PS#8 Due in Class Monday, March 5. Please pay good attention to describe the lens you are using and explain your method.

**** Make sure to consider the direction of acceleration to inform your choice of axis. Do you remember how to pick a good axis?

1. Please do exercise #1 in section 7.1: pulling child in sled with energy considerations.

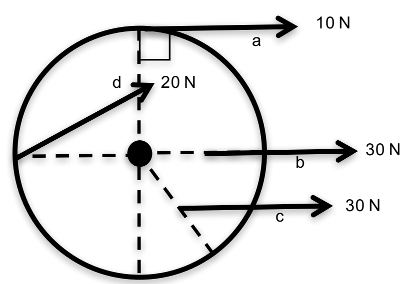
We will use an energy lens because the work I do will be transferred to kinetic and thermal energy. We recognize that work is the parallel component of the force times the distance. It looks as though the 40 N tension on the rope provides a little less than 30 N forward and a little more than 30 N upward, Let's say 28 N forward, and 32 N upward. Thus, the 10 kg sled is in equilibrium in the vertical direction with 100 N of gravitational force downward, 32 N of vertical tension upward, and 68 N of normal force (upward). The work done is ~ 280 J

a) Without friction, this would yield 280 J of E_K, corresponding to a final speed of 7.5 m/s.

b) The force of friction with a coefficient of friction of 0.15 is about 10 N, yielding 100 Joules of heat (work of friction), leaving only ~180 J of E_K corresponding to a final speed of 6.0 m/s.

2. Exercise #2 in section 7.1: Torque Wheel.

Most of the "mistakes" for this were estimating or just getting the numbers wrong. For starters, a diameter of 1 m means a *radius* of 0.5 m. Then we can see that d) probably has a perpendicular force of only 10 m and c) has a perpendicular force of about 25 N.



   a) $\vec{\tau} = 5Nm\otimes$
   b) $\vec{\tau} = 0$
   c) $\vec{\tau} = 12.5Nm\odot$
   d) $\vec{\tau} = 10Nm\otimes$

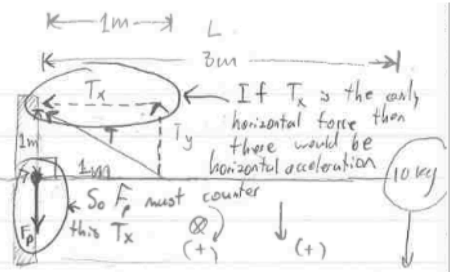
3. Exercise #3 in section 7.1: Tension in cable holding sphere.

PS #9 A-

B-

(1.) (7.1 #3) 10 kg ball at the end of 3 m pole (free to pivot at its base) is supported by a cable attached 1 m from the pivot. $\vec{T}$? $\vec{F}_{wall\ on\ pole}$?

"p" pivot



I use a dynamics / rotational dynamics lens because I see forces / torques causing acceleration / angular acceleration. $\Rightarrow \Sigma \vec{F} = m\vec{a}$ & $\Sigma \vec{\tau} = I\vec{\alpha}$. This is also a statics problem so $\Sigma \vec{F} = 0$ & $\Sigma \vec{\tau} = 0$.

$\Sigma \vec{\tau} = I\vec{\omega} = r\vec{F}_\perp = 0$

$(\Sigma \vec{\tau} = 0)$ $\vec{\tau}_g + \vec{\tau}_T + \vec{\tau}_p = 0$

$F_g \cdot L + (-T_y)L_0 = 0$

$(100N)(3m) + (-T_y)(1m) = 0$

$(100N)(3m) = T_y(1m)$

$T_y = 300N\,\hat{y}$

$T_y \sim 0.6\,T_x$ $\Sigma F_x = ma_x = 0$

estimate $T_x \approx \frac{1}{0.6}(300N)$

don't use trig $T_x \approx 500\ N$

$\therefore T = \sqrt{(T_x)^2 + (T_y)^2} \approx \boxed{583\ N = T}$

$F_p - T_y + F_g = 0$ $(\Sigma \vec{F} = 0)$

$F_p = T_y - F_g$

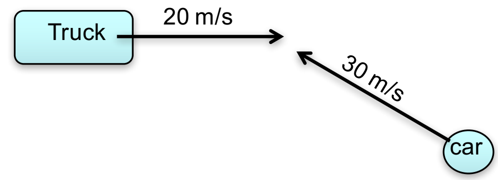
$F_p = 300N - 100N = \boxed{200N = F_p}$

