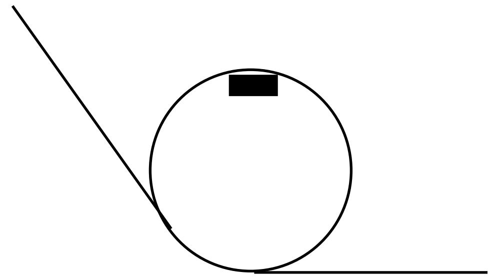
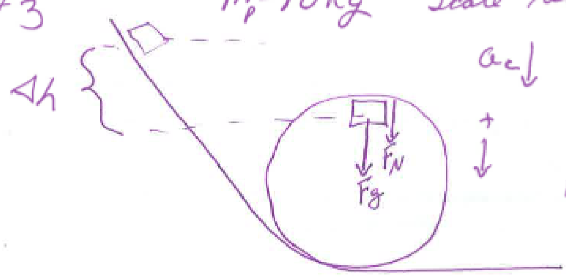
PS#9 Solutions and direction

1) There is a video about this firehose problem for week 11

2) *If you think you've already done this problem before, just skip down to part d*. I summarize a)-c) for the solutions below and focus on d)
   d) Let's say that the object is instead a hollow sphere that is rolling without slipping. How would this change the problem? Can you do part a) and c) above for this scenario? Essentially, this is what we're asking: for a) would the rolling sphere be going faster than the frictionless cart or slower – how much faster or slower – is the acceleration greater or less?; and for b) in order to make it around the loop without falling, would the hollow sphere need to start from higher or lower than the frictionless cart? How much higher or lower?



#3 $m_p = 70kg$ Scale reads F_N



Dynamics Σ

$\Sigma F_y = ma_c$

$F_g + F_N = ma_c$

or $a_c \geq g$

The way we solved this was

$\Delta PE \Rightarrow KE$ (energy)

$mg\Delta h = \frac{1}{2}mv^2$ and $a_c = \frac{v^2}{R} \geq g$

$2g\Delta h = v^2$

so $v^2 \geq Rg$

But Now we have to consider $KE_{rot} = \frac{1}{2}I\omega^2$ is needed to have the sphere roll.

$I_{\text{hollow sphere}} = \frac{2}{3}mR^2$, so now

$mg\Delta h = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{3}m\underbrace{R^2)\omega^2}_{v^2}$

$mg\Delta h = \frac{1}{2}mv^2 + \frac{1}{3}mv^2 = \frac{5}{6}mv^2$

$\cancel{m}\cancel{g}\Delta h = \frac{5}{6}\cancel{m}R\cancel{g}$

$\Delta h = \frac{5}{6}R$ which $> \frac{1}{2}R$ we got for a frictionless cart.

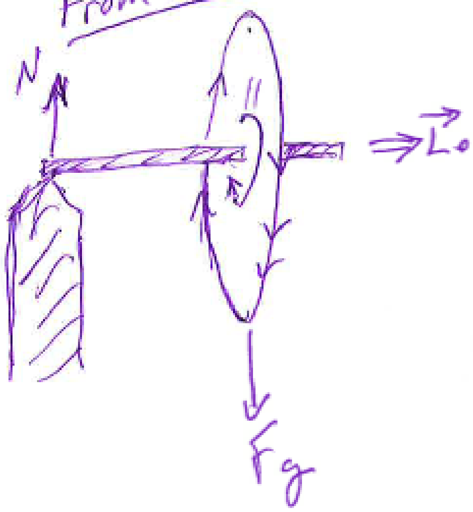
3) Take a bicycle wheel and spin it very fast, then support the horizontal axel only at one end (some distance "x" from the center of the wheel's hub, letting the other side "fall".

a) see video (Big Picture Rotation video at 8:59 to 9:25)

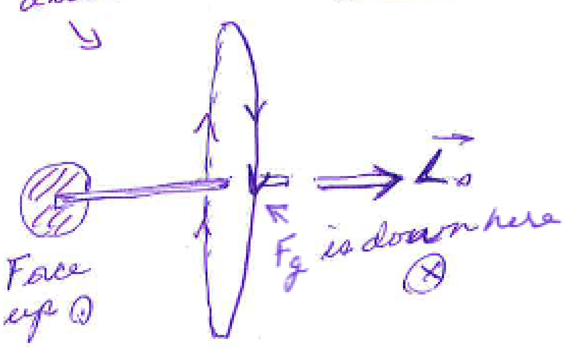
Look at it from above:

