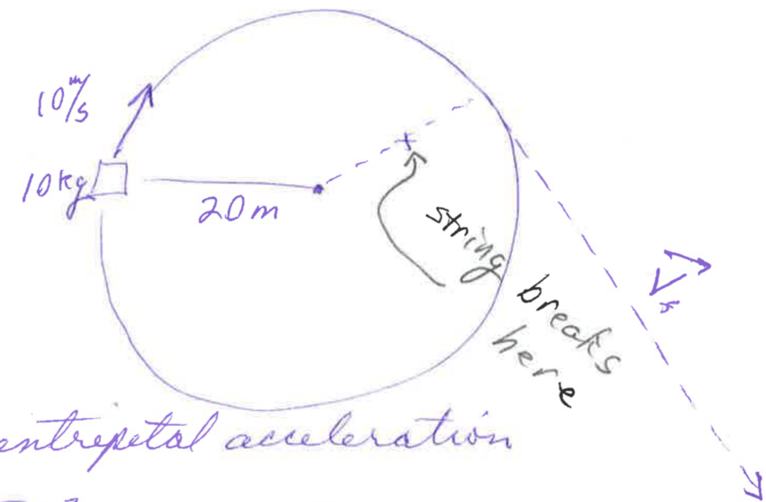
Problem set #6 Solutions

#1



a) The $\vec{V}$ is changing direction. This is centripetal acceleration

b) $a = \frac{V^2}{r} = \omega^2 r = \left(\frac{0.5}{s}\right)^2 20m = \underline{\underline{5\ m/s^2}}$ or $\frac{1}{2}g$

c) $F = ma = 10\,kg \cdot 5\ m/s^2 = 50N$

Direction: Inward

d) say it....

e) $T = F = ma = 50N$

f) when string ~~[illegible]~~ breaks, $\vec{F} = 0$, $\vec{a} = 0$, $\vec{V} = $ const straight line

like this

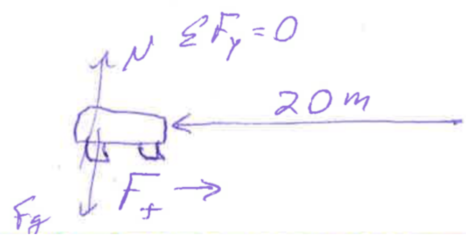
g) This must be gravity! $F = 50N = \frac{m_1 m_2}{r^2} G$ center planet

$$m_2 = \frac{50N\, r^2}{m_1 G} = \frac{50N\,(20m)^2}{10\,kg \cdot 6.67\times10^{-11} N\,m^2/kg^2} \approx 3\times10^{13}\,kg$$

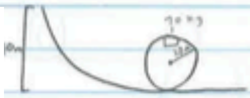
$$V_{sphere} = \frac{4}{3}\pi r^3 \simeq 4 \cdot 8000\,m^3 \simeq 32{,}000\,m^3 = 3.2\times10^4\,m^3$$

$$\rho = \frac{m}{V} \approx \frac{3\times10^{13}\,kg}{3\times10^4\,m^3} \approx 10^9\,\frac{kg}{m^3}$$ or 10^6 times the density of water.

h) $F_f = N\mu = 50N$ $\boxed{\mu = 0.5}$



#2



Here we have neglected to identify the lenses. We can see this is a combination of energy and dynamics: Energy because we are finding velocity by recognizing that the height lost will translate to PE => KE. Dynamics because in a circle we know that we undergo centripetal acceleration and recognize the sum of the forces (in this case gravity and normal force) = ma. Also notice that this student chose to calculate the energy in Joules. This is not necessary. You could skip this step by setting PE=KE and just solving for V^2, which is what you need for centripetal acceleration. In any case, we see that a "9g" force at the bottom is more than a pregnant woman (and many other people) should experience.

a)
$PE_I = (70\,kg)(10\,m/s^2)(40\,m) = 28000\,J$

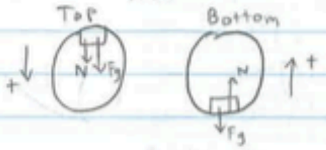
$PE_{Top\ of\ Loop} = (70\,kg)(10\,m/s)(20\,m) = 14000$

