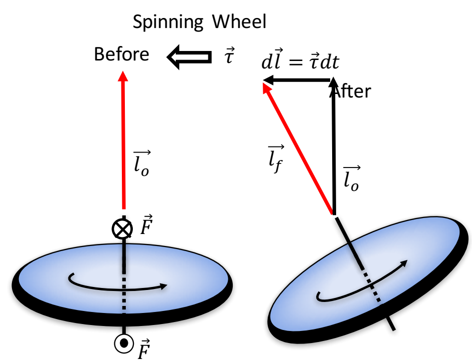
PS#9 Due Thursday, June 7 in class. Start each question with a description of the lens and method.

1. 7.5 Exercise 3, Precession of a bicycle wheel.
    - Predict what happens when the wheel is spinning in the opposite direction? Why?

If the wheel is spinning the opposite direction, the torque and change in angular momentum are the same, but the initial angular momentum is in the opposite direction. Thus, we see that the wheel will tip in the opposite direction. Thus it will precess in the opposite direction.

- What happens if the wheel is spinning faster? Can you explain why?

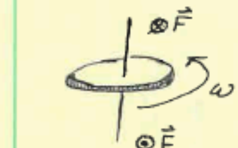
If the initial angular momentum is greater, then change of angular momentum from the same torque of gravity will result in a smaller change of angle of the spinning wheel. So it would precess slower.



- How does the rate of precession change when you push harder on the axle? Why?

If you push harder on the wheel the torque and change in angular momentum will be greater. In the same amount of time, the wheel will tilt by more. Thus, the rate of precession will increase.

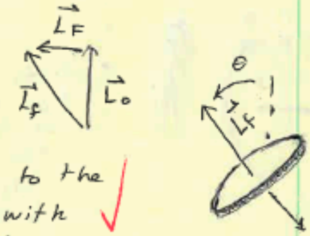
2. 7.5 Ex. 1



Lens | Angular Momentum: $\Sigma\vec{L} = \vec{L_o} + \Delta\vec{L}$

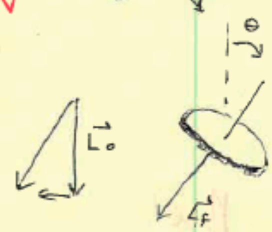
Dynamics

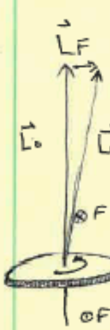
- The forces provide a ~~rightward~~ left-ward angular momentum $\vec{L}$.



→ The axle will turn to the left from the top with the forces provided. ✓

→ Opposite direction:
The $\vec{L_o}$ is downward, ~~So~~ the forces provide $\vec{L}$ to the left, so...
The axle turns to the right from top ✓





* less direction change

→ If the wheel is spinning faster, there's more angular momentum up or down, so there's a greater vector magnitude, which is more difficult to change. Axle direction will shift less. ✓

→ Rate of precession:
will increase with a larger force since a larger angular momentum ⊥ to the wheel is delivered, so it will ~~more~~ precess quicker. ✓

- What changes if you switch sides and support the axle on the other side? Why?

Supporting the wheel from the other side will reverse the torque that gravity provides. This will result in the wheel precessing in the opposite direction. Please prove this to yourself with a drawing.

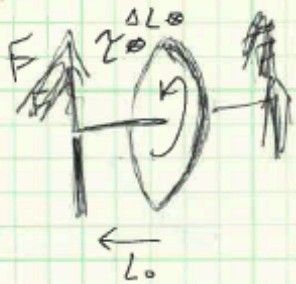
- What happens if you support the axle closer to the center of the wheel? Why?

Supporting the wheel close to the axle will reduce the torque from gravity. This will reduce the change in angular momentum, so the rate of precession will decrease.

Angular Momentum / Rotational Dynamics

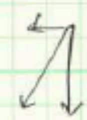
$\Delta \vec{L}$ is 0 so $\sum \vec{L}_{Total} = \vec{L}_0 + \Delta \vec{L}$

- Forces applied at a radius create torques, (impulse on angular momentum) $\vec{\tau} = \frac{d\vec{L}}{dt}$



- Supporting it on the ~~left~~ right side creates a torque and $\vec{L}$ into the paper. $\vec{L}_0$ is to the left. Summing these like vectors shows it will spin direction upward.