$I\vec{\omega} = \vec{L}$ in this direction →

From The side



From above



τ is ⊗ into page

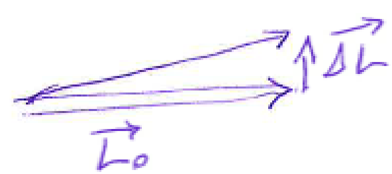
$\vec{\Delta L} = \vec{\tau} \cdot dt$ is ⊗

So wheel precesses into page or counter-clockwise from above.

τ is ↑

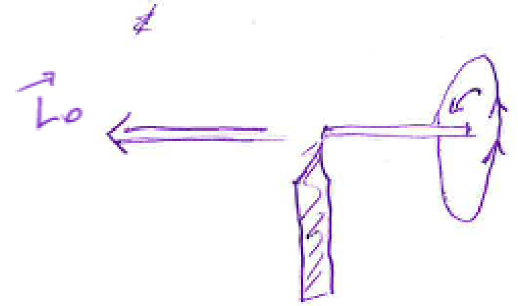
$\vec{\Delta L} = \vec{\tau} \cdot dt$ is in ⇑ direction

so $\vec{L} = \vec{L}_o + \vec{\Delta L}$



Precession is counter-clockwise when viewed from above

b) if we change the direction of $\vec{\omega}$, then $\vec{L}$ is in the other direction, but the torque from gravity is in same direction, so $\vec{\Delta L}$ is same direction



$\vec{\Delta L}$ is still $\otimes$, so the $\vec{L}$ would get pushed into the paper, and the wheel would come out, precessing in opposite direction.

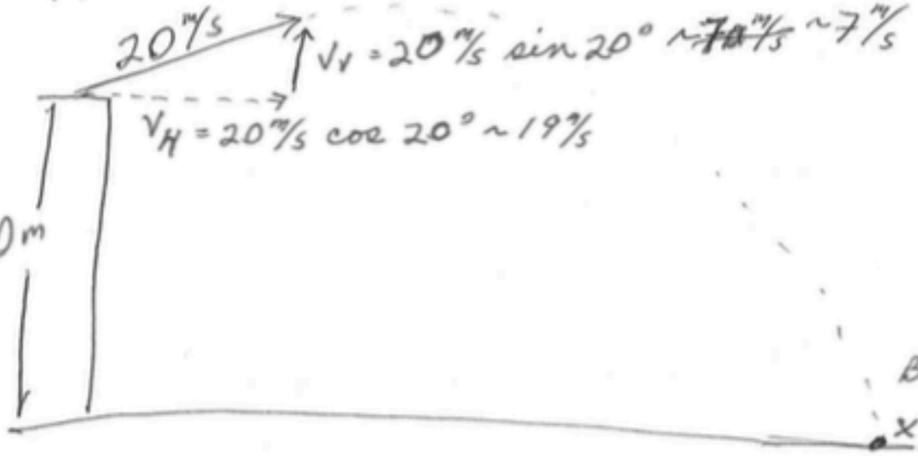
c) if $\vec{L}_0$ is bigger, then the same $\vec{\Delta L} = \vec{\tau}_g \Delta t$ would change the angle of $\vec{L}$ by less, so precession would decrease

d) if r_+ is less, $\tau_g \Downarrow$ so $\vec{\Delta L} = \vec{\tau} dt$ is less, so it would precess slower

e) $I \Downarrow$, so $\vec{L}_0$ decreases, but mass is the same, so τ_g is the same. So the same $\vec{\Delta L} = \vec{\tau} dt$ will rotate the ~~wett~~ wheel's axis more, so precession would increase in rate

4) If you hit a baseball at a 20 degree angle above the horizon, at an initial velocity of 20 m/s off the edge of a cliff 50 m high,