$\Sigma \vec{F}_x = 0$ also

T_x

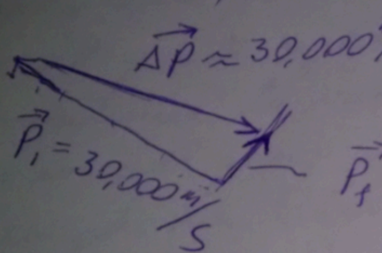
4. Exercise #4 in section 7.1: Collision on ice

On a very slippery road, a 1000 kg car and a 2000 kg truck have a collision and stick to each other as shown at right



- Could the wreckage have gone off in this ↓ direction? How do you know? We are going to conserve momentum here because there are negligible outside forces. Neither of the vehicles have a velocity component in the negative y direction, so this could not be the correct direction.
- Estimate the general direction that the wreckage went after collision. Because we need to conserve momentum, we need to turn this diagram from velocity to momentum. The truck has momentum of 40,000 kg m/s, and the car has 30,000 kg m/s. and ***NO***, this doesn't mean the total momentum is 70,000 kg m/s. If we draw this picture correctly and add the momenta correctly, we should end up with a total momentum of about 25,000 kg m/s in about this direction: ↗ Corresponding to a final velocity of about 8 m/s in the same direction.
- Find the impulse that the car received during the collision. Making a good momentum diagram, we can see the change in momentum of the car is about 30,000 kg m/s ↘
- If the collision lasted 1/15 of a second, estimate the average force on the car during the collision. About 200,000 N, or the force of gravity on 20 tons.

For the car, we need to find $\vec{\Delta p}$.



$\vec{p}_f \approx 1000\,kg \cdot 8\,m/s$

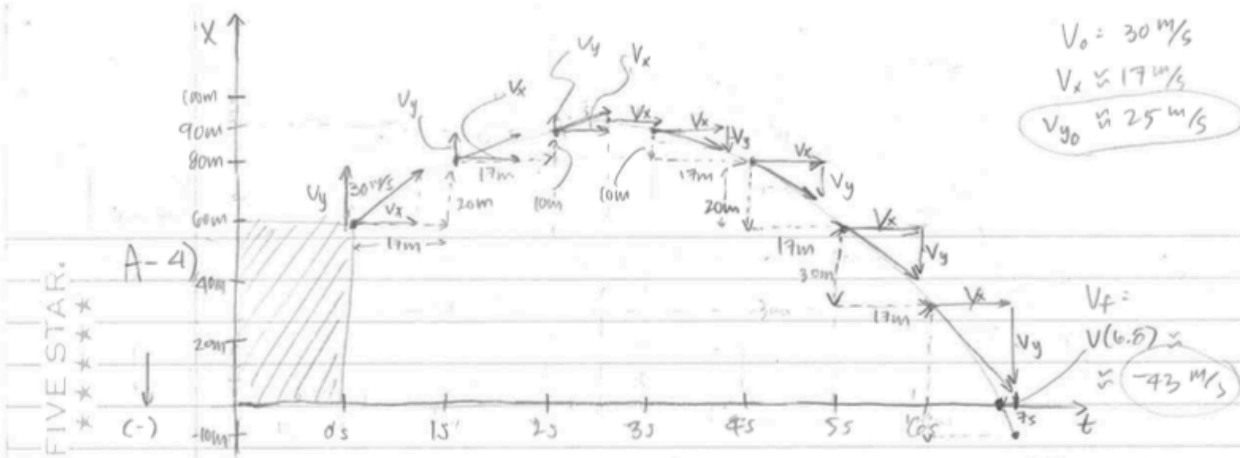
$= 8000\,kg\,m/s$ ↗

$\vec{\Delta p} \approx 30{,}000\,kg\,m/s \Rightarrow$

$$\vec{F} = \frac{d\vec{p}}{dt} \approx \frac{30{,}000\,kg\,m/s}{0.15\,s} \approx 200{,}000\,kg\,m/s^2$$

