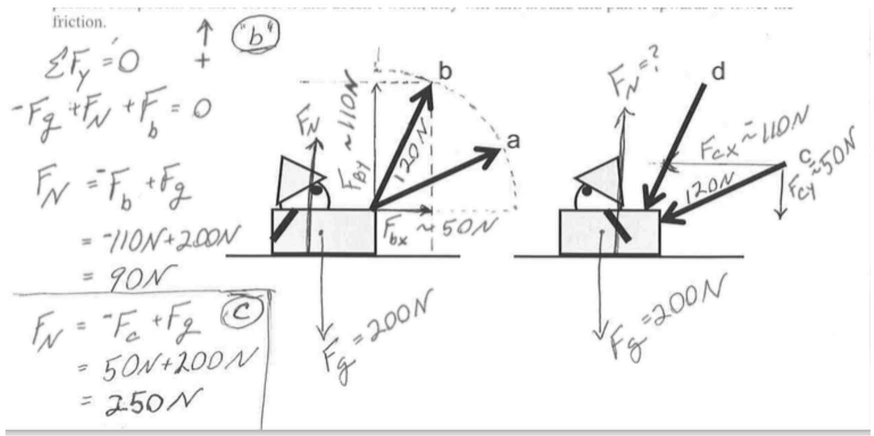
Problem Set #7 Solutions

1. MT#2 Solutions – Hand in Tuesday. Solutions are posted on the main webpage under tests.

2. My daughter is sledding (total mass = 20 kg), and I am applying a force of 120 N to her sled. I have 4 different options (pushing and pulling at two different angles) and I try all of them.

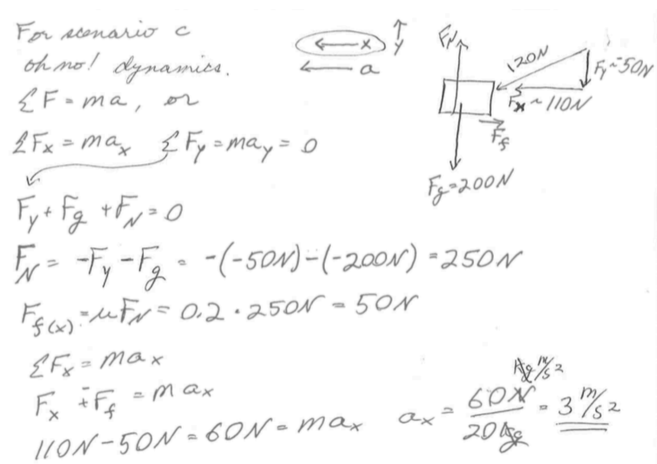
a) For each scenario, estimate both the acceleration of the sled and the normal force between the sled and the frictionless snow. This is a dynamics problem because I see there are forces and acceleration involved. It's crucially important here to separate the problem into the x and y components. Using no trigonometry, I'm estimating the components of the tension.

POOP! I don't know anything. WAIT, I know that the vector sum of the forces = ma. So I draw the force diagram (right) and ask if it's in equilibrium: NO, it's accelerating *to the right… horizontally.* I immediately know that I have to have an x – y coordinate system, and the sum of the y forces = zero, the sum of the x forces = ma_x. Above, I show how I estimate the components of the forces for b and c. Then I calculate the normal forces for b and c. Show the normal forces for abcd are 150N, 90N, 250N, 310N, respectively.



b) Now, please rank the different force scenarios in order of least acceleration to greatest acceleration. If some accelerations are the same, please indicate that. Because there's no friction, the normal force doesn't matter and I'm just looking at the forces in the x-component for the problem. There is only 1 force: the x component of the force. To find acceleration, just divide by the 20 kg mass. $F_{xa} = F_{xc} > F_{xb} = F_{xd}$ so the accelerations in increasing order are: b=d ~ 2.5 m/s < c=a ~ 5.5 m/s

c) Now, let's say that the coefficient of friction of the snow is *actually* 0.2. How does this change things? Please rank again the different force scenarios in order of least acceleration to greatest acceleration. Now that there's friction, I know that I have to include an additional frictional force, and because d and c have me pushing *downward*, they will have a higher normal force to be in equilibrium in the y direction, and thus a higher frictional force than a and b. Additionally d has a higher vertical component of compression than c because it is coming at a higher angle. So the order of *increasing* frictional force slowing us down is b < a < c < d. So clearly scenario "a" will accelerate faster than c; and b will accelerate faster than d. However, it's not clear until we do the math if a or b has higher acceleration, but d definitely has the lowest acceleration because it has the highest frictional force and the lower parallel force. The approximate accelerations I get are respectively for abcd: 4 m/s^2, 1.6 m/s^2, 3 m/s^2, - 0.6 m/s^2, because $F_x < F_{friction}$. See at right the calculation for scenario "c". Please repeat this for the other scenarios and make sure your answers make sense.



