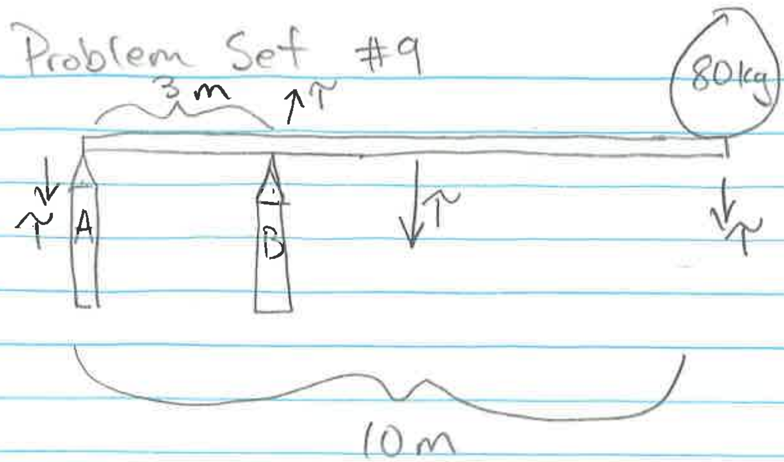
Problem Set #9

1)



a) "A is pivot point"

$\Sigma \tau = 0$

$(80\ kg)(10\ \frac{m}{s^2})(10\ m) = (F_B)(3\ m)$

$\boxed{F_B = 2667\ N \uparrow \text{ upwards}}$

"B" is pivot point

$\Sigma \tau = 0$

$-(80\ kg)(10\ \frac{m}{s^2})(7\ m) = (F_A)(3\ m)$

$\boxed{F_A = 1867\ N \downarrow \text{ downwards}}$

b) $m_{board} = 100\ kg$

"A is pivot", $\Sigma \tau = 0$

$(80\ kg)(10\ \frac{m}{s^2})(10\ m) + (100\ kg)(10\ \frac{m}{s^2})(5\ m) = (F_B)(3\ m)$

$\boxed{F_B = 4333.3\ N \uparrow \text{ up}}$

"B is pivot", $\Sigma \tau = 0$

$(F_A)(3\ m) = (100\ kg)(10\ \frac{m}{s^2})(2\ m) + (80\ kg)(10\ \frac{m}{s^2})(7\ m)$

$\boxed{F_A = 2533.3\ N \downarrow \text{ down}}$

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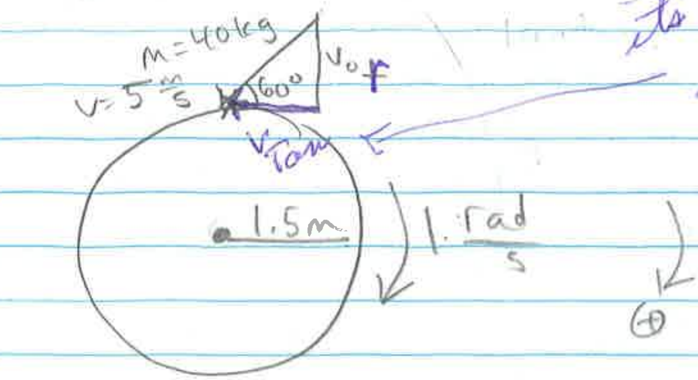
2)

$M = 100$ kg

its tangential velocity we need



a) conservation of angular momentum

$l_{oc} = \frac{1}{2}(100\,kg)(1.5^2\,m)(1\,\frac{rad}{s})$

$l_{o_{carousel}} = 112.5\ \frac{kg \cdot m^2}{s}$

$l_{o_{kid}} = (40\,kg)(1.5\,\cancel{m^2})\left(\frac{(\cos 60)(-5\,\frac{m}{s})}{\cancel{1.5\,m}}\right)$

$l_{o_{kid}} = -150\ \frac{kg \cdot m^2}{s}$

$l_{oc} + l_{ok} = l_{final}$

$112.5\ \frac{kg \cdot m^2}{s} - 150\ \frac{kg \cdot m^2}{s} = -37.5\ \frac{kg \cdot m^2}{s} = l_{final}$

(counterclockwise)

$-37.5\ \frac{kg \cdot m^2}{s} = (I_k + I_c)(\omega_f)$

$-37.5\ \frac{kg \cdot m^2}{s} = \left(\frac{1}{2}(100\,kg)(1.5^2) + (40\,kg)(1.5^2)\right)(\omega_f)$

$\omega_f = -.19\ \frac{\text{radians}}{\text{sec}}$ ⊙

b) $l_{oc} = -112.5\ \frac{kg \cdot m^2}{s}$ (counterclockwise), $l_{ok} = -150\ \frac{kg \cdot m^2}{s}$ (c-clockwise)