$= 200\,kN$, or the F_g on $\approx$ 20 Tons

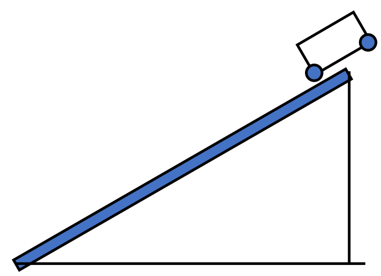
5. Please do Exercise #4 in 7.0: Throwing a rock off a cliff.



$g = -10$ m/s² I use a kinematics lens because I see x(t), v(t), and a(t) all as explicit f(t). Because I know F_g is the only force acting on the rock once it leaves my hand, I know that it's only acceleration is $g = -10$ m/s².

$V_f = V_0 + gt$	$V_{avg} = (V_i + V_f)\frac{1}{2}$	$\Delta x = (V_{avg})(t)$
1 s : $V_y = 25$ m/s $+ (-10$ m/s²$)(1s) = 15$ m/s	$\frac{1}{2}(25$ m/s $+ 15$ m/s$) = 20$ m/s	$(20$ m/s$)(1s) = 20$ m
2 s : $V_y = 25$ m/s $+ (-10$ m/s²$)(2s) = 5$ m/s	$\frac{1}{2}(15$ m/s $+ 5$ m/s$) = 10$ m/s	$(10$ m/s$)(1s) = 10$ m
2.5 s : $V_y = 25$ m/s $+ (-10$ m/s²$)(2.5s) = 0$ m/s	$\frac{1}{2}(5$ m/s $+ 0$ m/s$) = 2.5$ m/s	$(2.5$ m/s$)(1s) = 2.5$ m
3 s : $V_y = 25$ m/s $+ (-10$ m/s²$)(3s) = -5$ m/s	$\frac{1}{2}(0$ m/s $- 5$ m/s$) = -2.5$ m/s	$(-2.5$ m/s$)(1s) = -2.5$ m
4 s : $V_y = 25$ m/s $+ (-10$ m/s²$)(4s) = -15$ m/s	$\frac{1}{2}(-5$ m/s $- 15$ m/s$) = -10$ m/s	$(-10$ m/s$)(1s) = -10$ m
5 s : $V_y = 25$ m/s $+ (-10$ m/s²$)(5s) = -25$ m/s	$\frac{1}{2}(-15$ m/s $- 25$ m/s$) = -20$ m/s	$(-20$ m/s$)(1s) = -20$ m
6 s : $V_y = 25$ m/s $+ (-10$ m/s²$)(6s) = -35$ m/s	$\frac{1}{2}(-25$ m/s $- 35$ m/s$) = -30$ m/s	$(-30$ m/s$)(1s) = -30$ m
7 s : $V_y = 25$ m/s $+ (-10$ m/s²$)(7s) = -45$ m/s	$\frac{1}{2}(-25$ m/s $- 45$ m/s$) = -40$ m/s	$(-40$ m/s$)(1s) = -40$ m

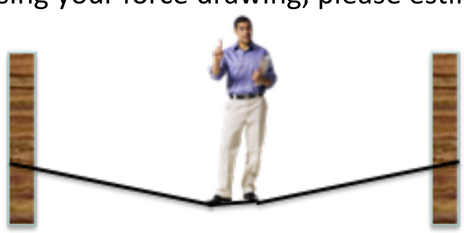
6. Please do section 7.2 and consider the cart of mass m_o at right, released from rest on a low friction track.