$28000\,J = \frac{1}{2}mv^2,\ V_{bottom} = 28.28\,m/s$

$14000\,J = \frac{1}{2}mv^2,\ V_{top} : 20\,m/s$

$a_{c_{bottom}} = \frac{(28.28\,m/s)^2}{10\,m} = 80\,m/s^2$

$a_{c\ top} = \frac{(20\,m/s)^2}{10\,m} = 40\,m/s^2$



$\Sigma F_c = ma_c$

$N + F_g = ma_c$

$N = ma_c - F_g$

$N = (70\,kg)(40\,m/s^2) - (70\,kg)(10\,m/s^2) = \boxed{2100\,N}$

$\Sigma F_c = ma_c$

$N - F_g = ma_c$

$N = ma_c + F_g$

$N = (70\,kg)(80\,m/s^2) + (70\,kg)(10\,m/s^2) = \boxed{6300\,N}$

→ As you round the bottom of the loop, you feel more force pushing down on you. This is not a good ride for a pregnant woman!

b) If you start at the height as the top of the loop, you won't have enough velocity to make it around the top of the loop, because if energy is conserved, then KE will be 0 as you approach the top of the loop

c)
$PE_I = PE_T + KE_T$

$mgh = mg(20m) + \frac{1}{2}mv^2$

$gh = g(20m) + \frac{1}{2}v^2$

$\Downarrow$

$(10\,m/s^2)h = (10\,m/s^2)(20\,m) + \frac{1}{2}(10.01\,m/s)^2$

$\boxed{h > 25\,m}$

$\Sigma F = ma_c$

$F_g < ma_c$

$\cancel{m}g < \cancel{m}a_c$

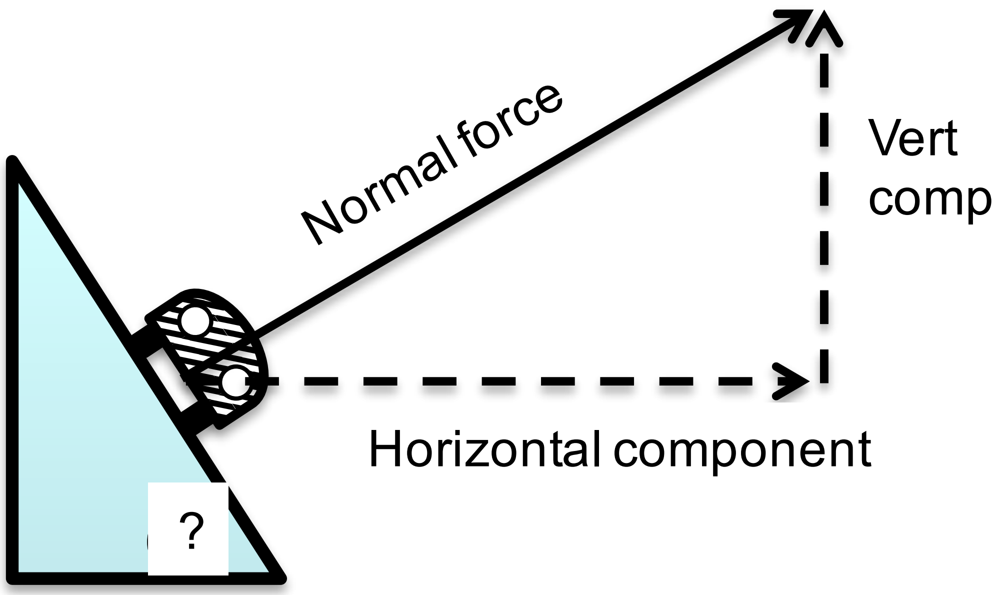
$a_c > g$

$\frac{v^2}{r} > g$

$V > \sqrt{gr}$

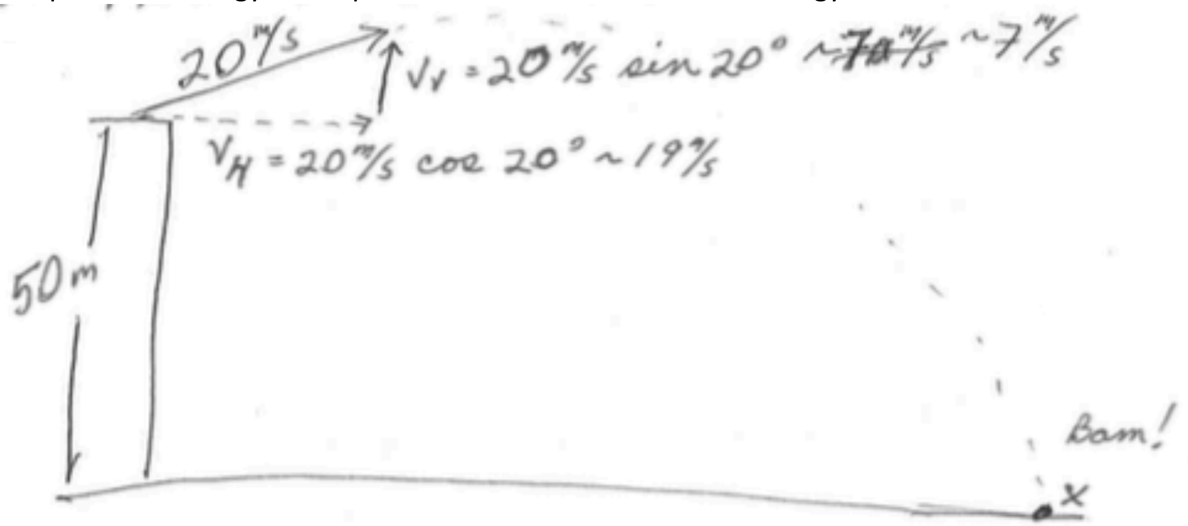
$V > 10\,m/s$

#3. This a dynamics problem because I'm undergoing centripetal acceleration as a result of two forces: normal force and gravity. We write the vector sum of the forces = ma, and draw a good picture showing the forces on the car. We as ourselves that very important question and realize