- Spinning the wheel the opposite way creates a L_0 to the right. The ↑ forward sums with the downward $\vec{L}_0$, so spins downward.



Nice!

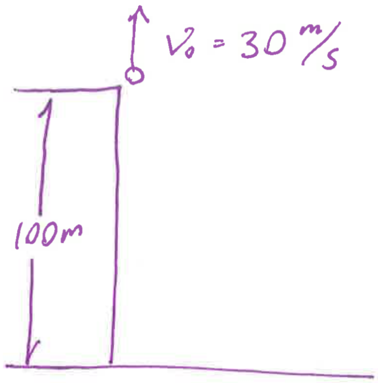
- Faster spin = slower precession since there is a larger L_0, so $\Delta \vec{L}$ will be more minimal compared to the large $\vec{L}_0$

- Switching sides of the supporting axle, it will change the direction of τ applied ~~so it will precess opposite~~ so it will precess opposite

- Closer to the center, the torque applied by gravity will be less since $\tau = F_\perp (r)$, so a smaller r = smaller τ. This means slower precession.

2. 7.6 Exercise 3, Throwing a rock upwards off the edge of a cliff.

I use a kinematics lens because we have motion: an explicit $f(t)$



$$Y(t) = Y_0 + V_0 t + \tfrac{1}{2} a t^2$$

$$0 = \underbrace{100m}_{c} + \underbrace{30\,m/s}_{b}\, t \underbrace{- 5\,m/s^2}_{a}\, t^2$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-30\,m/s \pm \sqrt{(30\,m/s)^2 - 4(-5\,m/s^2)(100m)}}{2(-5\,m/s^2)}$$

multiply through by (−1)

$$= \frac{30\,m/s \mp \sqrt{900\,m^2/s^2 + 2000\,m^2/s^2}}{10\,m/s^2}$$

$$= 3s \pm \sqrt{29}\ s$$

$$\approx 3s \pm 5.4s = -2.4s,\ 8.4s$$

The negative value.. given this trajectory, if we went backwards in time, it would be at the bottom of the cliff, moving upwards at about 54 m/s ... $= 30\,m/s + g(2.4s)$

But we didn't need the quadradic equation. We knew all along how to find time:

$\Delta X = V_{ave}\,\Delta t \qquad \Delta t = \dfrac{\Delta X}{V_{ave}} \qquad V_{ave} = \dfrac{(V_0 + V_f)}{2}$

we can use this given constant acceleration (g)

We can find V_f using an energy lens because

$E_p \Rightarrow E_K$ $\qquad E_0 = E_f$

$$E_K + E_p = E_K$$

$$mgh_0 + \frac{1}{2}mv_0^2 = \frac{1}{2}mv_f^2$$

$$V_f = (V_0^2 + 2gh_0)^{\frac{1}{2}}$$

$$= \left[(30\,\text{m/s})^2 + 2(10\,\text{m/s}^2)100\,\text{m}\right]^{\frac{1}{2}}$$

$$\approx 54\,\text{m/s} \,, \qquad + \uparrow$$

$$V_{ave} = \frac{(30\,\text{m/s} + -54\,\text{m/s})}{2} \approx -12\,\text{m/s} \qquad \Delta x = -100\,\text{m}$$

$$\Delta t = \frac{\Delta x}{V_{ave}} = \frac{-100\,\text{m}}{-12\,\text{m/s}} \approx \underline{\underline{8.3\,\text{s}}} \quad \checkmark$$

3. 7.6 Exercise 4, Catching the Bus.