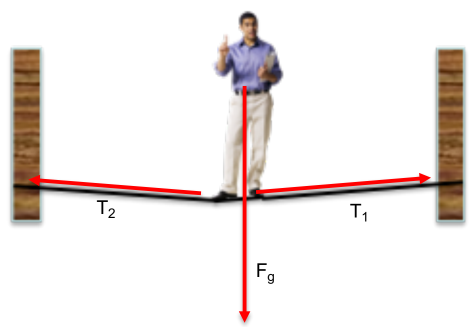


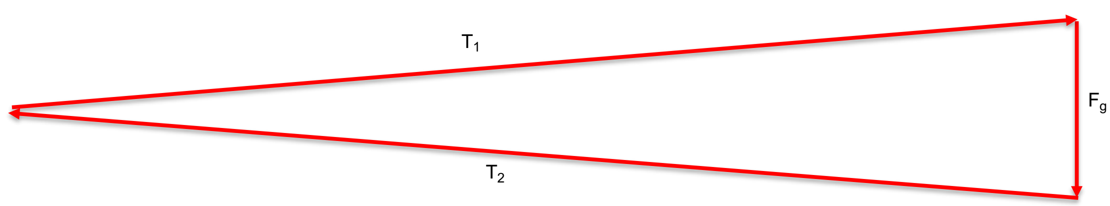
- Please find the resultant force on the cart in terms of constants that we know. Clearly outline your approach. We recognize this as a dynamics problem because the forces cause acceleration down the ramp. We do a good free body diagram and note that the interesting directions are parallel and perpendicular to the inclined surface because the cart accelerates parallel to this surface. We decompose the force of gravity into a parallel component and a perpendicular component. We see that the parallel component is about half of the full force of gravity or about ½mg.
- Please estimate the acceleration down the track. Recognizing that F=ma, the acceleration down the ramp is about half of gravity, or about 5 m/s².
- Repeat the above two questions if there is a coefficient of dynamic friction of 0.3 between the cart and the road. We need to find the normal force. If we did a good job with the FBD, we should see it's about 0.85 mg, yielding a force of friction of about 0.25 mg. Assuming the cart is moving downward, this force of friction is up the ramp, leaving only about 0.25mg down the ramp, for an acceleration of 2.5 m/s².

- What coefficient of friction would be necessary for the cart to move at a constant speed? Constant velocity means it's in equilibrium and the sum of the forces = 0. So, the force of friction needs to be about ½mg. Given a normal force of about 0.85, the frictional coefficient would need to be about 0.6.
- If the wheels on the cart had considerable mass, how would this affect the acceleration? I'd prefer to use an energy lens because when we add mass to the wheels, we'd see that in the energy transformation from potential energy to kinetic energy, some linear kinetic energy would be sacrificed to provide rotational kinetic energy of the wheels. Using a dynamics/rotational dynamics lens, we realize that in order to rotationally accelerate the wheels, torque is necessary requiring tangential force on the wheels in the upward direction… this force decreases the acceleration of the cart.

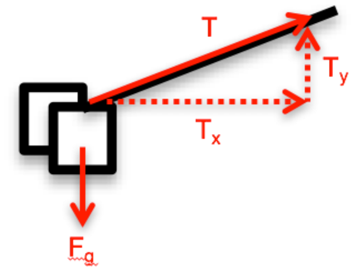
7. Slacklining is pretty fun, but you have to run some webbing between two trees first. Below, you see two pictures of me at 70 kg, slack lining.
   a) In which drawing is the line tighter? Please prove how you know this with a good force drawing and discussion. Lens? Using a dynamics lens… statics. If the vertical acceleration = 0, then the sum of the vertical forces = zero. So, the force of gravity = the vertical components of the tension. Thus, the strings that are more horizontal must have a greater tension. Please see how I have drawn the sum of the forces diagram below for the scenario at right. Notice that the tension is 5 to 6 times as great as the force of gravity. So for the figure at right, the tension is about 4000 N.
   b) Using your force drawing, please estimate the tension on the slack line at left.