#4

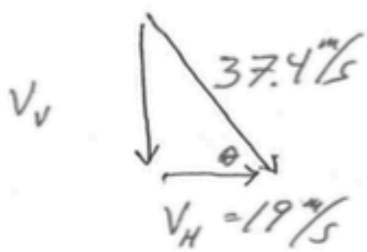


a) $E_0 = E_f$

$KE_0 + PE_0 = KE_f + \cancel{PE_f}^{\,0}$ solve ~~for~~ math and find $V_f \approx 37.4\,m/s$

$\frac{1}{2}\cancel{m}(20\,m/s)^2 + \cancel{m}\,10\,m/s^2 \cdot 50\,m = \frac{1}{2}\cancel{m}V_f^2$

b) $a_H = 0$, so $V_{H_{final}} = V_{H_{initial}}$ = const $\approx 19\,m/s$



$\cos\theta \approx \frac{19\,m/s}{37.4\,m/s} \approx 60°$

$V_V^2 + V_H^2 \approx V^2$

$V_V^2 + (19\,m/s)^2 = (37.4\,m/s)^2$

$V_V \approx 32\,m/s \Downarrow$

c) $\Delta V_V = -32\,m/s - 7\,m/s$

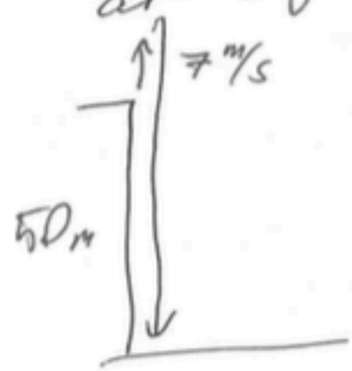
$\approx -40\,m/s = at$

$a \approx -10\,m/s^2$, so $\Delta t \sim 4s$

$\Delta x = V_H t + \cancel{\frac{1}{2}a_H t^2}^{\,0} \approx 19\,m/s \cdot 4\,s \approx \underline{75\,m}$

horizontal $\vec{a} = 0$

d) now I use straight kinematics.
Horizontally, it's just moving along at $V_H \approx 19\,m/s$ in the $\hat{x}$ direction;
vertically it's moving upward at $V_{V_0} \sim 7\,m/s$ and accelerates downward at $a_V = -10\,m/s^2$



find t w/

$$X_f = 0 = 50m + 7\,m/s + \tfrac{1}{2}(-10\,m/s^2)\,t^2$$

solve!

$t \approx 4s$ (and $\sim -2.5s$, but that's not relevant)

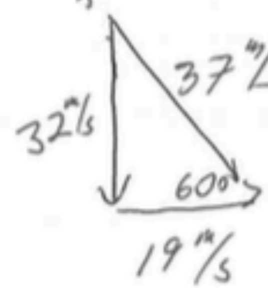
Then you can find $X_h = V_H t$ and

$$V_V = V_{0V} + at$$

to get same answers as before.

e) $X_f \approx 75m$

f



5) Solve the infamous "catching the bus" problem. The bus is at your stop, and you're running at a constant speed of 7 m/s from behind in order to catch it. However, just when you're 20 m behind it (or behind the bus driver to be exact), the bus begins accelerating away from you at 1 m/s^2, and will continue accelerating at 1 m/s^2 unless you can meet eyes with the driver.

