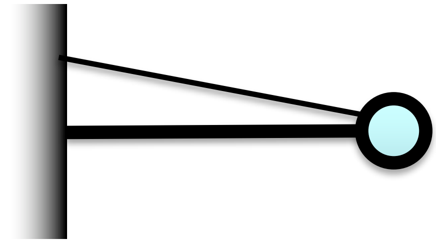
PS#8 solutions, Fall 2016

1) A 100 kg sphere is hung in front of a store (as an advertisement for ball bearings) using a very light (carbon fiber) bar and a thin cable as shown at right. The linkage between the bar and wall is a freely rotating hinge (shown below) so that without the cable, the ball would swing downward with no friction. **BUT**, the cable keeps it hanging there. Note that this solution has a different angle than your problem set angle. This tension is much greater than yours because the ratio of the vertical tension / horizontal tension is smaller and the horizontal tension = 1000 N if the ball is in equilibrium. Your answers are probably more like Tension ~ 2000 N and the horizontal tension = compression force ~ 1700 N

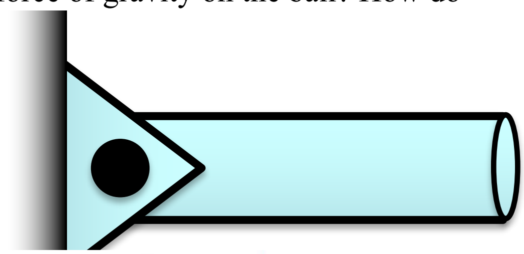


a) Is the tension in the cable more, less, or equal to the force of gravity on the ball? How do you know?

b) Is the bar pushing on the ball or pulling on the ball? How do you know?

c) Estimate the tension on the cable.

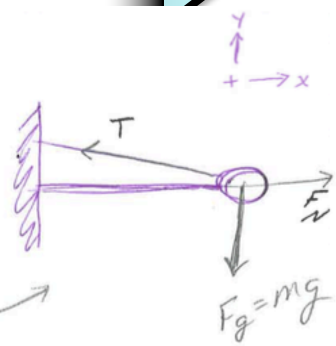
d) Estimate the force the hinge puts on the bar at the store front (include direction).



Because this deals w Forces + $\vec{a}$ (=0) I choose to use dynamics actually statics!

I know $\sum \vec{F}_{Ball} = m\vec{a}_{Ball} = 0$

FBD



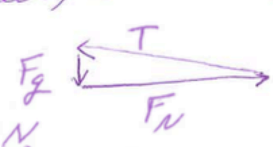
a) I know $\sum F_y = 0 = T_y - mg \Rightarrow T_y = F_g = mg$ (F_g)

$T \sim 5 \times T_y >> T_y = mg$

b) $\sum F_{x\,Ball} = 0 = \overset{compression}{\vec{F}_N} + \overset{\leftarrow}{T}_x$

Because the $\hat{x}$ component of the tension Pulls the ball to the left, the compressional force of the bar, F_N, must be =, opposite.

c) I will draw a $\sum \vec{F}$ diagram showing that it = 0 (bites its tail) because $\underline{a=0}$



so from inspection,

I estimate if $F_g = mg = 1000N$,

then $T \approx 5000N$

d) $F_N \approx 4500N$

2. On a surface of frictionless ice, a 1000 kg car driving 30 m/s westward collides with a 4000 kg truck at rest. The truck subsequently takes off at 10 m/s in the direction indicated. North is indicated
   a) Without using a calculator, please determine as best you can the subsequent velocity of the car. If the outside forces are zero, and force is the rate of change of momentum, then in a collision, momentum is conserved. Also, we know that momentum is a vector and we add it either component by component or with a vector drawing. So you see at right that the initial momentum of the car is equal to the vector sum of the final momentum of the car and the truck together.
   b) Was mechanical energy conserved in this collision? This is an energy problem. We just compare initial kinetic energy with final kinetic energy.