d) Have you ever pushed a lawn mower (or watched someone do it)… you are using force scenario d, pushing along the handle. When you run into some thick grass the "coefficient of friction" might be high enough to stop you cold. What scenario can you change to, and why does this work? From the above arguments and calculations, you should be able to watch someone pushing a lawn more: They start comfortably walking pushing as d. When they hit some thick grass, they will go down to c, lowering the friction and increasing the parallel component of their force. If this doesn't work, they will turn around and pull it upwards to lower the friction.

3. Consider pushing the sled above in scenario "c" on the 0.2 frictional snow for a total of four meters, please find the amount of work I do, the amount of heat produced and the final speed of the sled. Carefully lay out your lens discussion. I use an energy lens because I'm looking at work that I do goes to increase kinetic energy and heat (the work of friction =~ 100 J).

$E_o = E_f$

$\cancel{KE_o} + W = Heat + KE_f$ ($KE_o \to 0$)

$W_{Pete} = W_f + KE_f$

$KE_f = W_{Pete} - W_f = 120 J$ (↳ lost as heat)

(110N)

$W_p = \vec{F} \cdot d\vec{s} = F_x \cdot 2m = 220 J$

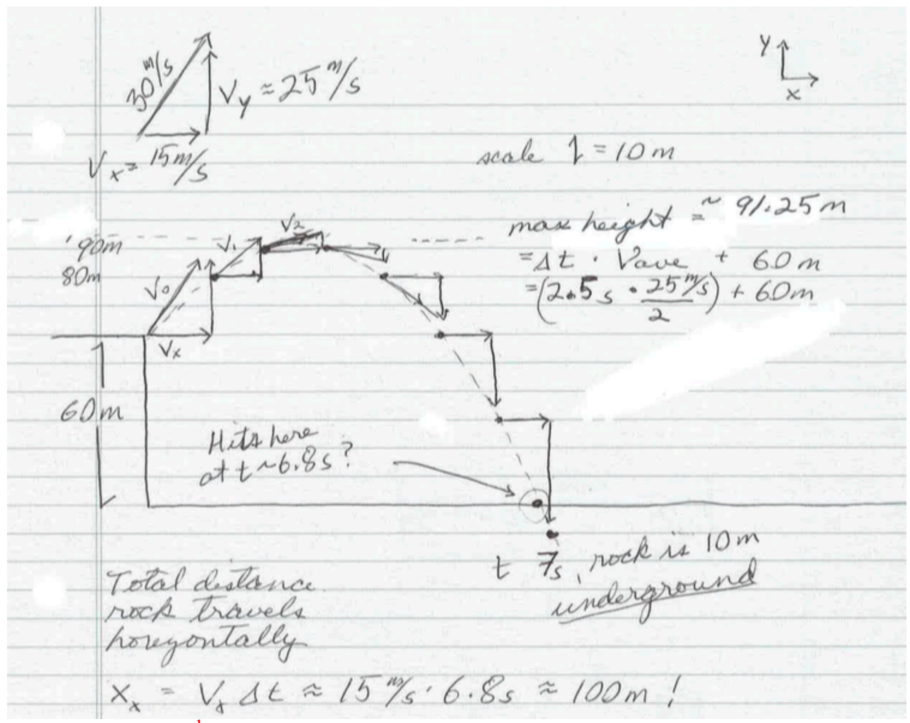
$W_f = \vec{F}_f \cdot d\vec{s} = 50N \cdot 2m = 100 J$

$KE_f = \frac{1}{2} m v_f^2$

$v_f = \left(\frac{2 KE_f}{m}\right)^{\frac{1}{2}} = \left(\frac{2 \cdot 120\, kg\, m^2/s^2}{20\, kg}\right)^{\frac{1}{2}}$