8. You are watching the fuzzy dice from the rearview mirror. As you take off on level ground, it makes an angle as shown at right.



   a) state how you will inform your choice of axis.

This is a dynamics lens because we have forces acting on the dice causing them to accelerate. We write $\sum \vec{F} = m\vec{a}$ and identify forces and direction of acceleration. Because the acceleration is horizontal, we break things up into horizontal and vertical components.

   b) Estimate the acceleration of the car.

We see that $T_y = F_g$ because $a_Y = 0$. This we can see that the x component of tension (which is what's accelerating the dice in the x direction) is about twice $3*T_y = 3*F_g$ So the acceleration must be about 3 gravities or 20 m/s^2.

   c) What must be the coefficient of friction of your tires for this to happen?

Because the normal force on the car = F_g between the car and road (because there's no vertical acceleration), we can see that the coefficient of friction would have to be an extraordinarily high value of 3.0 in order to attain such a high acceleration.

   d) Is this realistic?

It's possible, but very unlikely, and not possible for regular tires and cars.

   e) If the mass of the dice is 100 g, what is the tension in the string?

Looking back at the work we did for b) we can see that the tension should be a little more than $3*F_g$ or about 35 N.

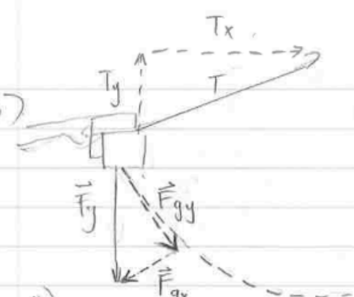
9. Consider the fuzzy dice above. Now the car is stationary and you are sitting it in. You grab the dice and pull them to one side exactly as in the diagram above. Then you let go of them.

   a) **** Choose a good axis. Is the direction of acceleration the same as above? State how this direction will inform your choice of axis.

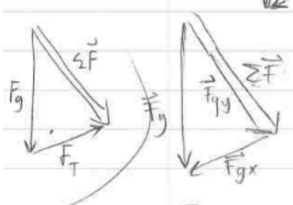
Now the acceleration is tangential, perpendicular to the radius, so our axes are radial and tangential, so we keep tension as a single force and decompose gravity into parallel (tangential) and perpendicular (radial) components.

   b) Again find the acceleration of the dice with direction.
   c) Again, if the mass of the dice is 100 g, please find the tension in the string. Is it the same as the string above? Why might this make sense?

B 3)



a) I use a dynamics lens since I see forces causing acceleration and since I see a body moving in a circular path; it has a_{cp} caused by some force; the $\vec{a}$ this time is in the direction of $\Sigma\vec{F}$ along the circle



b) I use the same lens for the same reason.

$\vec{a}_{radial} = 0$

$\therefore \vec{F}_{gx} = T$

$\therefore \Sigma\vec{F} = F_{gy}$

$F_{gy} \sim 0.9\ F_g = 0.9(mg)$

$\Sigma\vec{F} = m\vec{a} = F_{gy}$

$0.9\ \cancel{m}g = \cancel{m}\vec{a}_{tang}$

$\vec{a} = 0.9g$

$\boxed{\vec{a} = 9\ m/s^2}$

good

c) I use the same lens for the same reason.