PS #6

#1 - Catching Bus

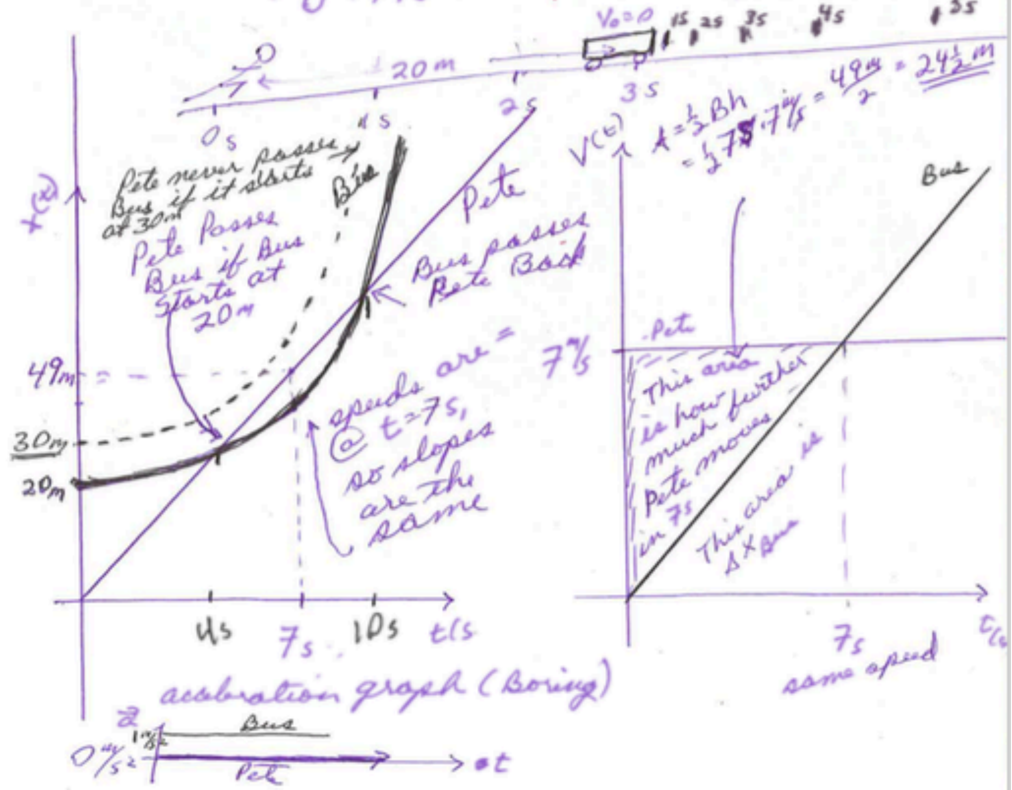
Pete: $V_P = 7\,m/s = \text{const}$, $x_0 = 0$

Bus: $V_{0B} = 0$, $a_B = 1\,m/s^2$, $x_0 = 20m$, $V = a_B t$

Kinematics - because we are dealing with exclusive use of position and its time derivatives as an explicit function of time. In particular: $X_P(t) \overset{?}{=} X_B(t)$ when and if are our displacements the same

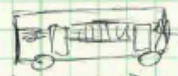
Pete: $X = X_0 + Vt$
$= 0 + 7\,m/s\; t$

Bus: $X = X_0 + V_0 t + \frac{1}{2} a t^2$
$= 20m + 0 + \frac{1}{2}\,m/s^2\; t^2$



⑥ 7.6 Ex. 4

$\vec{V} = 7\,m/s$ → 20m



$\vec{a} = 1\,m/s^2$

Lens: kinematics, we have velocity and $\vec{a}$ as a fn of time. ✓

$x(t) = \cancel{x_0 + V_0 t + \frac{1}{2} a t^2}$

$\cancel{20m + 7\,m/s\, t + \ldots}$

Person

$x(t) = x_0 + V_0 t + \frac{1}{2} a t^2$ ✓

$x(t) = 0 + 7\,m/s\, t + 0$

$x_p(t) = 7\,m/s\, t$

Bus

$x_B(t) = 20m + 0 + \frac{1}{2}(1\,m/s^2)t^2$

$x_B(t) = 20m + \frac{1}{2}\,m/s^2\, t^2$

$7\,m/s\, t = 20m + \frac{1}{2}\,m/s^2\, t^2$

$0 = \frac{1}{2}\,m/s^2\, t^2 - 7\,m/s\, t + 20m$

$t = \dfrac{7 \pm \sqrt{49 - 4(\frac{1}{2})(20)}}{1} = 7 \pm \sqrt{9}$

$t = 7 \pm 3$

$t = 4, 10$ ✓

$x_p(t) = 7\,m/s\, t$

$x_p(4) = 7\,m/s\,(4s) = \boxed{28\,m}$ $\boxed{@\ 4s}$

$x_p(10) = 7\,m/s\,(10s) = \boxed{70\,m}$ $\boxed{@\ 10s}$

You catch the bus

4. 7.6 Exercises 5 – 7 (Pulling sled, Hitting a baseball, Torque on a wheel.

