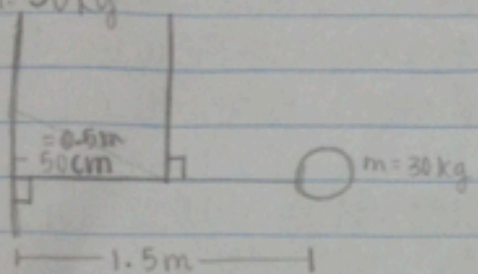
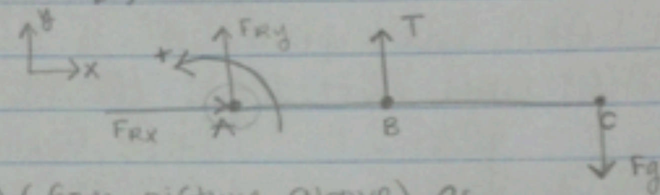
Problem Set #8

1. m: 30kg



a) This is a dynamics problem because we are finding tension which is a force. This is rotational because there is a pivot involved which means torque is present.

b)



c) You will use point A (from picture above) as the center for calculating torque because it is the point where the pivot point is located.

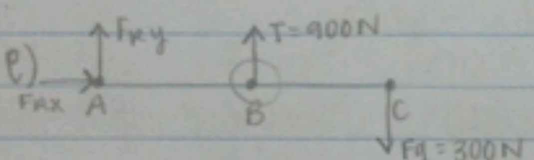
d) T?

$$\Sigma\tau_A = 0 = T\times\vec{AB} - F_g\times\vec{AC}$$

$$F_g\vec{AC} = T\vec{AB}$$

$$T = \frac{mg\vec{AC}}{\vec{AB}} = \frac{(30kg)(10\frac{m}{s^2})(1.5m)}{(0.5m)} = \boxed{900N}$$

e)



$$\Sigma\tau_B = 0 = F_g\times\vec{BC} - F_{Ry}\times\vec{BA}$$

$$F_{Ry} = \frac{-F_g\times\vec{BC}}{\vec{BA}} = \frac{(-300N)(1m)}{0.5m}$$

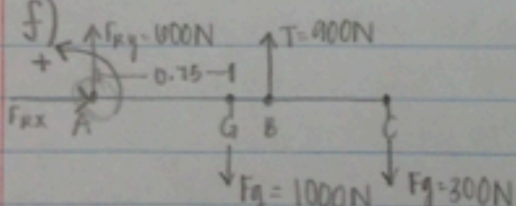
$$= -600N$$

$$\Sigma F_x = 0 = F_{Rx}$$

$$F_{Rx} = 0N$$

$F_R = \boxed{600N \downarrow}$ $\quad (\vec{F}_R = -600N\,\hat{j})$

f)



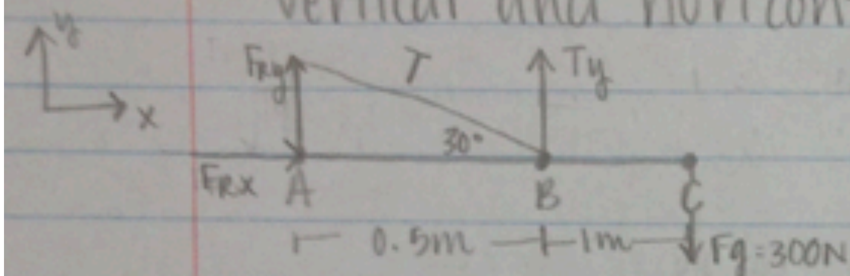
$$\Sigma\tau_A = 0 = -F_{g_G}(\vec{AG}) + T(\vec{AB}) - F_{g_C}(\vec{AC})$$

$$T = \frac{F_{g_G}(\vec{AG}) + F_{g_C}(\vec{AC})}{\vec{AB}} = \frac{(1000N)(0.75m) + (300N)(1.5m)}{0.5m}$$

$$= \frac{750N\cdot m + 450N\cdot m}{0.5m} = \boxed{2400N}$$

g) The tension would increase because distance between each force would be less and there's less torque so the tension needs to increase to maintain equilibrium.