$l_{oc} + l_{ok} = l_f$

$-112.5\ \frac{kg \cdot m^2}{s} + -150\ \frac{kg \cdot m^2}{s} = -262.5\ \frac{kg \cdot m^2}{s} = l_f$

$-262.5\ \frac{kg \cdot m^2}{s} = \left(\frac{1}{2}(100\,kg)(1.5^2) + (40\,kg)(1.5^2)\right)(\omega_f)$

$\omega_f = -1.3\ \frac{\text{radians}}{s}$ ⊙

☆This is for part a

c) $KE = \frac{1}{2} I \omega^2$

$KE_k = \frac{1}{2}\left((40\,kg)(1.5m^2)\right)\left(\frac{(\cos 60)(5\,\frac{m}{s})}{1.5\,m}\right)^2$

☆ $KE_k = 125\ J$

$KE_C = \frac{1}{2}\left(\frac{1}{2}(100\,kg)(1.5^2)\right)\left(1\ \frac{rad}{s}\right)^2$

☆ $KE_C = 56.25\ J$, $KE\ total = 181.25\ J$

$KE_{(k+c)} = \frac{1}{2}\left((40\,kg)(1.5\,m^2) + \frac{1}{2}(100\,kg)(1.5\,m^2)\right)\left(.19\ \frac{rad}{s}\right)^2$

$\boxed{KE_{(k+c_f)} = 19.24\ J}$, KE is not conserved

☆KE is not conserved b/c energy is lost through heat

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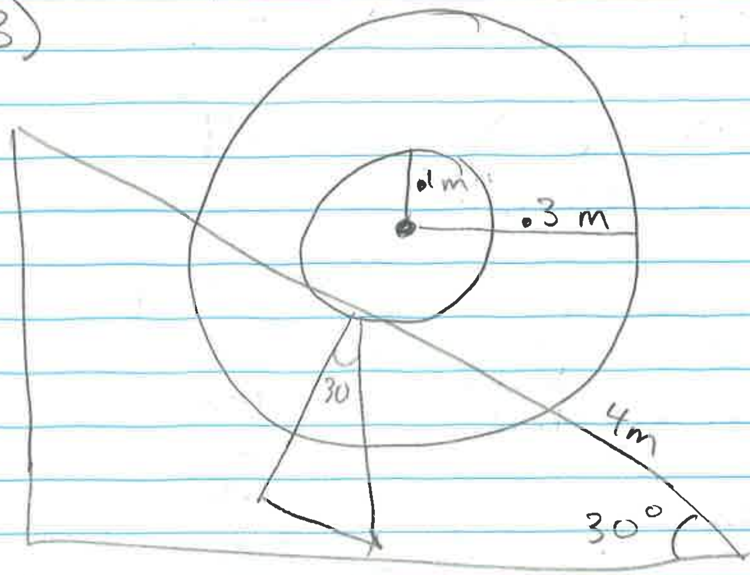
3) $M = 3\,kg$



a) $PE_{lost} = KE_{gained}$

$KE = I\omega^2$, $I = \frac{1}{2}mr^2$

$PE_{lost} = PE_0 - PE_f$

$PE_{lost} = (3kg)(10\,\frac{m}{s})((\sin 30)(4m)) - 0$

$\boxed{PE_{lost} = 60\ J}$

b) $\Delta PE = \Delta KE_{rot} + \Delta KE_{trans}$ $\quad \vec{\omega} = \frac{v}{r}$

$60J = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$

$60J = \frac{1}{2}\left(\frac{1}{2}(3kg)(.3m)^2\right)\left(\frac{v}{.1m}\right)^2 + \frac{1}{2}(3kg)v^2$

$60J = (6.75)(v^2) + (1.5)(v^2)$

$(7.27\,\frac{m^2}{s^2}) = v^2$

$\boxed{v_f = 2.7\,\frac{m}{s}}$

$v = (\omega)(r)$

$\left(\frac{2.7\,\frac{m}{s}}{.1\,m}\right) = \omega$ $\quad \boxed{\omega_f = 27\,\frac{radians}{s}}$

c) avg vel $= \frac{(2.7\,\frac{m}{s}) + (0\,\frac{m}{s})}{2} = \boxed{\text{avg vel} = 1.35\,\frac{m}{s}}$