7.6 Exercise 5. We would solve this problem exactly as we did before we used trigonometry. The only difference is now we could calculate the components rather than just eyeball (estimate) them. Of course, we recognize this as a dynamics problem whereby the acceleration is horizontal, so we choose x-y components and break the tension into horizontal and vertical components.

$T_x = T\cos(30^o) \sim 40\ N*(0.866) = 34.6\ N$

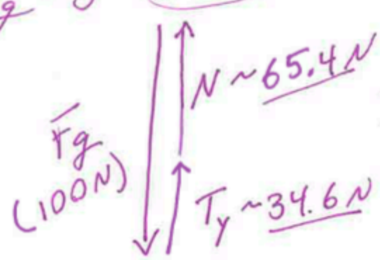
$T_x = T\sin(30^o) \sim 40\ N*(0.5) = 20\ N.$

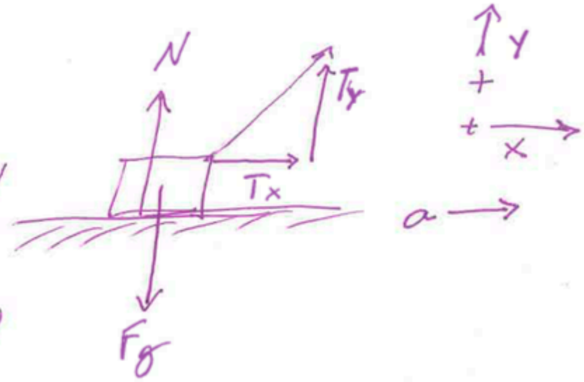
$W = \vec{F} \cdot \overrightarrow{dx}$, We take the x-component of the tension (force) to find that the work I do is *$20\ N * 5\ m = 100\ J.$*

$\Sigma F_y = 0$

$T_y + N - F_g = 0$

$\Sigma F_y = 0$





$\Sigma F_x = ma$

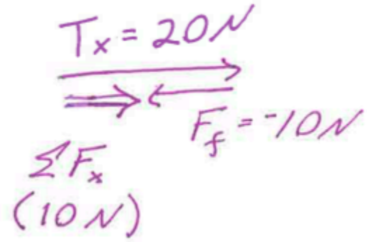
$T_x - F_f = ma$

$20N - 10N = ma$

$\frac{10N}{m} = a \approx 1\ m/s^2$

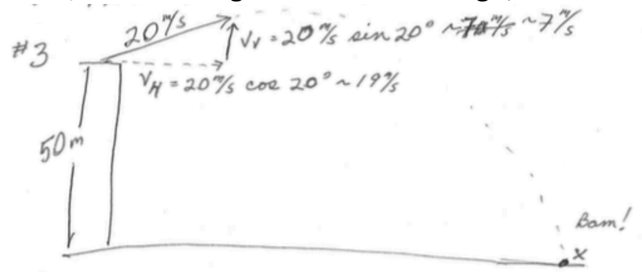
$F_f = \mu N$

$= 0.15 \cdot 65.4N \sim 10N$



finding the acceleration requires us to use a dynamics lens because the force cause the acceleration. We do a good FBD as always and identify that the forces in the x direction are the horizontal tension and the friction force. To find the force of friction, we need the normal force. We recognize that we are in equilibrium in the y direction because we are (likely) not accelerating off the surface of the earth. Gravity provides 100 N of force (downward), and the vertical component of tension is 34.6 N upward. In order to be in equilibrium in the y direction, the normal force must be 65.4 N (upward). This yields a friction force of about 10 N in the direction opposite to our motion. Assuming that we are moving forward as I pull the sled, the net force is the sum of the x-component of tension minus the frictional force *20 N – 10 N = 10 N* in the positive direction. This yields an acceleration of the 10 kg sled and girl of 1 m/s^2.