h) The tension would increase because the line is now the hypothenuse of a triangle made with the bar and store front and the hypothenuse is always greater than the vertical and horizontal components. The reactionary force (force supplied by the wall) would be greater because there is now a horizontal component involved as well as the existing vertical component



$$\Sigma \tau_A = 0 = (T\sin 30°)(\vec{AB}) - Fg(\vec{AC})$$

$$T = \frac{mg(\vec{AC})}{(\sin 30°)(\vec{AB})} = \frac{(300N)(1.5m)}{(\frac{1}{2})(\frac{1}{2}m)} = \frac{450N}{\frac{1}{4}} = \boxed{1800N}$$

$$F_{Rx} = T\cos\theta = (1800N)(\cos 30°) = (1800N)(\tfrac{\sqrt{3}}{2}) \approx 1350N$$

$$\Sigma \tau_B = 0 = -Fg(\vec{BC}) - F_{Ry}(\vec{BA})$$

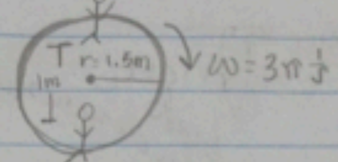
$$F_{Ry} = \frac{-Fg(\vec{BC})}{\vec{BA}} = \frac{-(300N)(1m)}{0.5m} = -600N$$

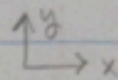
$$F_R = \sqrt{F_{Ry}^2 + F_{Rx}^2} = \sqrt{(-600N)^2 + (1350N)^2} = \sqrt{360000N^2 + 1822500N^2}$$

$$= \sqrt{2182500N^2} \approx \boxed{1500N}$$

2. $m_c = 100\,kg$ $d = 3m \rightarrow r = 1.5m$ $\omega = 1.5\,\frac{rev}{s} \times \frac{2\pi\,rad}{rev} = 3\pi\,\frac{1}{s}$

$m_x = 30\,kg$ $X = 1m$





22.5
×5
112.5

a) $I_c = \frac{1}{2}mr^2 = \frac{1}{2}(100kg)(1.5m)^2 = (50kg)(2.25m^2) = \boxed{112.5\,kg \cdot m^2}$

$I_x = mr^2 = (60kg)(1m)^2 = \boxed{60\,kg \cdot m^2}$

b) ω?

$\omega_i = 1.5\,\frac{rev}{s} \times \frac{2\pi\,rad}{1\,rev} = 3\pi\,\frac{1}{s}$

$\rightarrow -3\pi\,\frac{1}{s} \approx \boxed{-10\,\frac{1}{s}}$

c) The rotation rate will slow down because rotational momentum is conserved and inertia increases due to the greater mass so angular velocity will decrease. this is therefore a momentum problem.

d) $L_i = L_f$

$I_i\omega_i = I_f\omega_f$

$\omega_f = \frac{I_i\omega_i}{I_f} = \frac{(112.5\,kg \cdot m^2)(-10\frac{1}{s})}{(112.5\,\cancel{kg \cdot m^2} + 60\,\cancel{kg \cdot m^2})} = \frac{-1125\,\frac{1}{s}}{172.5} \approx \frac{-220\,\frac{1}{s}}{35} = \frac{-44\,\frac{1}{s}}{7}$

$\approx \boxed{-6\,\frac{1}{s}}$

e) $KE_i = \frac{1}{2}I_i\omega_i^2 = \frac{1}{2}(112.5\,kg \cdot m^2)(-10\frac{1}{s})^2 = \frac{1}{2}(11250\,\frac{kg \cdot m^2}{s^2}) = \boxed{5625\,J}$

172.5
×18
13800
+1725.0
3105.0

$KE_f = \frac{1}{2}I_f\omega_f^2 = \frac{1}{2}(112.5\,kg \cdot m^2 + 60\,kg \cdot m^2)(-6\frac{1}{s})^2 = \frac{1}{2}(172.5\,kg \cdot m^2)(36\,\frac{1}{s^2})$

$= 18(172.5J) = \boxed{3105\,J}$

Kinetic energy was not conserved. Kinetic energy ~~could~~ was have been u~~sed in work ouput or~~ changed into heat from friction in this inelastic rotational collision

112.5
×6
675.0

f) $L_{ic} = I_c\omega_i = (112.5\,kg\,m^2)(-10\frac{1}{s}) = -1125\,\frac{kg \cdot m^2}{s}$