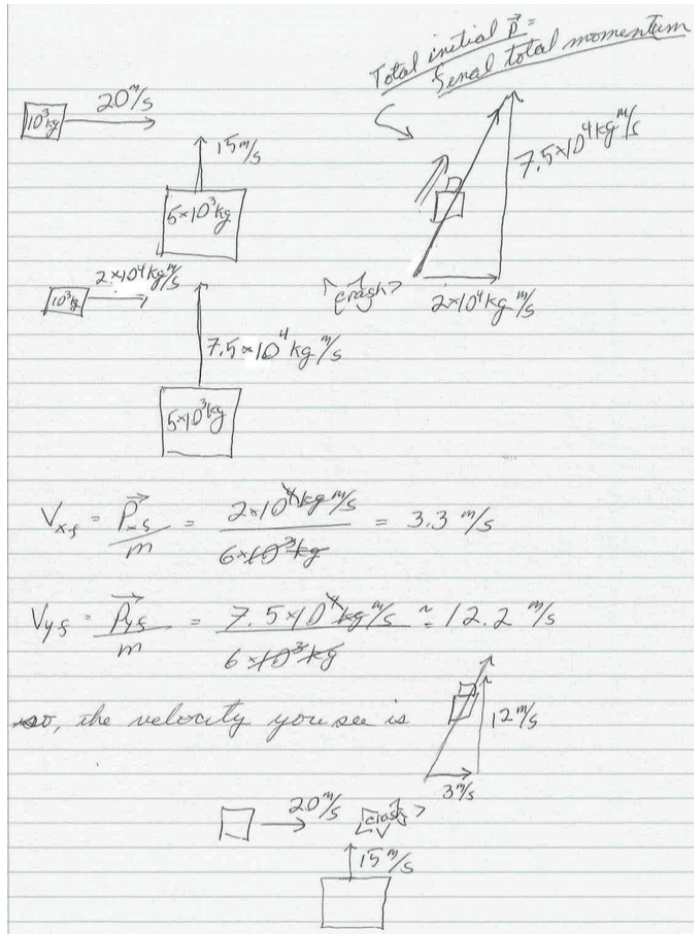
$\approx 3\frac{1}{2}\, m/s$

4. Consider throwing a rock from the edge of a 60 m high cliff at a speed of 30 m/s in the direction indicated by force vector "b" above.
   a) Please make a drawing showing the rock at each second until it hits the ground. You may not use a calculator, as we are making simple approximations here. For each second elapsed, estimate where the rock is and its velocity. Draw the velocity vector at that point. This is straight up kinematics – displacement, velocity changes over time (and the explicit reference to time is a strong indicator that this is kinematics). I'm estimating that 30 m/s at this angle corresponds to about $v_x = \sim 15$ m/s, and $v_y = \sim 25$ m/s. We know that the acceleration is only in the y direction (~ - 10 m/s^2), so that v_x will not change, and v_y will be 15 m/s after 1 second, 5 m/s after 2 seconds… etc. ALSO, we know that the *average* speed for the first second will be 20 m/s (half way between 25 m/s and 15 m/s) so it will move 20 m upward in the first second); and for the 2nd second, the average speed will be 10 m/s, and during the 3rd second it will turn around and have an average velocity of 0… etc. All the while, the rock will move 15 m further in the x direction *each second*.