7.6 Exercise 6: If you hit a baseball at a 20 degree angle above the horizon, at an initial velocity of 20 m/s off the edge of a cliff 50 m high,

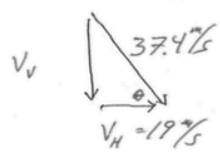


a) $E_o = E_f$

$KE_o + PE_o = KE_f + \cancel{PE_f}^{\,0}$ solve ~~for~~ math and find $V_f \approx 37.4\,\text{m/s}$

$\frac{1}{2}\cancel{m}(20\,\text{m/s})^2 + \cancel{m}\,10\,\text{m/s}^2 \cdot 50\,\text{m} = \frac{1}{2}\cancel{m}V_f^2$

b) $a_H = 0$, so $V_{H_{final}} = V_{H_{initial}}$ = const $\approx 19\,\text{m/s}$



$\cos\theta \approx \frac{19\,\text{m/s}}{37.4\,\text{m/s}} \approx 60°$

$V_V^2 + V_H^2 \approx V^2$

$V_V^2 + (19\,\text{m/s})^2 = (37.4\,\text{m/s})^2$

$V_V \approx 32\,\text{m/s}$ ⇓

c) $\Delta V_V = -32\,\text{m/s} - 7\,\text{m/s}$

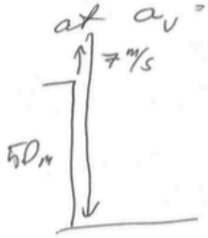
$\approx -40\,\text{m/s} = at$

$a \approx -10\,\text{m/s}^2$, so $\Delta t \sim 4\,\text{s}$

$\Delta x = V_H t + \cancel{\frac{1}{2}a_H t^2}^{\,0} \approx 19\,\text{m/s} \cdot 4\,\text{s} \approx \underline{\underline{75\,\text{m}}}$

horizontal $\vec{a} = 0$

d) now I use straight kinematics.
Horizontally, it's just moving along at $V_H \approx 19\,\frac{m}{s}$ in the $\hat{x}$ direction.
vertically it's moving upward at $V_{V_0} \sim 7\,\frac{m}{s}$ and accelerates downward at $a_V = -10\,\frac{m}{s^2}$



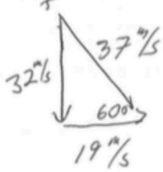
find t w/
$X_f = 0 = 50m + 7\frac{m}{s} + \frac{1}{2}(-10\frac{m}{s^2})t^2$
solve!
$t \approx 4s$ (and $\sim -2.5s$, but that's not relevant)

Then you can find $X_h = V_H t$ and
$V_V = V_{0V} + at$

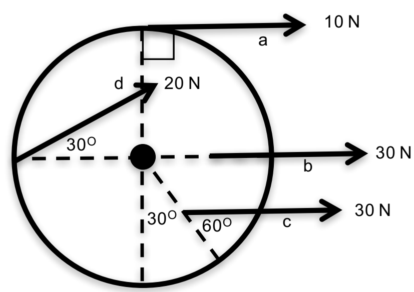
to get same answers as before!

e) $X_f \approx 75\,m$

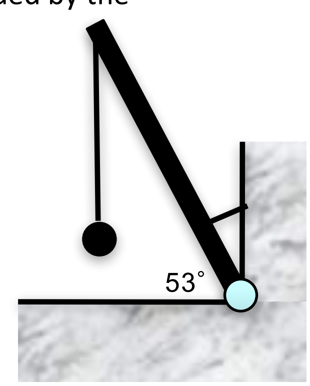
f



a. Exercise 7: Using some geometry, please show that I have correctly labled the algne to be 60º, that it is the complement of the 30º central angle. We know $\vec{\tau} = F_{\perp}r = Fr_{\perp} = Fr(\sin\theta_{included})$. THUS!
$a)\ \vec{\tau} = 15\ Nm\otimes$
$b)\ \vec{\tau} = 0$
c) $\vec{\tau} = 0.75\ m * 30\ N(\sin 60^o) = 19.5\ Nm\ \odot$
d) $\vec{\tau} = 1.5\ m * 20\ N(\sin 30^o) = 15\ Nm\ \otimes$