$L_{fc} = I_c\omega_f = (112.5\,kg \cdot m^2)(-6\frac{1}{s}) = -675\,\frac{kg \cdot m^2}{s}$

1125
−675
450

$\Delta L_c = L_f - L_i = -1125\,\frac{kg \cdot m^2}{s} + 675\,\frac{kg \cdot m^2}{s} = \boxed{+450\,\frac{kg \cdot m^2}{s}}$ $\frac{kg}{s}$

$L_{ix} = I_x\omega_i = (60\,kg \cdot m^2)(0\,\frac{1}{s}) = 0\,\frac{kg \cdot m^2}{s}$

$L_{fx} = I_x\omega_f = (60\,kg \cdot m^2)(-6\frac{1}{s}) = -360\,\frac{kg \cdot m^2}{s}$

$\Delta L_x = L_f - L_i = -360\,\frac{kg \cdot m^2}{s} + 0\,\frac{kg \cdot m^2}{s}\ \boxed{-360\,\frac{kg \cdot m^2}{s}}$

these two $\Delta\vec{L}$ should be = and opposite because $\vec{L}$ is conserved. your numbers are different because of rounding errors.

because the carousel ~~slows~~ $\Delta\omega, \Delta L, \tau$
in the - direction
is all positive.

g) $t = 0.05\,s$

τ?

$\tau = \frac{\Delta L}{t} = \frac{-360\ \frac{kg \cdot m^2}{s}}{0.05\,s} = \frac{-360\ kg \cdot m^2}{s} = \boxed{-7200\ kg \cdot m^2} \frac{}{s^2} = \cancel{70}$ $= 7200\ NM$

both in denominators

h) $r = 2\,m$

ω?

$I_k = mr^2 = (60\,kg)(2\,m)^2 = 240\ kg \cdot m^2$

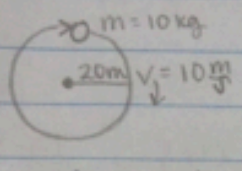
$L_i = L_f$

$I_i \omega_i = I_f \omega_f$

$\omega_f = \frac{I_i \omega_i}{I_f} = \frac{(112.5\ kg \cdot m^2)(-10\ \frac{1}{s})}{(112.5\ kg \cdot m^2 + 240\ kg \cdot m^2)} \approx \frac{-1125\ \frac{kg \cdot m^2}{s}}{350\ kg \cdot m^2} \approx \boxed{-3\ \frac{1}{s}}$

much slower.

3. $m = 10kg$ $\quad v = 10\frac{m}{s}$ $\quad r = 20m$



a) ~~circumference = 2πr~~

~~r = circumference / 2π = 30m / 2π ≈ 5m~~

$v = r\omega$

$\omega = \frac{v}{r} = \frac{10\frac{m}{s}}{20m} \approx \boxed{0.5\frac{1}{s}}$

b) The velocity is constantly changing direction when going in a circle so we know the rock is accelerating. The force of gravity keeps it moving in a circular motion.

c) a_c?

$a_c = \omega^2 r = (0.5\frac{1}{s})^2(20m) = (0.25\frac{1}{s^2})(20m) = \boxed{5\frac{m}{s^2}}$

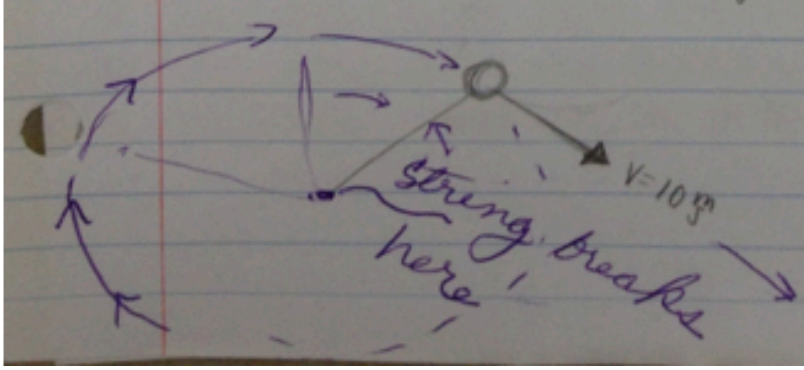
d) $F = ma_c = (10kg)(5\frac{m}{s^2}) = \boxed{50N}$

e) I have no idea what force is acting on it, because I can't see anything that the rock is interacting with so I have to look around at what object must be applying a force of 50N on the rock to make it accelerate at $5\frac{m}{s^2}$.

f) This is the force of tension that you are applying.