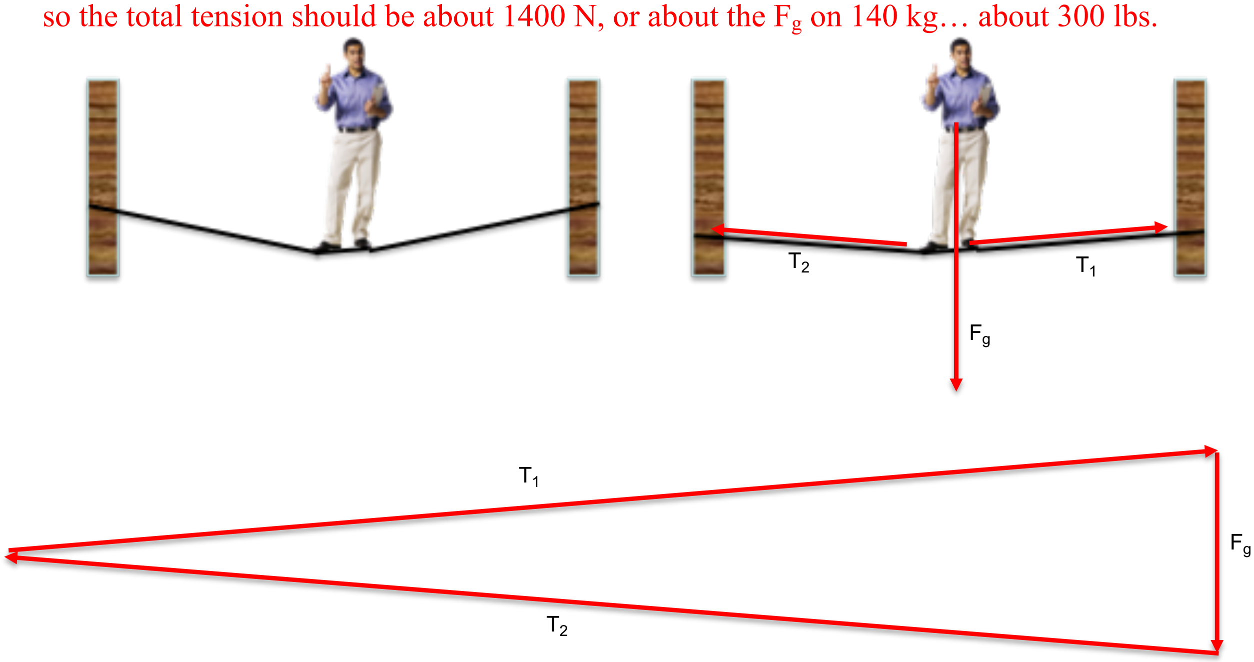


   b) Use an energy lens to judge if your final speed is reasonably close to what you would expect. Here we use an energy lens because the final kinetic energy is the sum of the initial kinetic energy + the potential energy that the rock lost. According to my drawing, the final speed would be 15 m/s *x* and in the y direction, a little less than 45 m/s … with a total speed of about 45 m/s corresponding to kinetic energy of about 1000 J for a 1 kg rock. How does this compare to the initial energy? Please show that a 1 kg rock with *v=30 m/s* has 450 J of kinetic energy, and at an elevation of 60 m, has 600 J of kinetic energy for a total mechanical energy of 1050 J. We can see that my estimates are reasonable: The final kinetic energy ~ total initial mechanical energy. Please repeat this calculation.

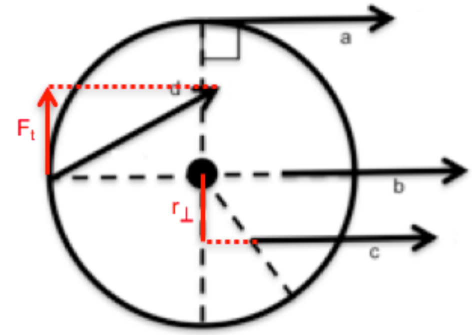
5. On a surface of frictionless ice, a 1000 kg car driving 20 m/s eastward collides and sticks to a 5000 kg truck driving 15 m/s northward. The vehicles stick together and slide:
   a) Please draw and indicate the final velocity of the vehicles. This is clearly a momentum problem because we have an inelastic collision, so the only thing we know is that momentum is conserved… remember that momentum is a vector. A velocity diagram won't do us much good because velocity isn't conserved, so I immediately draw a momentum diagram (in the middle at right). Then, conserving momentum, I can find the final momentum. I find the final velocity by dividing the final momentum by the total mass of the two vehicles stuck together.
   b) Please calculate the amount of energy turned to heat in the collision.



6. Slacklining is pretty fun, but you have to run some webbing between two trees first. At right, you see two pictures of me at 70 kg, slack lining.
   a) In which drawing is the line tighter? Please prove how you know this with a good force drawing and discussion. Lens? I use a Statics lens because I know that this isn't moving and I'm interested in the forces, so the vector sum of the forces = ma = 0. I show below how to add the forces so that we get a resultant vector, a net force of zero. I did it for the scenario at right. You can do it for the scenario at left and see that the tension at right is greater.
   b) Using your force drawing, please estimate the tension on the slack line at left. OK, I did the wrong one below. I can see that the tensions are about 6 times F_g, or about 4200 N, the force of gravity on 420 kg, or about 1000 lbs. Please repeat this process for the scenario at right. Another way to look at it is that each rope carries about half the 700 N of the person, so the vertical component of the tension for each rope is about 350 N. For the scenario at left, the horizontal component is about 4 times that of the total tension, so the total tension should be about 1400 N, or about the F_g on 140 kg… about 300 lbs.



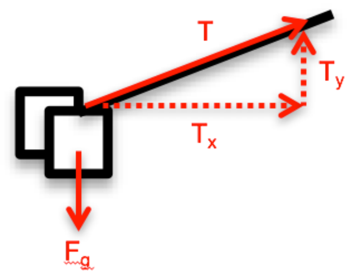
7. My bicycle wheel has a radius of 50 cm and seems to be locked up around the hub. I want to get it to turn. Please find the torque when I put a force of 200 N on it in the different ways shown at right. Please rank the torques from highest torque to lowest torque and estimate the torque of each.