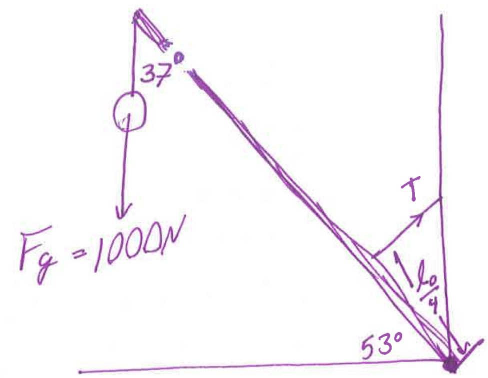
5. In the diagram at right, a post of some length supports a 100 kg ball. The length of the tilted rod is 10 m and the cable is attached 2.5 m from the pivot. From the drawing at right (make your own better drawing), estimate the tension on the cable and the force provided by the foundation at the pivot.



I assign directions

$\longrightarrow +x \qquad \uparrow +y$

$\otimes$ + rotation



There are unknown forces at the pivot so I use the pivot as the center of rotation, leaving the only unknown torque that of the Tension:

length of rod

$$\Sigma \tau_{pivot} = {}^{+}T \cdot \frac{l_0}{4} \sin 90^\circ + {}^{-}F_g \cdot l_0 \sin 37^\circ = 0$$

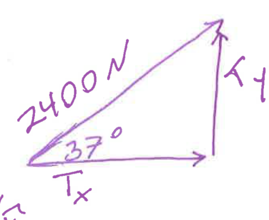
l_0 cancels and $\sin 37^\circ \sim \frac{3}{5} = 0.6$

$$T \sim 4 \cdot F_g \cdot \sin 37^\circ \sim 4 \cdot 1000N \cdot 0.6 = \underline{\underline{2400N}}$$

In order to find the reaction force provided by the pivot, we set $\Sigma F = ma = 0$. I choose to decompose the Tension into T_x and T_y

