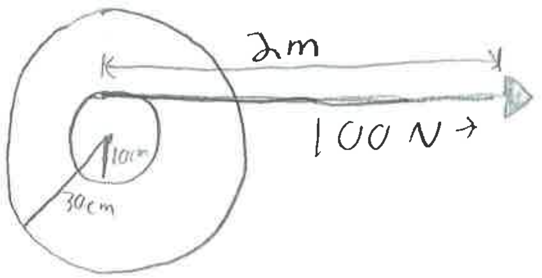
Problem Set #7

1.)



$m = 3\,kg$

$\ell = 2\,m$

a.) $I_{disk} = \frac{1}{2} m r^2 = \frac{1}{2}(3\,kg)(.3\,m)^2 = \boxed{.135\ kg\ m^2}$

b.) $W = F \cdot d$

$W = (100N)(2m) = \boxed{200\ N{\cdot}M}$ J

This work went into the rotational kinetic energy of the disk.

c.) $200\,N{\cdot}M = \frac{1}{2} I \omega^2$

$200\,N{\cdot}M = \frac{1}{2}(.135\,kg\,m^2)\omega^2 \rightarrow \boxed{\omega = 54.43\ rad/sec}$

D.) $\theta = \frac{\Delta s}{r}$ $\Delta s = 2m$ $r = .1\,m$

$\theta = \frac{2m}{.1m} = \boxed{20\ rads}$

E.) $\omega_0 = 0$ $\frac{\omega_F - \omega_0}{2} = \omega_{Avg} = \boxed{27.2\ rad/sec}$

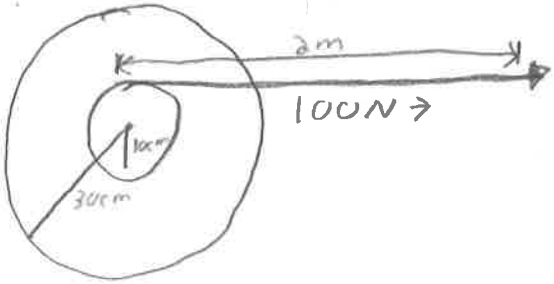
$\omega = \frac{\Delta\theta}{\Delta t} \rightarrow \Delta t = \frac{\Delta\theta}{\omega} \rightarrow \Delta t = \frac{20\,rads}{27.2\,rad/sec} = \boxed{.74s}$

F.) $\alpha = \frac{\Delta\omega}{\Delta t} = \frac{54.43\,rad/sec}{.74\,sec} = \boxed{73.5\ rad/s^2}$

G.) $\tau = I\alpha$

$\tau = (.135\,kg\,m^2)(73.5\,rad/sec^2) = \boxed{10\ N{\cdot}M}$

2.)



$m = 3\,kg$

$I = .135\,kg\,m^2$

a.) $\tau = rF\sin\theta$

$\tau = (.1m)(100N)(\sin 90) = \boxed{10\,N\cdot M}$

b.) $\tau = I\alpha$

$10\,N\cdot M = (.135\,kg\,m^2)\alpha \rightarrow \alpha \frac{10\,N\cdot M}{.135\,kg\,m^2} = \boxed{74\,rad/sec^2}$

C.) Rotational Work = $\tau\theta$

linear work = rotational work???

$F\cdot d = \tau\theta$

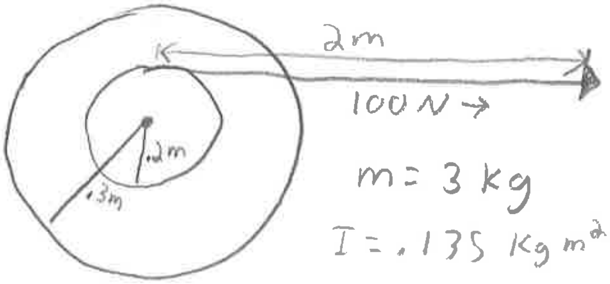
$(100N)\cdot(2m)\cdot = (10N\cdot M)(20\,rads)$

$\boxed{200NM = 200N\cdot M}$

Yes, linear work = rotational work

D.) I like the dynamics method best.

3.)