that the car is accelerating horizontally toward the middle of the circle and the vertical acceleration is zero. Because the vertical acceleration = 0, the y component of the normal force = force of gravity, mg… and the horizontal component of the normal force = ma_x = $ma_{centripetal}$. The centripetal acceleration ~ 30 m/s^2, or 3 times that of gravity, so I know that this ramp surface has a slope of about 3, so I'm estimating the angle at 70 degrees. A 90 kg driver would feel a normal force of mg + ma_c, but we'd have to add these like vectors, so the magnitude of this force would only be a little more than 3 gravities or about 3000 N. Then I get out my calculator and see that the centripetal acceleration is actually 30.6 m/s^2, yielding an angle of about 72 degrees, with a total force of about 2900N on the driver's body. The driver would feel this as a strong compression.



#4 is a straight up kinematics problem because we are explicitly looking at position and motion as a function of time. ***However***, we can use conservation of energy to find the speed at the bottom because the loss of gravitational potential energy corresponds to an increase in kinetic energy

#4



a) $E_0 = E_f$

$KE_0 + PE_0 = KE_f + \cancel{PE_f} \to 0$ solve ~~for~~ math and find $V_f \approx 37.4\,m/s$

$\frac{1}{2}\cancel{m}(20\,m/s)^2 + \cancel{m}\,10\,m/s^2 \cdot 50\,m = \frac{1}{2}\cancel{m}V_f^2$

b) $a_H = 0$, so $V_{H_{final}} = V_{H_{initial}} = \text{const} \approx 19\,m/s$