$T_y = 2400N \sin 37^\circ \sim 1440N$

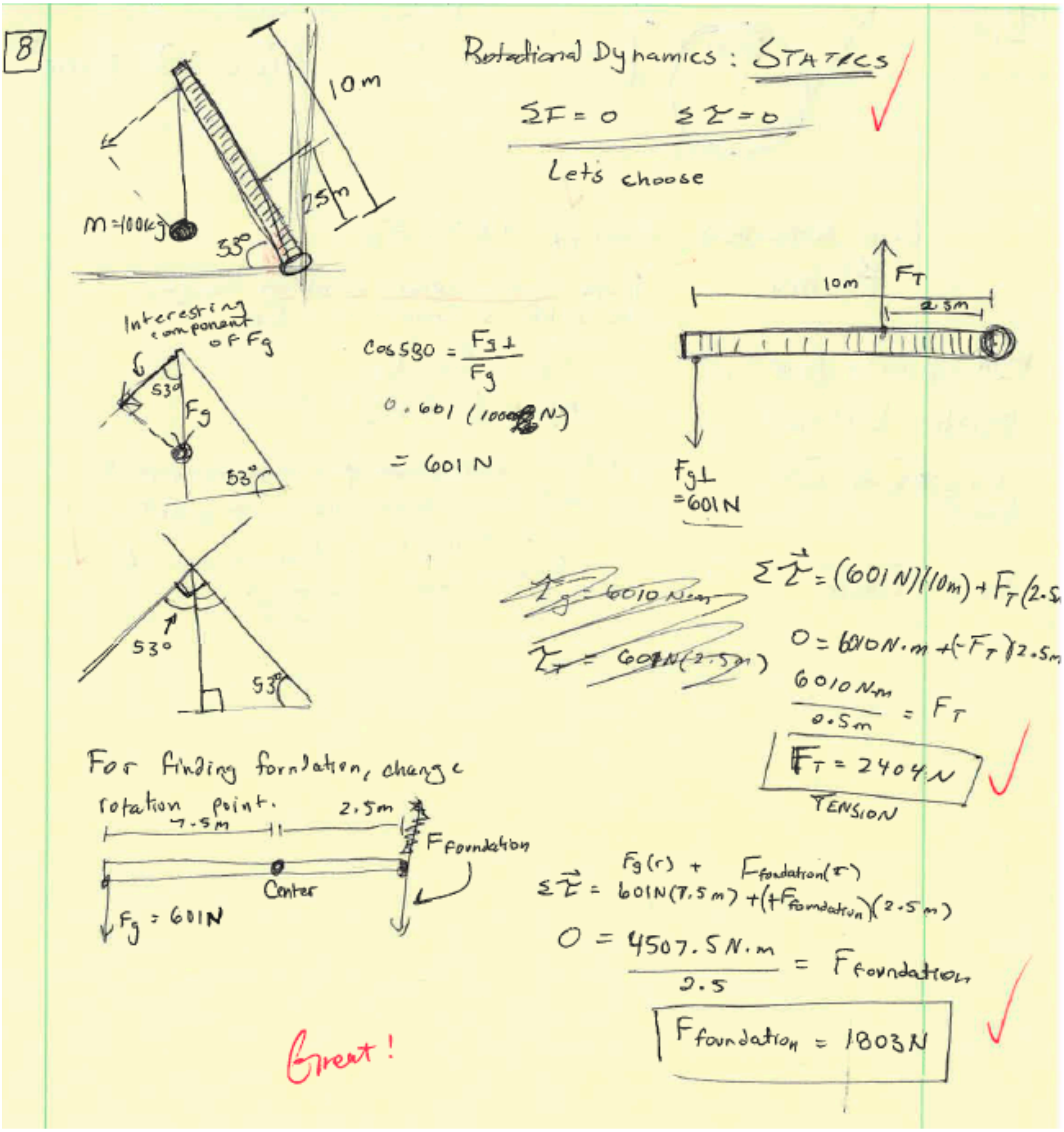
$T_x = 2400N \cos 37^\circ \sim 1920N$



$\Sigma F_x = F_{px} + T_x = 0; \; F_{px} = -1920N\hat{x}$

$\Sigma F_y = F_{py} + T_y + F_g = 0; \; F_{py} = -440N\hat{y}$

We see that the reaction force that the pivot provides is downward and to the left.
Please show yourself that if the cable was connected in the *middle* of the supporting rod, the tension on the cable would be only 1200 N, resulting in a pivot force that would be 280 N upward and 960 N in the negative x direction.



8

Rotational Dynamics : STATICS

$\Sigma F = 0 \quad \Sigma \tau = 0$

Lets choose

10 m

2.5 m

$m = 100\,kg$

53°

Interesting component of F_g

F_g

53°

$\cos 53^\circ = \frac{F_{g\perp}}{F_g}$

$0.601\,(1000\,N)$

$= 601\,N$

10 m

F_T

2.5 m

$F_{g\perp} = 601\,N$

~~$\tau_g = 6010\,N\cdot m$~~

~~$\tau_T = 601\,N(2.5\,m)$~~

$\Sigma \vec{\tau} = (601\,N)(10\,m) + F_T(2.5$

$0 = 6010\,N\cdot m + (-F_T)2.5\,m$

$\frac{6010\,N\cdot m}{2.5\,m} = F_T$

$\boxed{F_T = 2404\,N}$

TENSION

For finding foundation, change rotation point.

7.5 m

2.5 m

$F_{foundation}$

Center

$F_g = 601\,N$

$\Sigma \vec{\tau} = F_g(r) + F_{foundation}(r)$

$= 601\,N(7.5\,m) + (+F_{foundation})(2.5\,m)$

$0 = \frac{4507.5\,N\cdot m}{2.5} = F_{foundation}$

$\boxed{F_{foundation} = 1803\,N}$

Great!

6. A bicycle is a beautiful thing to me! Imagine that I can put a constant force (perpendicular to the radius of rotation) of 200 N onto the pedal that is 20 cm long, and am able to maintain that force for some time as I pedal along. Let's say that I am rotating the pedals at 60 rotations per minute. Imagine that I am riding up at constant speed against wind friction.

A) Find the torque my legs put on the pedals and the omega of the pedals.
B) Find the power I'm putting out.