$v = \frac{m}{t}$, $1.35\,\frac{m}{s} = \frac{(4m)}{(t)}$ $\quad \boxed{t = 2.96\ s}$

lin. accel $= \frac{\Delta v}{\Delta t} = \frac{2.7\,\frac{m}{s}}{2.96} = \boxed{\text{lin accel} = .91\,\frac{m}{s^2}}$ $\quad \alpha = \frac{\Delta\omega}{\Delta t} = \left(\frac{27\,\frac{rad}{s}}{2.96\ s}\right)$ $\boxed{\alpha = 9.12\,\frac{rad}{s^2}}$

d) $\alpha = 9.12 \ \frac{rad}{s^2}$, $\tau = I\alpha$

$\tau = \left(\frac{1}{2}(3\ kg)(.3\ m)^2\right)\left(9.12 \ \frac{rad}{s}\right)$

$\boxed{\tau = 1.23\ N\cdot m}$

$\tau_{Fr} = (F_{Fr})(r)$ ← r of hub

$(1.23\ N\cdot m) = (F_{Fr})(.1\ m)$

$\boxed{F_{Fr} = 12.3\ N}$

e) $\sum F_x = ma = F_{gx} - F_{Fr}$

$(\sin 30)(3\ kg)(10\ \frac{m}{s^2}) - (12.3\ N) = (3\ kg)(a)$

$\boxed{a = .9\ \frac{m}{s^2}}$

✓ yes they are the same!

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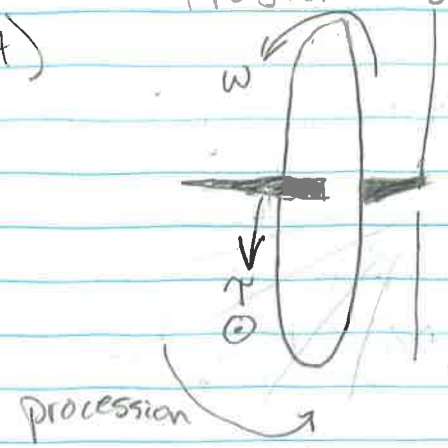
4)



a) $\leftarrow \omega$

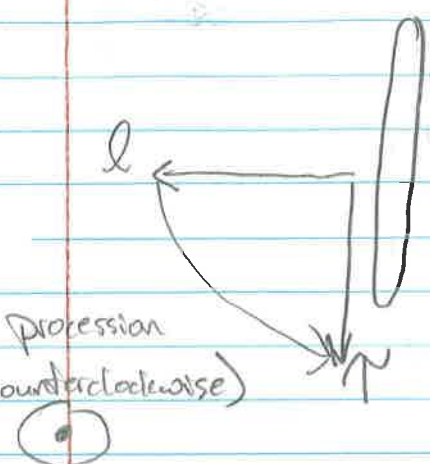
$l = (I)(\omega)$

$\leftarrow l$

τ ⊙

direction of procession ↑
counterclockwise
as viewed from above

Top View:



b) If you initially spin the wheel the other direction, it will process the other way. Procession is the resultant vector of l and τ, and if the direction of l changes, then the resultant procession vector will go in the other direction.

c) It will process ~~less~~ slower with a larger "ω" because if "ω" is larger, then "l" is larger, and the τ vector will affect the larger "l" vector less, causing it to process less. slower

d) $\tau = (F_g)(\Delta x)$. If "Δx" or the distance between the finger and the wheel decreases, then τ decreases, causing the wheel to process less. slower

e) $\ell = I\omega$

$I_0 = mr^2$, $I_f = \frac{1}{2}mr^2$

$I_f < I_0$

If the moment of inertia decreases as shown above, the "ℓ" or angular momentum will decrease. If the "ℓ" vector decreases, then the torque vector will have a greater effect on it, causing the wheel to process ~~more~~ faster

(5) 

a) $PE_I = (70\text{ kg})(10\text{ m/s}^2)(40\text{ m}) = 28000\text{ J}$

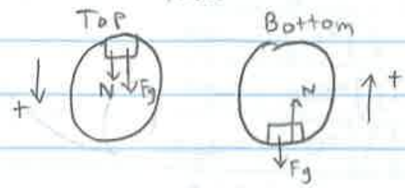
$PE_{\text{Top of Loop}} = (70\text{ kg})(10\text{ m/s})(20\text{ m}) = 14000\text{ J}$

$28000\text{ J} = \frac{1}{2}mv^2, \quad V_{\text{bottom}} = 28.28\text{ m/s}$

$14000\text{ J} = \frac{1}{2}mv^2, \quad V_{\text{top}} = 20\text{ m/s}$

$a_{c_{\text{bottom}}} = \frac{(28.28\text{ m/s})^2}{10\text{ m}} = 80\text{ m/s}^2$

