work of heat? Work is F*dx, so the force of friction on the top block is 1 N, so it does 1 Joule of work, so the total kinetic energy is now only 1.5 J. I get a little over 1.5 m/s.
b) How would this change the dynamics considerations of the system? Find the new acceleration and tension in the string. Now there would be two forces acting on the system. 2.5 N in the positive direction, and 1 N in the negative direction for a total force of 1.5 N acting on 1.25 kg, yielding an acceleration of 1.2 m/s^2

4. In a tug-o-war, each team pulls as hard as they can on the rope. However, the tension on the rope must be the same on both sides, pulling each team in opposite directions with the same force. So then, how can a team win? A team wins when they can accelerate backwards, when the vector sum of the forces is in the backwards direction.

a) Please look at a single team through a dynamics lens and do a good free-body diagram. What can you say about the forces necessary for this team to win? Draw a good diagram. The tension is pulling the team in the "losing" direction. Their feet are pushing the ground in the losing direction as well, pushing their bodies in the winning direction. So they must push harder with their feet than the tension in the rope.

b) Then look at the two connected teams as a system and do a good free-body diagram. What must be true for one team to win? Look at the forces on the system. The tension is not a force on the system. There are only two sets of feet pushing the ground toward the other team. The team that pushes harder with their feet wins… as long as they hang onto the rope.

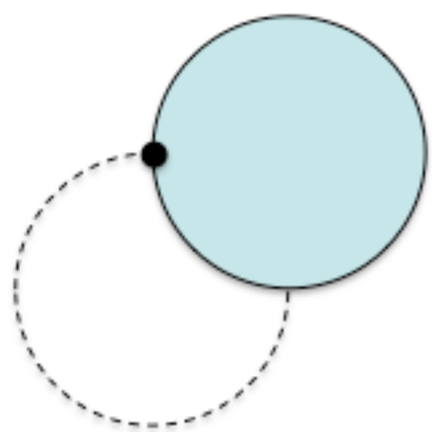
5. Consider the system at right where the 1 kg box is on a very slippery table inclined such that if the system moves one meter, the box changes elevation by half a meter.



a) How does this change the energy balance equation you set up for problems in 6.0? Now we see that the 1 kg mass will gain potential energy as the 250 g mass loses potential energy. In fact, we find that if the system moves to the right, it will be in a *higher* potential energy state because the 1kg gains more (twice as much) potential energy than the 250 g mass loses.

b) Can you tell me which way the system will acceleration (if at all)? How can you be sure? The system will accelerate to the left because then it will lose energy. In fact, you should find that it will accelerate to the left at the same rate… 2 m/s^2, because moving to the left 1 m results in the same potential energy loss as moving 1 m to the right did in the system with the 1 kg on a level track.

6. A disk of uniform mass, total mass m_o, and radius, R is secured to a wall with a frictionless pivot that allows rotation as shown at right. It is started in the higher position where the center of the circle is at the same height as the pivot and allowed to drop and swing. We want to find the force on the pin when the disk is swinging at the bottom. In order to solve this complicated, multidimensional problem, please consider:



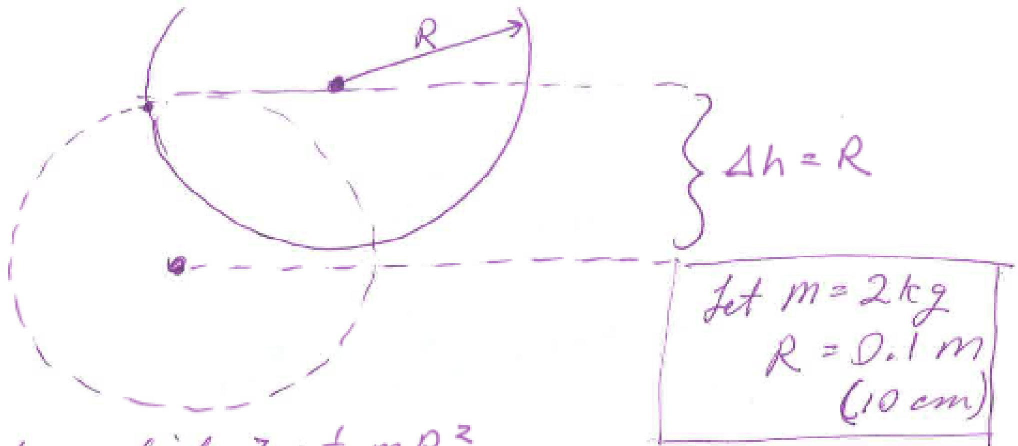
a) What is the complete energy transition happening as the disk rotates from top to bottom? Gravitational potential energy transitions to kinetic energy of the disk moving and rotating about its center of mass… or just rotating about the pivot point. You can look at the kinetic energy either way. If you use the latter, be mindful that the moment of inertia requires the use of the parallel axis theorem.

b) What is the complete dynamics going on when the disk is at the bottom of the swing? Is the force on the pivot just equal to *mg*? This is circular motion, so the disk is accelerating into the center of the circle… upward, so the force on the pivot must be upwards and greater than the force of gravity.

c) Find the force on the pivot when the disk is in full swing at the bottom. Include direction. Check

#5 The solution is for a slightly different problem…. please find:

a) Omega, the angular velocity of the disk about the pivot.
b) The ***angular momentum*** of the disk about the pivot.
c) The force that the pivot is providing to the disk. Include direction.