C) I’m in my highest gear, the diameter of the pedal gear is 20 cm, and the diameter of the gear driving the rear wheel is 4 cm. Please find the tension in the chain, and the torque the chain produces on the rear wheel.
D) Given the speed of the chain and the tension in the chain, what is the power I deliver to the chain?
E) What is omega of the rear wheel? What is the power the torque of the chain delivers to the rear wheel?
F) If the diameter of the rear wheel is 700 mm, what is the force that the torque on the rear wheel delivers to the road (assume that there is no slipping).
G) What must be the speed of the surface of the rear tire surface (which is equal to the speed of the bike)? And what is the power that this surface delivers to the bicycle?
H) At some time, I change gears, putting the chain on a rear gear cluster on a gear that is 8 cm in diameter (doubling the diameter of the rear gear), and I am able to continue putting the *same amount of force* on the pedals. What change to I experience? What do I notice in my pedaling? what would be the new:
   i) The torque on the rear wheel?
   ii) The power to the rear wheel?
   iii) The speed of the chain?
   iv) Omega of my legs?
   v) What will happen to the motion of my bike?
   vi) What will happen to the feeling in my body? (will I relax or do I have to work harder?)

This question is largely addressed through the “bicycle transmission” in Week 10. Please see the video again if you are so inclined. However, I will address right here question H) What happens when you change the rear gear to twice the original radius? This is downshifting by a factor of two. At that moment, there is no immediate change of speed, so the rear wheel has the same rotational velocity. With twice the radius on the rear cog (gear), the chain must move twice the speed to keep up with the rotating wheel. Because the chain has the same tension on it (you are pushing with the same force on your feet), the power supplied by the chain (*P = F*v*) is doubled. Because the chain tension acts on the rear wheel at twice the radius, the chain’s torque on the rear wheel doubles, doubling the force to the earth’s surface (and the force of the earth’s surface on the bike doubles). By doubling the force to the rear wheel, the bike will accelerate, and you’ve doubled the power delivered to the rear wheel. In order to move the chain twice as fast, you will need to spin your legs twice as fast, doubling your power output if you are able to continue pushing with the same force on your legs: $P = \tau\omega = Fv$. This is what we experience every day… if you are cruising at constant velocity on the freeway, your engine is not putting out very much power. But then you want to pass someone. You downshift to a lower gear (or your automatic transmission does that). The engine spins much faster, which you can hear. The power to the wheels increases greatly and you accelerate increasing the kinetic energy of your car. Same thing on a bicycle. Now can you answer the questions:

7. The classic “notorious ladder problem”: why does a ladder not slip when you stand on it at the bottom, but then it slips as you go higher? *Please don’t attempt this problem until you thoroughly understand the diving board problem from previous problem sets.* A 30 kg 5 m ladder leans up against a frictionless wall at an angle of 53° with respect to the ground. You are 50 kg, and the coefficient of

static friction with the floor is a dangerous 0.50. At first, you are standing at the base of the ladder on the bottom rung, essentially 0 meters from the bottom.

a) I hope you already drew a great diagram! … and labeled all the forces? And thought about all the torques? Do you have a lens? Can you group all the horizontal forces and make a statement about them? Can you do the same with all the vertical forces?

b) How much force can we depend on the friction to provide for us? Is this the actual force friction is providing, or don't we know? The actual amount of frictional force is going to depend on how hard we push on it. What forces are competing with this frictional force to keep the ladder in equilibrium?

c) We may notice that there's a normal force provided by the wall the ladder is leaning against? Why is this force necessary? Does this normal force supply a necessary torque to keep the ladder in equilibrium?

d) Please set up the torques. Which point to you use to be the center of rotation? Why do you choose that point? Can you use this equation to find the normal force of the wall on the ladder. How do you calculate the torque when the forces are not perpendicular to the radius?

e) Can you use this information to show that the ladder does not slip when you are on the bottom rung of the ladder? Can you show that you can "test" the ladder by bouncing up and down on it and it won't slip?

f) Now that you are confident about the security of the ladder, you start walking up the ladder. Which of the equations does this change? How does it change them? How does the situation become more dangerous?

g) Will I make it to the top of the ladder? Find out by doing an analysis with me at the top of the ladder, and see if the ladder will slide.

h) Please find my location when the ladder slides. Is it bad for me?

i) If we were to do this problem again, and we changed the inclination angle of the ladder to 60°, would this make the situation safer, or more dangerous? How do you know?

Please see the dedicated video.