$T = ma_c = (10kg)(5\frac{m}{s^2}) = \boxed{50N}$

g) The rock will move in the direction tangent to the curve where the string broke at a velocity of $10\frac{m}{s}$.



h) The force of gravity would be acting on the rock now.

$F_g = \frac{-m_1 m_2 G}{r^2}$

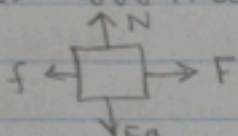
$m_2 = \frac{-\cancel{m_1} a_c r^2}{\cancel{m_1} G} = \frac{-(1.5\,\frac{m}{s^2})(20m)^2}{6.67\times10^{-11}\,\frac{N\cdot m^2}{kg^2}} \approx 300\times10^{11}\ kg = \boxed{3\times10^{13}\ kg}$

$\rho = \frac{m}{V} = \frac{3\times10^{13}\ kg}{\frac{4}{3}\pi(20m)^3} \approx \frac{1}{10}\times10^{10}\ kg/m^3 \approx \boxed{1\times10^{9}\ \frac{kg}{m^3}}$

a subject with this density is a neutron star. (well not quite

but it's more dense than anything we have on earth

i) Now the force of friction is acting on the car.



$m = 10kg \quad r = 20m \quad v = 10\,\frac{m}{s}$

μ?

$\Sigma F_y = N - F_g = 0$

$N = mg = (10kg)(10\,\frac{m}{s^2}) = 100N$

$\Sigma F_x = f$

$ma_c = \mu N$

$\mu = \frac{ma_c}{N} = \frac{(10kg)(5\,\frac{m}{s^2})}{100\,N} = \boxed{0.5}$

~~reasonable~~ fr

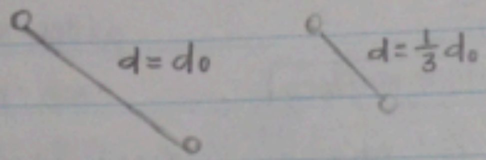
this is about the friction on the road

Water : $\rho_w = 10^3\,\frac{kg}{m^3}$

rock $\rho = 5 \cdot \rho_w$

metals $\rho \sim 10\rho_w \sim 10^4\,\frac{kg}{m^3}$

4.



a) there is no external torque on the system, angular momentum is conserved.

b) $I = mr^2$

$\boxed{I = \frac{1}{9} I_0}$

c) $\boxed{L = L_0}$

d) $\Delta l = l_f - l_0 = 0$ $\quad l_f = l_0$ $\quad \boxed{\omega = 9\omega_0}$

$I_f \omega_f = I_0 \omega_0$

$9\left(\frac{1}{9} I_0\right)\omega_f = I_0 \omega_0$

$9\omega\left(\frac{1}{9} I_0\right) = I_\omega \omega_0$

e) tension is the same regardless of length of string $\quad T \Rightarrow 3T_0$

g) $KE_{rot} = \frac{1}{2} I\omega^2$ $\quad KE_f = KE_0$

$\frac{1}{2} I_f \omega_f = \frac{1}{2} I_0 \omega_0^2$

$\frac{1}{2}\left(\frac{1}{9} I_0\right)(9\omega_0)^2 = \frac{1}{2} I_0 \omega_0^2$

$\boxed{KE = 9KE_0}$

h) work is done by the motor to pull the masses closer so that's where the extra KE came from

i) makes sense! as ~~the~~ she pulls in her arms her arms actually move faster ⇒ more KE!

$$T = \underline{\underline{m a_c}} = m\frac{v^2}{r} = m\omega^2 r$$

or $T = m\frac{L^2}{m^2 r^{\cancel{2}}}\cancel{r}$

so $L = m\omega r$, $\omega = \frac{L}{mr}$

$$T = \frac{L^2}{mr}$$

any way you look at it, as $d \Rightarrow \frac{1}{3} d_0$ $T \Rightarrow 3T_0$