V_V 37.4 m/s θ $V_H = 19\,m/s$

$\cos\theta \approx \frac{19\,m/s}{37.4\,m/s} \approx 60°$

$V_V^2 + V_H^2 \approx V^2$

$V_V^2 + (19\,m/s)^2 = (37.4\,m/s)^2$

$V_V \approx 32\,m/s \Downarrow$

c) $\Delta V_V = -32\,m/s - 7\,m/s$

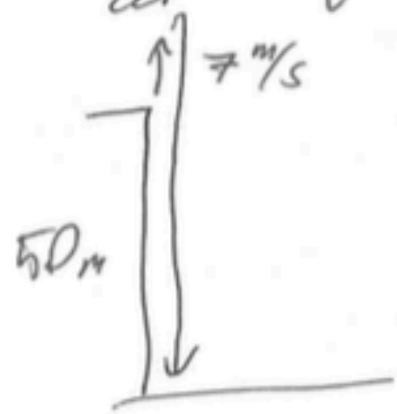
$\approx -40\,m/s = at$

$a \approx -10\,m/s^2$, so $\Delta t \sim 4\,s$

$\Delta x = V_H t + \frac{1}{2}\cancel{a_H}t^2 \approx 19\,m/s \cdot 4\,s \approx \underline{\underline{75\,m}}$

$\cancel{a_H} \to 0$ horizontal $\vec{a} = 0$

d) now I use straight kinematics. Horizontally, it's just moving along at $V_H \approx 19\,m/s$ in the $\hat{x}$ direction; vertically it's moving upward at $V_{V_0} \sim 7\,m/s$ and accelerates downward at $a_V = -10\,m/s^2$



find t w/

$X_f = 0 = 50\,m + 7\,m/s + \frac{1}{2}(-10\,m/s^2)\,t^2$

solve!

$t \approx 4\,s$ (and $\sim -2.5\,s$, but that's not relevant)

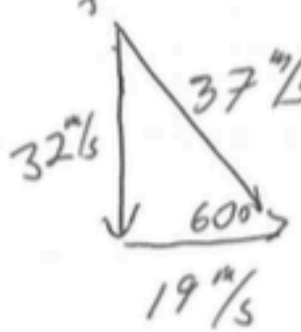
Then you can find $X_h = V_H t$ and

$V_V = V_{0V} + at$

to get same answers as before!

e) $X_f \approx 75\,m$

f



#5 is a straight up kinematics problem because we look at position as an explicit function of time.