$a_{c\text{ top}} = \frac{(20\text{ m/s})^2}{10\text{ m}} = 40\text{ m/s}^2$



$\Sigma F_c = ma_c$

$N + F_g = ma_c$

$N = ma_c - F_g$

$N = (70\text{ kg})(40\text{ m/s}^2) - (70\text{ kg})(10\text{ m/s}^2) = \boxed{2100\text{ N}}$

$\Sigma F_c = ma_c$

$N - F_g = ma_c$

$N = ma_c + F_g$

$N = (70\text{ kg})(80\text{ m/s}^2) + (70\text{ kg})(10\text{ m/s}^2) = \boxed{6300\text{ N}}$

→ As you round the bottom of the loop, you feel more force pushing down on you. This is not a good ride for a pregnent woman!

b) If you start at the height as the top of the loop, you won't have enough velocity to make it around the top of the loop, because if energy is conserved, then KE will be 0 as you approach the top of the loop.

c) $PE_I = PE_T + KE_T$

$mgh = mg(20m) + \frac{1}{2}mv^2$

$gh = g(20m) + \frac{1}{2}v^2$

$\Downarrow$

$(10\,m/s^2)h = (10\,m/s^2)(20m) + \frac{1}{2}(10.01\,m/s)^2$

$\boxed{h > 25\text{ m}}$

$\Sigma F = ma_c$

$F_g < ma_c$

$mg < ma_c$

$a_c > g$

$\frac{v^2}{r} > g$

$v > \sqrt{gr}$

$v > 10\,m/s$

d) $KE = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$

$= \frac{1}{2}I\frac{v^2}{r^2} + \frac{1}{2}mv^2$

$= \frac{1}{2}\cdot\frac{2}{3}m\cancel{r^2}\frac{v^2}{\cancel{r^2}} + \frac{1}{2}mv^2$

$= \frac{2}{6}mv^2 + \frac{1}{2}mv^2$

$= \frac{5}{6}mv^2$

PE = 28000 J at top

So, $KE_{bottom} = 28000\,J = \frac{5}{6}(70kg)v^2$, $v = 21.9\,m/s$, $a_c = 48\,m/s^2$

$KE_{top} = 14000\,J = \frac{5}{6}(70kg)v^2$, $v = 15.49\,m/s$, $a_c = 24\,m/s^2$

Bottom

$\Sigma F = ma_c$

$-F_g + F_N = (70kg)(48\,m/s^2)$

$F_N = 3360\,N + 700\,N = \boxed{4060\,N}$

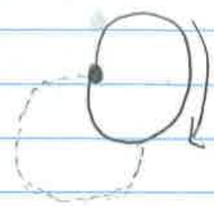
Top

$\Sigma F = ma_c$

$F_g + F_N = ma_c = (70kg)(24\,m/s^2)$

$F_N = 1680\,N - 700\,N = \boxed{980\,N}$

6



a. We can use energy to solve this, knowing energy is conserved.
PE is converted all into KE. Let $h = r$, so:

$$mgr = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$$

$$mgr = \frac{1}{2}I\omega^2 + \frac{1}{2}mr^2\omega^2$$

$$mgr = \frac{1}{2}\omega^2[I + mr^2]$$

$$mgr = \frac{1}{2}\omega^2\left[\frac{1}{2}mr^2 + mr^2\right]$$

$$\cancel{m}gr = \left(\frac{1}{2}\omega^2\right)\left(\frac{3}{2}\cancel{m}r^2\right)$$

$$g = \frac{3}{2}\cdot\frac{1}{2}\omega^2 r$$

$$\omega = \sqrt{\frac{4g}{3r}} = \boxed{\frac{2\sqrt{g}}{\sqrt{3}\sqrt{r}}\ \frac{rad}{sec.}}$$

b. $I = \frac{1}{2}mr^2$, $L = I\omega$, so $L = \frac{1}{2}mr^2\left(\frac{2\sqrt{g}}{\sqrt{3r}}\right) = \frac{1}{\sqrt{3}}\frac{mr^2\sqrt{g}}{\sqrt{r}}$

$$\boxed{L = \frac{m\sqrt{r^3 g}}{\sqrt{3}}}$$

c.

→ dynamics, because we know there is a centripetal acceleration from the nail, as well as a force of gravity and a force from the nail.

$\Sigma F_c = ma_c$ $\quad F_{nail} = ma_c + F_g$

$= m\left(\frac{4}{3}\frac{g}{\cancel{r}}\right)\cancel{r}$ $\quad = m\omega^2 r + F_g$

$= m\frac{4}{3}g + mg$

$= mg\left(\frac{4}{3}\right)$ $\quad \boxed{F_{nail} = \cancel{\frac{1}{3}mg}} \rightarrow 2\frac{1}{3}(mg)$