Torque = $F*r_\perp = F_\perp*r$

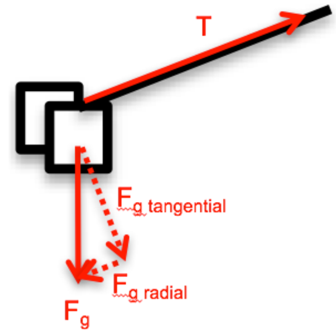
a) $T = 200N*0.5m = 100\ Nm \otimes$
b) $T = 200\ N*0 = 0\ Nm$
c) $T \sim 200N*.2m = 40\ Nm\ \odot$
d) $T \sim 100\ N*.5m = 50\ Nm \otimes$

8. You are watching the fuzzy dice from the rear view mirror. As you take off on level ground, it makes an angle as shown at right.



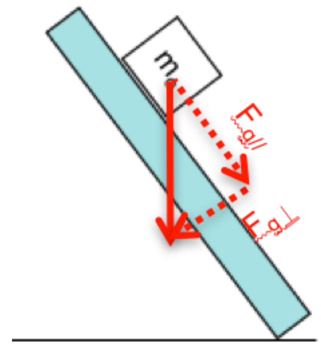
a) Ask yourself **** and state how this will inform your choice of axis. Because the acceleration is to the right, horizontally, we should use standard x-y coordinates
b) Estimate the acceleration of the car. This is a dynamics lens because we're looking for acceleration and we know forces, so I'll use $\sum \vec{F} = m\vec{a}$. Because the dice are in equilibrium in the y direction, $T_y = mg$. We can see from the angle that the string makes, that $T_x \sim 3T_y = 3\ mg$. $\sum F_x = \sim 3mg = ma_x$ so the acceleration is about 3g, or 30 m/s^2.
c) What must be the coefficient of friction of your tires for this to happen? This is a dynamics lens we need a friction force of *3mg*, so the coefficient of friction must be about 3.
d) Is this realistic? Few surfaces have a coefficient of friction this high. However, it is possible. Most coefficients between tire and road are slightly less than 1, but drag racers have a rubber and road surface that can get more than a coefficient of 4.
e) If the mass of the dice is 100 g, what is the tension in the string? The tension is slightly more than 3 times T_y, which is equal to mg. This would yield a tension of about 3 N.

9. Consider the fuzzy dice above. Now the car is stationary and you are sitting it in. You grab the dice and pull them to once side exactly as in the diagram above. Then you let go of them.



a) Ask yourself that question again****. Is the direction of acceleration the same as above? State how this direction will inform your choice of axis. This is again a dynamics problem because we're dealing with forces (gravity and tension) and acceleration. Because the acceleration is along the arc of the string, this question is now more like an inclined plan question.
b) Again find the acceleration of the dice with direction. The component of force of gravity along the direction of travel (acceleration) is about $0.9*F_g$. This is the net force on the object. So the acceleration of the dice downward along the arc is about 9 m/s^2.
c) Again, if the mass of the dice is 100 g, please find the tension in the string. Is it the same as the string above? Why might this make sense? Now we see that the dice are in equilibrium in the radial direction so the tension = the radial component of gravity. From the F_g vector triangle, we can see that the radial component of gravity ~ 0.4 mg. Thus T ~ 0.4 N, much less than the tension of the accelerating car. NOTE! The dice are in equilibrium in the radial direction the moment I let the dice go because the dice are not yet moving. Once the dice are moving, $T > F_g$ with the net inward force providing the centripetal acceleration.

10. Consider the mass on the inclined plane shown at right. Dynamics for the same reasons above.



a) Ask ****, and then state how will this inform you to set up the axis. Because the block is accelerating down the incline, this is the natural direction for the axis because the acceleration is produced by the // component of gravity and the acceleration in the perpendicular direction is 0, so $F_N = F_{g\perp}$

b) Estimate the acceleration if the surface is frictionless. $F_{g//} \sim 0.8$ mg = ma, so a ~ 8 m/s^2.

c) What coefficient of friction is necessary for the block to slide at a constant speed? First we look at the dynamics in the perpendicular direction and see that $F_{g\perp} \sim 0.6$ mg = F_N, because the block is in equilibrium in the perpendicular direction. Then we can look at the dynamics in the parallel direction and see that in order for there to be no acceleration, $F_f == F_{g//}$. We know $F_f = \mu F_{g\perp}$, so $\mu \sim 0.8/0.6$ or about 1.3, a pretty high coefficient of friction, but something we have seen before.