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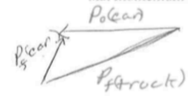
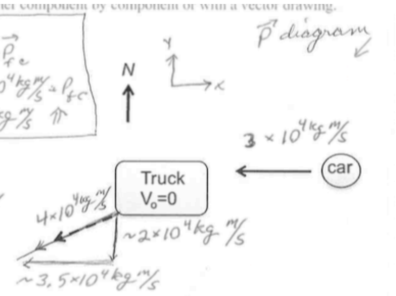


Ydirection
$\vec{P}_o = 0 = \vec{P}_{ft} + \vec{P}_{fc}$
$= -2\times10^4\,kg\,m/s + P_{fc}$
$P_{fc} = +2\times10^4\,kg\,m/s$ ⇑



Xdirection
$\vec{P}_o = 3\times10^4\,kg\,m/s = \vec{P}_{ft} + \vec{P}_{fc}$
$\vec{P}_{fc} = \vec{P}_o - \vec{P}_{fT} = -3\times10^4\,kg\,m/s + 3.5\times10^4\,kg\,m/s$
$= 0.5\times10^4\,kg\,m/s = 5000\,kg\,m/s$
$\vec{V} = \frac{\vec{p}}{m} = +5\,m/s\,\hat{x} + 20\,m/s\,\hat{y}$ $\vec{v}_{f(car)}$

$KE = \frac{p^2}{2m}$ $KE_o = \frac{(3\times10^4\,kg\,m/s)^2}{2\ 10^3\,kg} = \frac{9\times10^{\cancel{8}\,5}\,\cancel{kg}\,m^2/s^2}{2\times\cancel{10^3}\,\cancel{kg}} = 4.5\times10^5\,J$

$KE_f = KE_{car} + KE_{Truck}$

$KE_{\cancel{car}\ Truck} = \frac{(4\times10^4\,kg\,m/s)^2}{2\cdot4\times10^3\,kg} = \frac{16\times10^{\cancel{8}\,5}\,\cancel{kg}\,m^2/s^2}{8\times\cancel{10^3}\,\cancel{kg}} = 2\times10^5\,J$

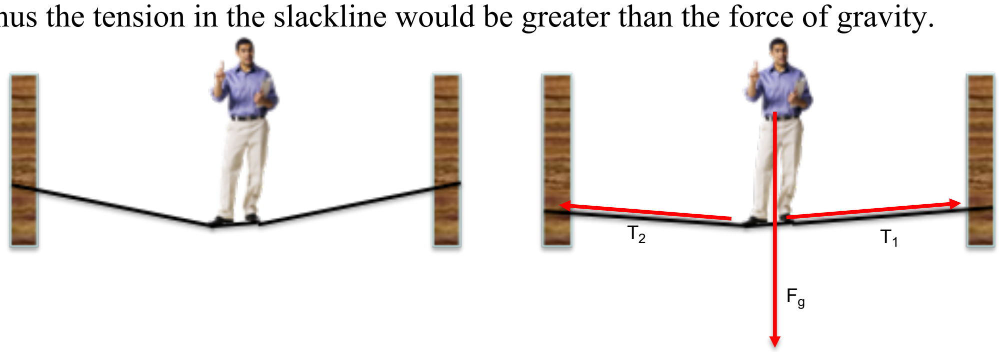
$KE_{car} = \frac{p^2}{2m} = \frac{P_x^2+P_y^2}{2m} = \frac{(0.5\times10^4\,kg\,m/s)^2+(2\times10^4\,kg\,m/s)^2}{2\times10^3\,kg} = \frac{4.25\times10^8\,\cancel{kg}\,m^2/s^2}{2\times10^3\,\cancel{kg}}$

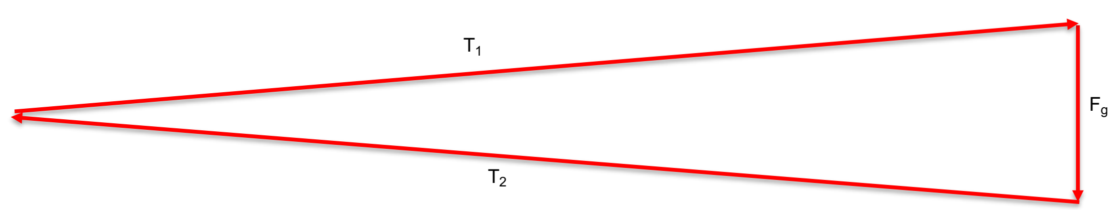
$= 2.125\times10^5\,J$

$KE_f \cong 4.125\times10^5\,J \approx KE_o$

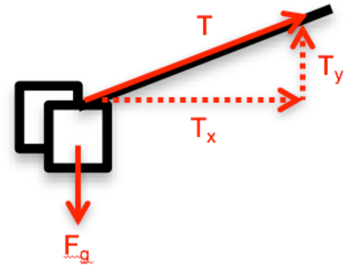
so this collision is quite elastic! very little Kinetic Energy was lost! – remember I was estimating – so, within my uncertainties, $KE_o = KE_f$