I dont understand

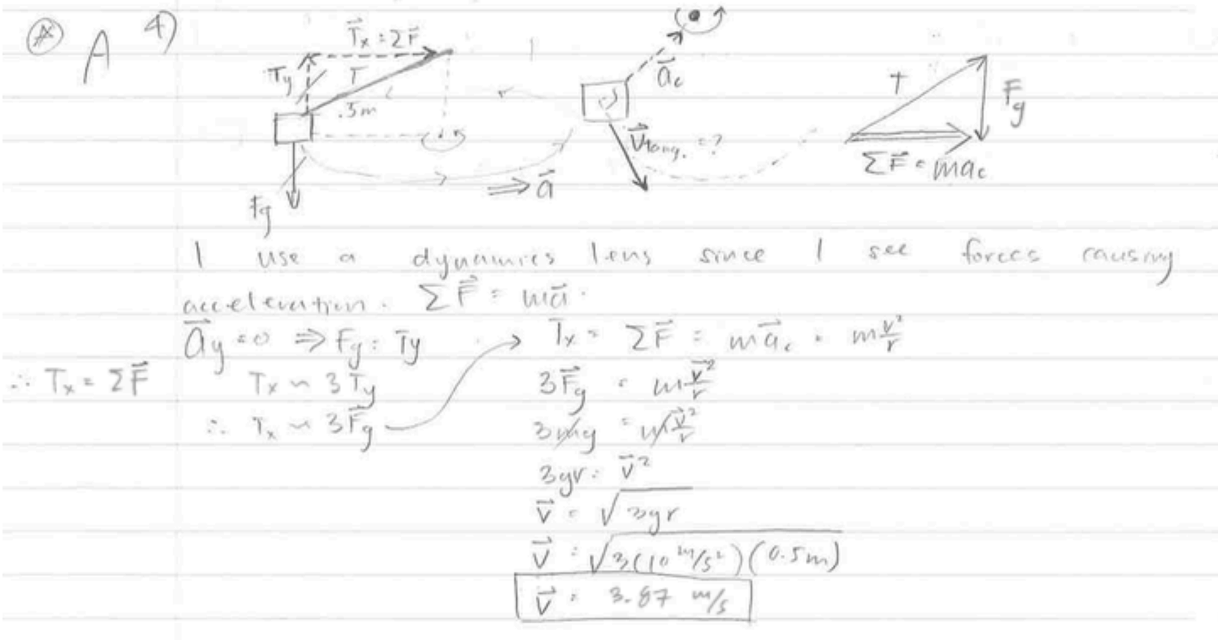
$T = F_{gx} \sim 0.4 F_g$

$T = 0.4(mg)$

$T = 0.4(0.1\ kg)(10\ m/s^2)$

$\boxed{T = 0.4\ N}$

10. Consider the fuzzy dice above. Now you are holding them from the end of the 50 cm string, and spinning the dice around in a circle. The path of the dice is a circle in the horizontal plane. Estimate the speed of the dice and the tension in the string.



I use a dynamics lens since I see forces causing acceleration. $\Sigma \vec{F} = m\vec{a}$.