#5

Catching Bus

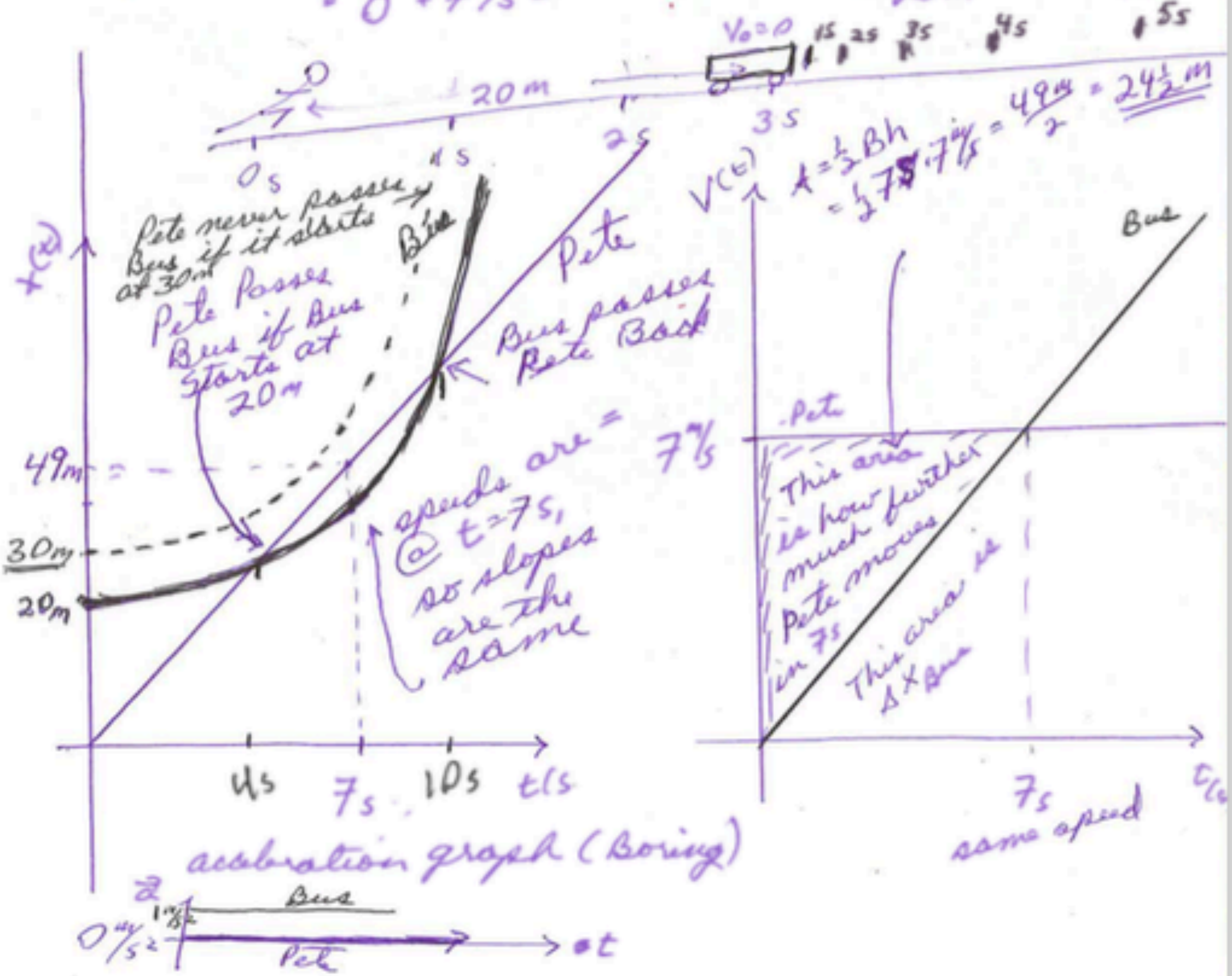
Pete: $V_P = 7\,m/s = \text{const}$, $x_0 = 0$

Bus: $V_{0B} = 0$, $a_B = 1\,m/s^2$, $x_0 = 20m$, $V = a_B t$

Kinematics - because we are dealing with exclusive use of position and its time derivatives as an explicit function of time. In particular: $x_P(t) \overset{?}{=} x_B(t)$ when and if are our displacements the same

Pete: $x = x_0 + Vt$
$= 0 + 7\,m/s\, t$

Bus: $x = x_0 + V_0 t + \frac{1}{2}at^2$
$= 20m + 0 + \frac{1}{2}\,m/s^2\, t^2$



#6 is a combination of kinematics (projectile motion) and (circular) dynamics. It requires two drawings, which I expect of you, but I am not supplying right now:

Using dynamics, make a good FBD and follow the protocol and you can see that the force of friction is the only radial force, so you set it equal to mass*centripetal acceleration. This would provide us with the coefficient of friction, except that we don't know the speed… but we know how far it moves horizontally when it falls, so we go to the kinematics lens.

Please show the parabolic trajectory as the coin falls from the edge of the spinning turntable. We want the initial horizontal velocity = dx/dt. dx = 0.5 m, but how about dt? This is revealed in the vertical component because the coin is falling from rest (vertically speaking) and hits the ground. I could use energy to find v_{final} and then use $v_{average} = ½v_{final}$ = dx/dt. Or I could just use kinematics that dx = $½at^2$. You should find that it takes about 0.45 s to fall 1 m from rest, so the initial horizontal speed is about 1.1 m/s.
Now we can go back to our circular dynamics lens and see that the centripetal acceleration at this speed at a radius of 20 cm is about 6 m/s^2, requiring a frictional force of mass*6 m/s^2, or a coefficient of friction of 0.6.

Could we test this? Sure! Please show that if we slowly tipped the surface, an object with a coefficient of friction of 0.6 will slide when the surface is inclined at about 30 degrees.