3. In a classic physics problem, a car is stuck in the mud, so you tie a rope to a tree on the other side of the road as tight as you can and then push the rope – do you pull it along the rope, or push it perpendicular? Would it be a good idea to slack line on it? If you were slack lining on it, would it be a good idea to jump on it? Because we are dealing with forces and acceleration (= unless we are bouncing) we know that this is a statics problem, so we must do a FBD and add all the forces to be zero. We *showed before* that if you push perpendicular to the rope, the tension will be greater than the perpendicular force you apply…. As long as the rope is inelastic enough to not deflect to high angles… That is, for the slackliner example, please show that if the slack line made an angle of 30 degrees with the horizon, that the tension would be the same as the force of gravity on the man. If you bounced on the slackline, then at the bottom of your oscillations, you would be accelerating upward, and thus the tension in the slackline would be greater than the force of gravity.





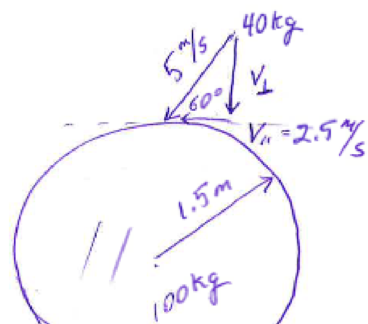
4. You are watching the fuzzy dice from the rear view mirror. As you take off on level ground, it makes an angle as shown at right.
   a) Please refer to PS#7 and see how we estimated acceleration is about 3g, or 30 m/s^2.
   b) What about if this is a 2 kg (heavy!!) tetherball at the end of a 2 m string? We have the exact same FBD and same acceleration ~ 3g to the right for the FBD I drew. The horizontal radius of the circle is just a little under 2 m because the string is nearly horizontal. Knowing that this 3g acceleration is centripetal acceleration = v^2/r, we can solve for the speed, meticulously canceling units, I get ~ 7m/s



   c) How is part b) like the F-22 in the conical pendulum video? This is really the same problem, but instead of tension acting on the person's body, there is the normal force of the seat. Or if we're talking about the plane itself, it's the lift of the wings that acts on the plane the same way the tension acts on the tether ball.

5. A child's carousel has a mass of 100 kg and a diameter of 3 meters. Assume that the mass is uniformly distributed over the circular area and is at rest. One kid, a 40 kg point mass, runs as fast as she can (5 m/s), jumps onto and grabs the edge of the carousel as shown. Please find the following… Please note that I answer d) and e) with one part d). Yes, if there was no post to absorb the momentum into the mass of the earth, then the final speed of the carousel and girl can be calculated by conserving momentum.

#$\boxed{5}$ Conserve $\vec{L}$, which comes from the running girl (a point mass)



$\vec{L}_o = mvr_\perp = m v_{tangential} r$

$= 40\,kg(2.5\,m/s) \cdot 1.5\,m$

$\vec{L}_o = 150\,kg\,\frac{m^2}{s}$ out of paper $\odot$ Positive $\vec{L}$ of girl

$\vec{L}_f = I\omega_f = (I_{girl} + I_{carousel}) I_f$

$I_{girl} = I_{pt.\,mass} = mr^2 = 40kg \cdot (1.5m)^2 = \underline{90\,kg\,m^2}$

$I_{carousel} = \frac{1}{2}mr^2 = \frac{1}{2}(100kg)(1.5m)^2 = \underline{112.5\,kg\,m^2}$

$I_{total} = I_{girl} + I_{carousel} \approx 200\,kg\,m^2$

a) $\vec{L}_f = I_f\vec{\omega}_f = \vec{L}_o$ $\quad \omega_f = \frac{L_o}{I_f} = \frac{150\,kg\,m^2/s}{200\,kg\,m^2} \approx 0.75/s$

b) The collision would either slow the rotation or ~~turn the~~ make the object rotate in the opposite direction because the ~~p~~ childs $\vec{L}$ is in the opposite direction of the inital $\vec{L}_{carousel}$.