$\vec{a}_y = 0 \Rightarrow F_g = T_y$

$T_x \sim 3T_y$

$\therefore T_x \sim 3F_g$

$\therefore T_x = \Sigma \vec{F}$

$T_x = \Sigma F = m\vec{a}_c = m\frac{v^2}{r}$

$3F_g = m\frac{\vec{v}^2}{r}$

$3mg = m\frac{\vec{v}^2}{r}$

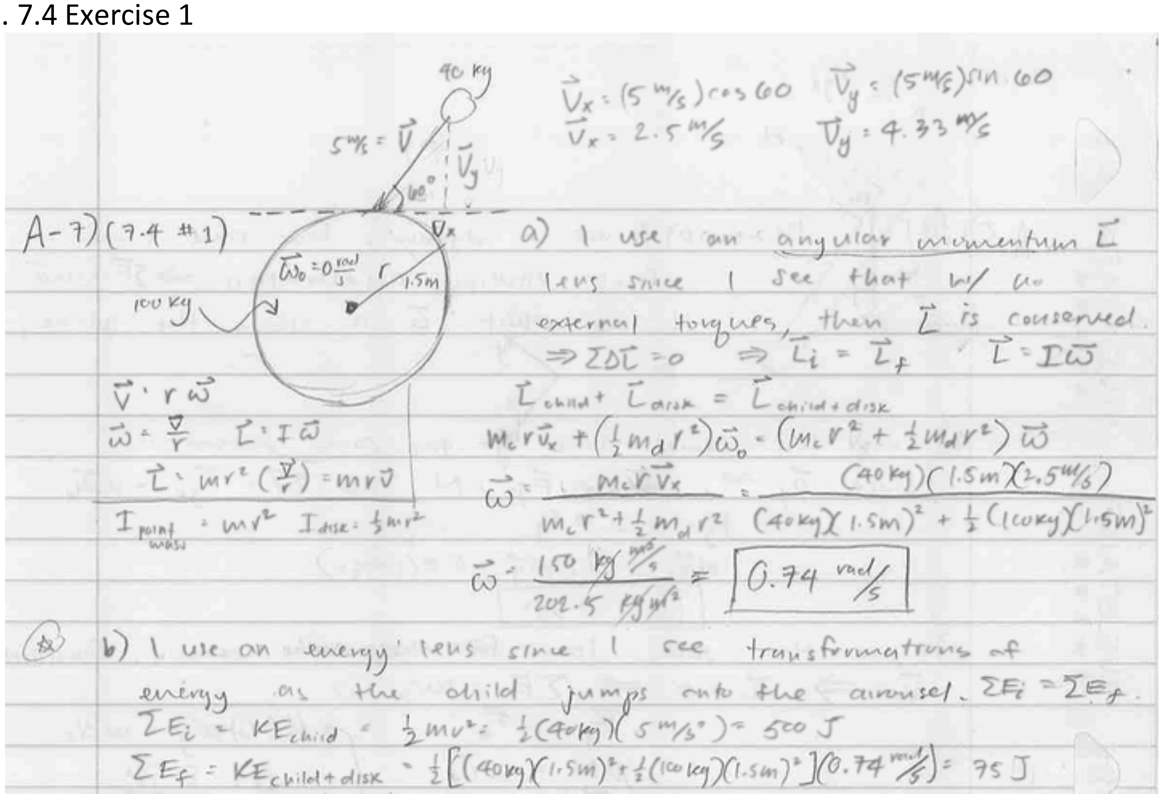
$3gr = \vec{v}^2$

$\vec{v} = \sqrt{3gr}$

$\vec{v} = \sqrt{3(10\ m/s^2)(0.5\ m)}$

$\boxed{\vec{v} = 3.87\ m/s}$

11. 7.4 Exercise 1



$\vec{V}_x = (5\ m/s)\cos 60$ $\quad \vec{V}_y = (5\ m/s)\sin 60$

$\vec{V}_x = 2.5\ m/s$ $\quad \vec{V}_y = 4.33\ m/s$

A-7) (7.4 #1)

a) I use an angular momentum $\vec{L}$ lens since I see that w/ no external torques, then $\vec{L}$ is conserved.

$\Rightarrow \Sigma\Delta\vec{\tau} = 0 \Rightarrow \vec{L}_i = \vec{L}_f \quad \vec{L} = I\vec{\omega}$

$\vec{L}_{child} + \vec{L}_{disk} = \vec{L}_{child+disk}$

$m_c r\vec{v}_x + (\frac{1}{2}m_d r^2)\vec{\omega}_0 = (m_c r^2 + \frac{1}{2}m_d r^2)\vec{\omega}$

$\vec{\omega} = \frac{m_c r\vec{v}_x}{m_c r^2 + \frac{1}{2}m_d r^2} = \frac{(40\ kg)(1.5\ m)(2.5\ m/s)}{(40\ kg)(1.5\ m)^2 + \frac{1}{2}(100\ kg)(1.5\ m)^2}$

$\vec{\omega} = \frac{150\ kg\ m^2/s}{202.5\ kg\ m^2} = \boxed{0.74\ rad/s}$

$\vec{V} = r\vec{\omega}$

$\vec{\omega} = \frac{\vec{V}}{r} \quad \vec{L} = I\vec{\omega}$

$\vec{L} = mr^2(\frac{\vec{V}}{r}) = mr\vec{V}$

$I_{point\ mass} = mr^2 \quad I_{disk} = \frac{1}{2}mr^2$

b) I use an energy lens since I see transformations of energy as the child jumps onto the carousel. $\Sigma E_i = \Sigma E_f$.