PS #6

#1 - Catching Bus

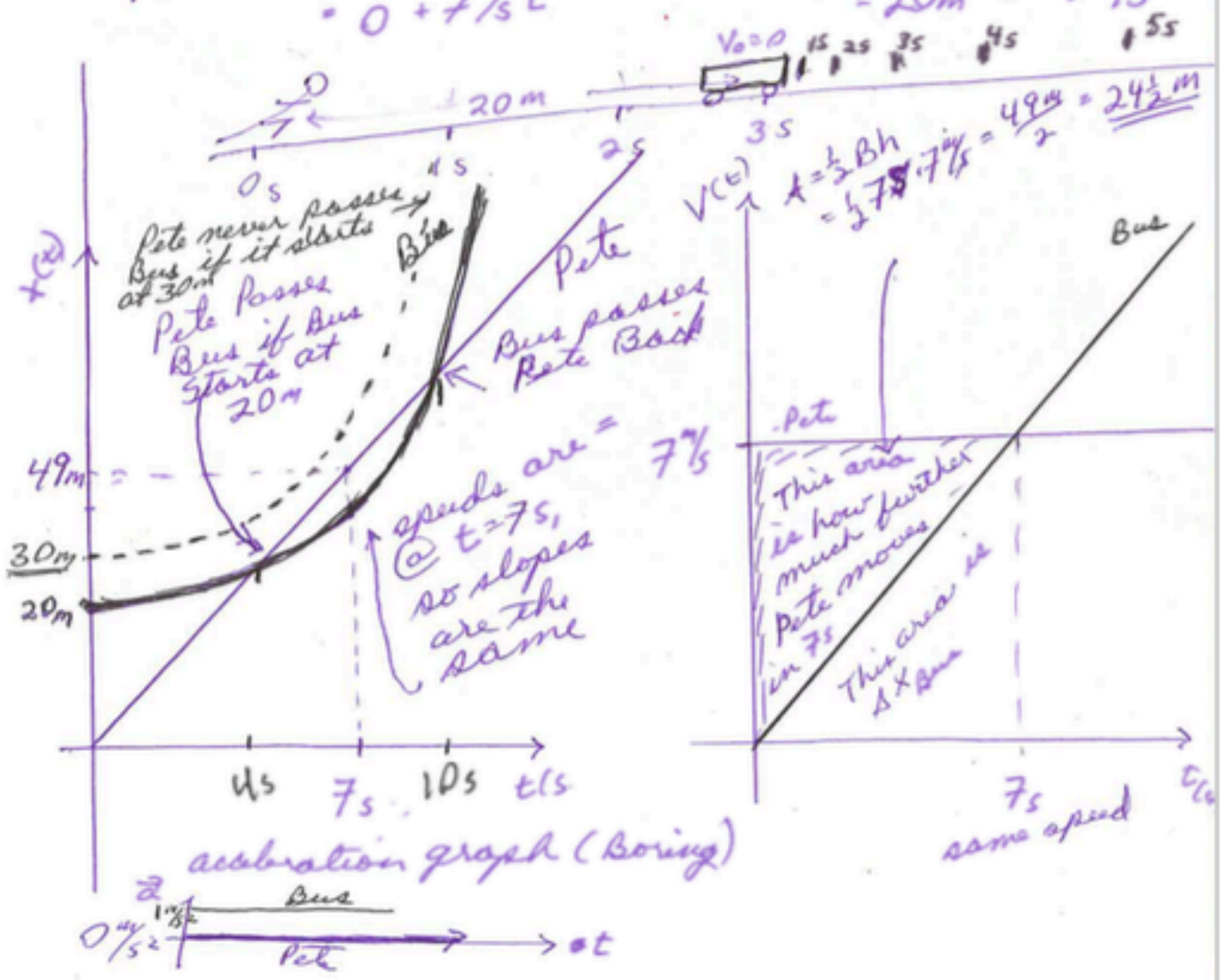
Pete: $V_p = 7\,m/s$ = const, $x_0 = 0$

Bus: $V_{0B} = 0$, $a_B = 1\,m/s^2$, $x_0 = 20m$, $V = a_B t$

Kinematics - because we are dealing with exclusive use of position and its time derivatives as an explicit function of time. In particular: $x_p(t) \stackrel{?}{=} x_B(t)$ when and if are our displacements the same

Pete: $x = x_0 + Vt = 0 + 7\,m/s\; t$

Bus: $x = x_0 + V_0 t + \frac{1}{2} a t^2 = 20m + 0 + \frac{1}{2}\,m/s^2\; t^2$

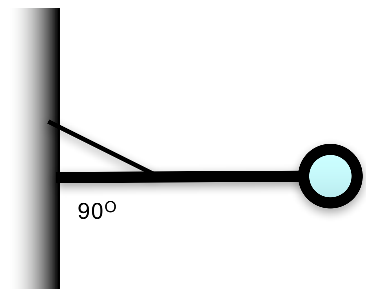


6) is a combination of kinematics (projectile motion) and (circular) dynamics. It requires two drawings, which I expect of you, but I am not supplying right now:

Using dynamics, make a good FBD and follow the protocol and you can see that the force of friction is the only radial force, so you set it equal to mass*centripetal acceleration. This would provide us with the coefficient of friction, except that we don't know the speed… but we know how far it moves horizontally when it falls, so we go to the kinematics lens.