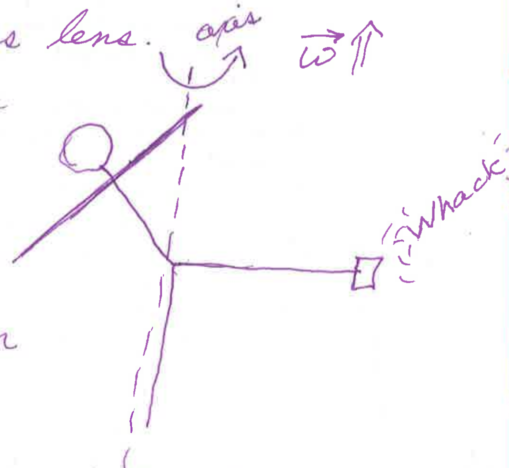
c) This was an inelastic collision. KE was changed to heat as she landed on the carousel

d) in space, we wouldn't have earth to absorb $\vec{p}_{girl}$ through the axel, so the carosel + girl would conserve $\vec{p}$ ~~by going~~ moving in same direction ~~of~~ the girl was going

6.

6 In doing a roundhouse kick (where the kicker spins about a vertical axis), explain with proper physics reasoning how the kicker should move her arms, at the moment she kicks, and why this works.

I can use either an $\vec{L}$ lens or a rotational dynamics lens.



In both cases, we recognize that we are already spinning about a vertical axis (upward in this drawing), but we want the kicking leg to spin as fast as possible! in both cases, we separate the body into an upper half + a lower half and recognize that if only 1 foot is on the ground, $r \sim 0$, $\tau \sim 0$, so $\vec{L}$ is conserved.

Angular Momentum $\Rightarrow \vec{L}_{Body} = \vec{L}_{Top} + \vec{L}_{Bottom}$

so $\Delta \vec{L}_{Body} = 0$, because $\tau = 0$

so $\Delta \vec{L}_{Top} = -\Delta \vec{L}_{Bottom}$, so, we spin the top half in the downward direction + impart upward $\Delta \vec{L}$ to the bottom half.

Rotational Dynamics $\vec{\tau} = I\vec{\alpha}$, a Torque is a turning force between 2 bodies, affecting each in the opposite direction: "For every torque, there's an = + opposite torque"... So there is a torque between the upper and lower body.

7. Dynamics because we're dealing with forces and acceleration! I immediately remember that the sum of the forces = ma, and I start to draw the forces in. I realize that it is accelerating down the incline, so this will be my "x" direction and I will decompose gravity into these two directions. If gravity is $m_o g$, then the x component must be about 0.9 mg, and the y component must be about 0.4 mg as judged from the drawing. The normal force is equal and opposite to the "y" component so there is no acceleration in the "y" direction, and the x component accelerates the block downward.

a) If the block is frictionless, a =

b) If 1 m/s^2, μ_k= …for this one, please add a $F_{friction}$ vector to my FBD, oriented in the $-x$ direction.

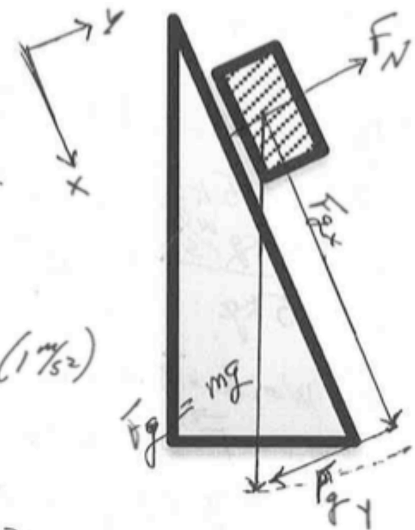
c) Does mass matter?

$\Sigma F_x = ma_x$ $\qquad$ $\Sigma F_y = ma_y = 0$

$F_{gx} = ma_x$ $\qquad$ $F_N - F_{gY} = 0$

$\sim 0.9mg = ma_x$ $\qquad$ $F_N = F_{gy} \approx .4mg$

$a_x \sim 9 \, m/s^2$



b) if $a = 1 \, m/s^2$, Then $\Sigma F_x = ma = m_o(1 \, m/s^2)$

now we have F_f in $-\hat{x}$.

$F_{gx} + F_f = ma_x \therefore F_f = F_{gx} - ma_x$

$= (9 \, m/s^2 - 1 \, m/s^2) m_o$

$F_f = 8 \, m/s^2 \, m_o$

$F_f = \mu F_N \approx \mu \, 0.4 m_o g$

$10 \, m/s^2$

$8 \, m/s^2 \, m_o \approx \mu \cdot 0.4 m_o g$

$\boxed{2 \approx \mu}$ very high coefficient of friction

c) We see from above that mass cancels. If you double mass, you also double F_g, F_N, F_f so the acceleration stays the same.