Uniform disk, $I = \frac{1}{2} m R^2$

as this swings down, I know that $PE \Rightarrow KE$

if Friction = 0, no energy is converted to thermal energy.

KE = Translational + Rotational energy.

so now we have 2 unknowns: ω, v, but $v = \omega R$, so we're OK.

OR you can view the pivot as the center of rotation and just say $PE \Rightarrow KE_{rotation}$ But you have to use the // axis theorem to calculate I about the rotation pt.

Either way, I get $\omega = \left(\frac{4}{3}\frac{g}{R}\right)^{\frac{1}{2}}$

or, if R = 10 cm $\omega \approx 11/s$ This is almost 2 times around per second!

6 $\vec{L} = I\vec{\omega}$, but I need to recognize that we are not rotating about the center of mass, so $I = MR^2 + \underbrace{I_{cm}}_{\frac{1}{2}mR^2} = \frac{3}{2}mR^2$

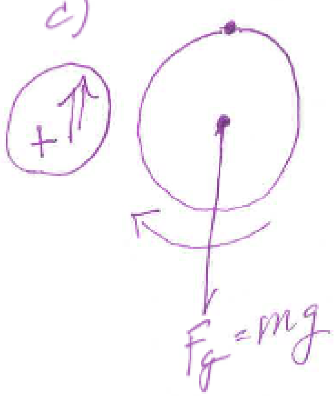
if $m = 2\,kg$

$I = 0.003\,kg\,m^2$

$$L = \frac{3}{2}mR^2\left(\frac{4}{3}\frac{g}{R}\right)^{\frac{1}{2}} = (3m^2R^3g)^{\frac{1}{2}}$$

or, if $m = 2\,kg$ + $R = 0.1\,m$, $L \approx 0.35\,kg\,\frac{m^2}{s}$

c) $a \Uparrow$



$$\omega = \left(\frac{4}{3}\frac{g}{R}\right)^{\frac{1}{2}}$$

This is dynamics as there is $a_c \Uparrow$ caused by the $\Sigma\vec{F}$

$$a_c = \frac{V^2}{R} = \frac{\omega^2 R^{\cancel{2}}}{\cancel{R}} = \omega^2 R$$

$$= \frac{4}{3}\frac{g}{\cancel{R}}\cancel{R} = \frac{4}{3}g$$

because we have uniform circular motion, a_c is up $\Uparrow$. F_g is down $\Downarrow$, so $F_{pivot} \Uparrow$

$\Sigma\vec{F} = m\vec{a}_c$

$F_p - F_g = ma_c$

$$F_p = ma_c + F_g$$
$$= ma_c + mg$$
$$= m\left(\frac{4}{3}g + g\right) = m\left(\frac{7}{3}g\right)$$

if $m = 2\,kg$ $\quad F_p \approx 47\,N \Uparrow$ The key here is that the force is $> 2 \times F_g$!

Again, to me the interesting thing here is that the force on the pivot is more than twice the force of gravity because the object is accelerating upward with centripetal acceleration.

7. You need to build a massive slingshot that propels a 100 kg object (you in a capsule) at 13 km/s so you can go into space (infinity)! For ***<u>each question</u>***, start with a statement of which of the 4 mechanics concepts is central to this problem and why.

a) How fast will you be going when you get to deep space?

b) How fast will you be going when you are 1 earth radius above the earth's surface?

c) If you passed near the moon, what effect would this have on your speed? I'm just looking for *speed* here. Direction is not what I'm asking about. Support your answer with a concept.

d) If your slingshot is a massive spring that compresses 10 m, please find the spring constant that gives you a speed of 13 km/s.

e) What would be the maximum acceleration of your body at launch? How would this work for you?

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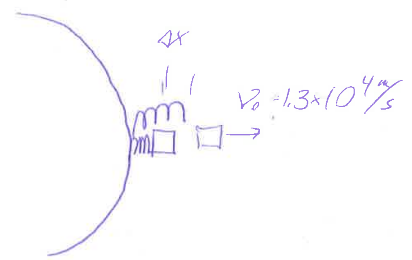

a) I conserve Energy because

$$E_s \Rightarrow E_K \Rightarrow E_K + \Delta E_p$$

$$\frac{1}{2}mv_0^2 \Rightarrow \frac{1}{2}mv_f^2 + (E_{p_s} - E_{p_o})$$

$$\frac{m_1 m_2 G}{r} - \frac{m_1 m_2 G}{\infty} \quad 0$$

$$\left[\frac{1}{2}\cancel{m}V_0^2 = \frac{1}{2}\cancel{m}V_f^2 + \frac{\cancel{m}\, m_e G}{r_e}\right]\times 2$$

escape velocity doesn't depend on mass.

$$V_f^2 = V_0^2 - \frac{2m_e}{r_e}G$$

$$= (1.3\times10^4 \,\text{m/s})^2 - 2\cdot\frac{6.0\times10^{24}\,\text{kg}}{6.4\times10^6\,\text{m}}\, 6.67\times10^{-11}\,\frac{\text{Nm}^2}{\text{kg}^2}$$

$\text{kg}\,\text{m/s}^2$

$$= (1.3\times10^4 \,\text{m/s})^2 - (1.1\times10^4\,\text{m/s})^2$$

$$V_f^2 \approx (6.6\,\text{km/s})^2 \qquad V_f \approx 6.6\,\text{km/s}$$

More to come.