$m = 3\text{ kg}$

$I = .135\text{ kg m}^2$

$r_{pulley} = 2r_0$

a.) $\theta = \frac{\Delta s}{r} = \frac{2\text{m}}{.2\text{m}} = 10\text{ rads}$

$\theta_0 = 20\text{ rads}$ so $\boxed{\theta \Rightarrow \frac{1}{2}\theta_0}$

b.) Work still $= 200\text{ N·M}$

so $200\text{ N·M} = \frac{1}{2}(.135\text{ kg m}^2)\omega^2$

$\omega = 54.43\text{ rads/sec}$

$\omega_0 = 54.43\text{ rads/sec}$

so

$\boxed{\omega \Rightarrow \omega_0}$

c.) $\tau = rF\sin\theta$

$\tau = (.2\text{m})(100\text{N})(\sin 90)$

$\tau = 20\text{ N·M}$

$\tau_0 = 10\text{ N·M}$

$\boxed{\tau \Rightarrow 2\tau_0}$

D.) $\tau = I\alpha$

$20\text{ N·M} = (.135\text{ kg m}^2)\alpha$

$\alpha = \frac{20\text{ N·M}}{.135\text{ kg m}^2} = 148\text{ rad/sec}^2$

$\alpha_0 = 74\text{ rad/}$ so $\boxed{\alpha = 2\alpha_0}$

$\alpha = \frac{\Delta w}{\Delta t}$

$\alpha = \frac{54.43\text{ rad/sec}}{.367\text{ sec}}$

$\alpha = 148\text{ rad/sec}^2$ ✓

E.) $\omega = \frac{\Delta\theta}{\Delta t}$

Avg $\omega = \frac{\omega_F - \omega_0}{2}$

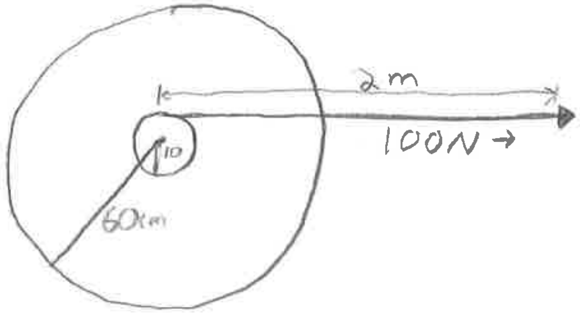
Avg $\omega = 27.2\text{ rad/sec}$

$\Delta t = \frac{\Delta\theta}{\omega_{Avg}} = \frac{10\text{ rads}}{27.2\text{ rad/sec}} = .367\text{ s}$

$t_0 = .74\text{ s}$ so

$\boxed{t = \frac{1}{2}t_0}$

4.)



a.) $A_0 = \pi(.3m)^2 \quad A = \pi(.6m)^2$

$A_0 = .28m^2 \quad A = 1.13m^2$

$A = 4A_0$ so we can say $\boxed{m = 4m_0}$

b.) $I = \frac{1}{2}mr^2$

$I = \frac{1}{2}(12kg)(.6m)^2$

$I = 2.16\ kg\,m^2$

$I_0 = .135\ kg\,m^2$

so

$\boxed{I = 16I_0}$

c.) $\tau = rF\sin\theta$

$\tau = (.1)(100N)(\sin 90)$

$\tau = 10N{\cdot}M$

$\tau_0 = 10N{\cdot}M$

so

$\boxed{\tau = \tau_0}$

D.) $200N{\cdot}M = \frac{1}{2}(2.16kg\,m^2)\omega^2$

$\omega = 13.6\ rad/sec$

$\omega_0 = 54.43\ rad/sec$

so

$\boxed{\omega = \frac{1}{4}\omega_0}$

E.) $\tau = I\alpha \rightarrow \alpha = \tau/I$

$\alpha = \frac{10N{\cdot}M}{2.16kg\,m^2} = 4.63\ rad/sec^2$

$\alpha_0 = 74\ rad/sec^2$

so

$\boxed{\alpha = \frac{1}{16}\alpha_0}$

F.) $\theta = \frac{\Delta s}{r} \quad \theta = \frac{2m}{.1m} = (20)\ rads$

$\theta_0 = 20\ rads$ so

$\boxed{\theta = \theta_0}$

Just to check: $C_{small} = 2(\pi)(.1m) = .628$

$\frac{2m}{.628}$ = # of rotations = 3.18 → $(3.18)(2\pi.6) = 12$ = arc length → $\frac{12}{.6} = (20)$ rads

(Big r)

G.) $\omega = \frac{\Delta\theta}{\Delta t}$

$\Delta t = \frac{\Delta\theta}{\omega_{Avg}}$

$t_0 = .74s$

so

$\omega_{Avg} = \frac{13.6\ rad/sec}{2}$

$\Delta t = \frac{20\ rads}{6.8\ rads/sec}$

$\omega_{Avg} = 6.8\ rad/sec$

$\Delta t = 2.94s$

$\boxed{t = 4t_0}$