$\Sigma E_i = KE_{child} = \frac{1}{2}mv^2 = \frac{1}{2}(40\ kg)(5\ m/s^2) = 500\ J$

$\Sigma E_f = KE_{child+disk} = \frac{1}{2}[(40\ kg)(1.5\ m)^2 + \frac{1}{2}(100\ kg)(1.5\ m)^2](0.74\ rad/s) = 75\ J$

oops, while this student did a great job, there is a little mistake here. The final kinetic energy is $E_{K-Rot} = \frac{1}{2}I\omega^2, where\ I = \frac{1}{2}m_{disk}R^2 + m_{girl}R^2$. This student forgot to square the rotational velocity. I get a final rotational kinetic energy of only 55 J., so close to 90% of the kinetic energy is transformed to thermal energy. Also, please note that although the kinetic energy is correctly calculated, the square for the speed needs to be outside the parenthesis.

Angular momentum lens

c) The collision would decrease the rotation rate, since the carousel is rotating in the opposite direction that the child would hit and provide angular momentum.

$$\vec{L}_o = \vec{L}_f$$

$$mvr_\perp - I\vec{\omega} = L_f$$

d) momentum lens. Momentum is always conserved since there are no outside forces. The girl's momentum is transferred to the Earth, but since the Earth's mass is so large, velocity is negligable.

e) momentum lens because no outside forces.

$$P_o = P_f$$

$$\overset{0}{\cancel{P_{co}}} + P_{po} = P_f$$

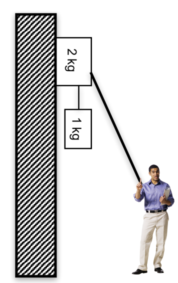
$$m v_{po} = m_{c+p} V_f$$

$$40kg(5\,m/s) = 140kg\, V_f$$

$$V_f = 1.4\,m/s \text{ southwest}$$

Also, again note that momentum is conserved (*all components* of momentum) independent of conserving the angular momentum (where only the tangential component contributes).

12. Please see a picture of me pushing a system of masses up a vertical wall with a coefficient of friction of 0.4. If I push the stick as shown with a force of 80 N, please find the approximate acceleration of the system of masses, and the tension in the string holding the 1 kg mass.

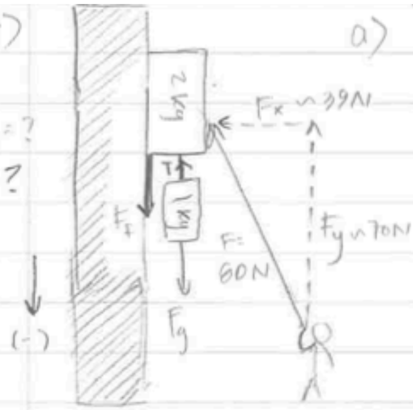


In this problem, we use a dynamics lens and recognize that the acceleration is upward (or downward), so we choose x-y coordinates. a_x=0, so we know the normal force = the x component of the force I put on the stick.

A 9)

$\vec{a}_{sys} = ?$

$T = ?$



a) I use a dynamics lens since I see forces causing acceleration. $\Rightarrow \Sigma F = m\vec{a}$

$$\vec{F}_y - F_g - F_f = (m_1 + m_2)\vec{a}_{sys}$$

$$\vec{a}_{sys} = \frac{F_y - (m_1+m_2)g - \mu F_x}{m_1 + m_2} \qquad \mu = 0.4$$

$$\vec{a}_{sys} = \frac{70N - (3kg)(10\,m/s^2) - (0.4)(39N)}{3\,kg}$$

$$\boxed{\vec{a}_{sys} \approx 8.2\ m/s^2}$$

b) I use a dynamics lens for the same reason.

$$T - F_{g_2} = m_2\vec{a}$$

$$T = m_2\vec{a} + m_2 g$$

$$T = (1\,kg)(8.2\,m/s^2 + 10\,m/s^2) \approx \boxed{18.2N}$$

We're glad to see that the tension in the string > F_g because the mass attached to the string is accelerating upwards.