8. At right, you see that I pull a 5 kg mass down a 2 m long incline with a 20 N, horizontal force. With good communication and without a calculator or angle measuring, please calculate:

a) For a frictionless surface, please calculate the acceleration of the block, and the normal force of the surface on the block.
b) For a frictionless surface, please calculate the total work I do, and the block's final speed.
c) If the coefficient of friction is 0.25, please calculate the acceleration of the block.
d) If the coefficient of friction is 0.25, please calculate the amount of heat produced (in Joules), and please calculate the final speed of the block.
e) If I double mass now like I did in question 1, would the acceleration of the block remain the same? What makes this different from the above problem? In this problem, there is a 20 N force that does not depend on gravity. So doubling the mass would change the problem: the acceleration would decrease because the effect of the tension would be less on the greater mass.

a) Dynamics because we have forces and want $\vec{a}$

$\Sigma F_{\parallel} = m a_{\parallel}$

$T_{\parallel} + F_{g\parallel} = m a_{\parallel}$

$18N + 20N = 5kg \, a_{\parallel}$

$a_{\parallel} \approx \frac{38 \, kg \, m/s^2}{5 kg} = 7.6 \, m/s^2$

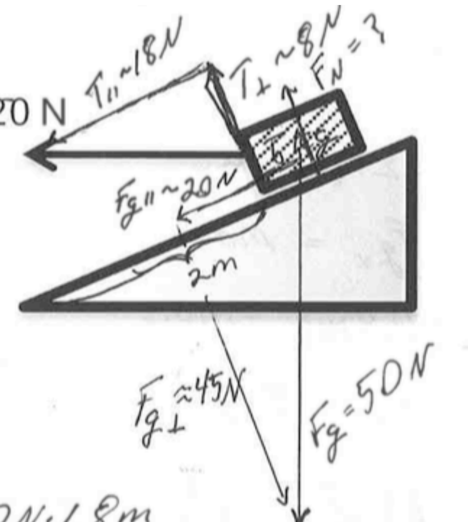
$\Sigma F_{\perp} = m a_{\perp} = 0$

$F_N + T_{\perp} + F_{g\perp} = 0$

$F_N = F_{g\perp} - T_{\perp}$

$\approx 45N - 8N$

$\approx 37N$



b) Work + Energy

$W_p = \vec{F} \cdot \vec{dx} \approx 18N \cdot 2m$, or $\approx 20N \cdot 1.8m$

$= 36J$, but we are also losing PE_g $mg\Delta h$

$PE_g \approx 5kg \cdot 10 \, m/s^2 (\sim 0.8 \, m)$

$\Delta PE \sim 40J$

Conserving Energy: $E_0 = E_f$

$PE_0 + \cancel{KE_0} + W_{Pete} = KE_f$

$\sim 76J = \frac{1}{2} m v^2$ $\quad V_f \approx 5.5 \, m/s$

c) now we must consider $F_f = \mu F_N \approx 0.25 \cdot 37N \approx 9N$

$\oplus \; T_{\parallel} + F_{g\parallel} + F_f = 18N + 20N - 9N = 29N = ma \quad a \sim 6 \, m/s^2$

d) W_f Work/Energy lens. The work of friction = heat.

$W_f = \vec{F_f} \cdot \vec{\Delta x} \approx 9N \cdot 2m = 18J \quad F_f \parallel \Delta x$ so just multiply

Now the total energy at the end is only 58J because 18J ⇒ heat. $\frac{1}{2} m v_f^2 \approx 58J$ $\boxed{V_f \approx 5 \, m/s}$

2. 9. . n problem set #3, we were able to solve for the acceleration by using the energy lens:
However, now I'd like you to find the acceleration using dynamics in one line: $a_S = \frac{\sum \vec{F}_S}{m_S}$. The only difference that the incline presents is that we don't care about the whole gravitational force on the 1 kg mass. We just care about the parallel component pulling the system in the positive direction (as indicated by the curved arrow and plus sign. So the total forces on the system in the parallel direction is + 2.5 N with a total mass of 1. 25 kg, yielding an acceleration of 2 m/s^2.