Please show the parabolic trajectory as the coin falls from the edge of the spinning turntable. We want the initial horizontal velocity = dx/dt. dx = 0.5 m, but how about dt? This is revealed in the vertical component because the coin is falling from rest (vertically speaking) and hits the ground. I could use energy to find v_{final} and then use $v_{average} = ½v_{final}$ = dx/dt. Or I could just use kinematics that dx = $½at^2$. You should find that it takes about 0.45 s to fall 1 m from rest, so the initial horizontal speed is about 1.1 m/s.
Now we can go back to our circular dynamics lens and see that the centripetal acceleration at this speed at a radius of 20 cm is about 6 m/s^2, requiring a frictional force of mass*6 m/s^2, or a coefficient of friction of 0.6.

Could we test this? Sure! Please show that if we slowly tipped the surface, an object with a coefficient of friction of 0.6 will slide when the surface is inclined at about 30 degrees.



7) Remember the problem #1 from PS#8? What happens to the tension as we bring the string closer to the pivot point? Please estimate the tension in the string at right for a mast supporting a 100 kg sphere and the reactive force at the pivot. There is no motion. This requires a dynamics lens, but in particular a statics lens. There is no acceleration (so the vector sum of the forces = 0) and there is no angular acceleration (so the vector sum of the torques = 0). Then I label my forces with a good FBD and begin to solve these two equations! Because there's two unkown forces at the pivot, I will make this point the center of rotation and start with the torque equation. I also know there's only two other torques – a known torque from the force of gravity on the ball, and an unknown torque from the *perpendicular* component of tension. These two torques must balance each other.

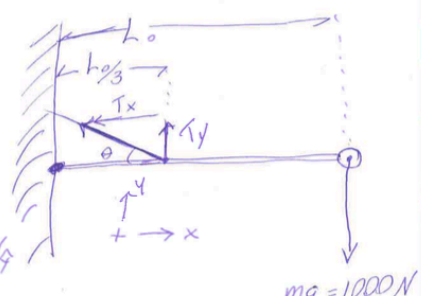
(+ ⊗

$\Sigma \vec{\tau} = 0$

$\tau_{gravity} + \tau_{tension} = 0$

$mgL + -T_y \frac{L}{3} = 0$

$T_y = 3mg = 3000N \hat{y}$



$\tan\theta = \frac{T_y}{T_x} \approx \frac{1}{2}$, so $T_x \approx 2T_y \approx -6000 N \hat{x}$

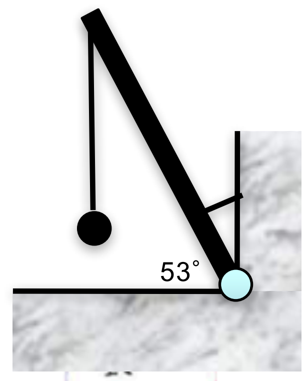
$\Sigma \vec{F} = 0$, so $\Sigma F_x = 0$ ∴ There is a reactive force from the wall on the pivot of $+6000 N \hat{x}$

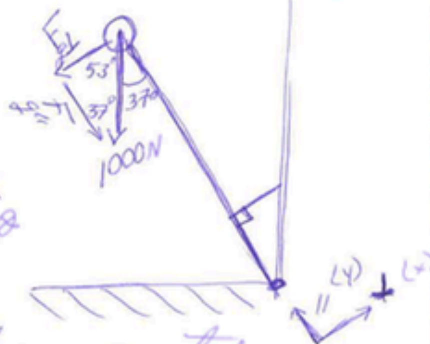
$\Sigma F_y = 0 = F_{pivot} + T_y + F_{g Ball}$

$0 = F_{pivot} + 3000N - 1000N$

$F_{pivot} = 2000 \Downarrow$

8) In the diagram at right, a post of some length supports a 100 kg ball. From the drawing at right (make your own better drawing), estimate the tension on the string and the reactive force at the pivot. There is no motion. This requires a dynamics lens, but in particular a statics lens. There is no acceleration (so the vector sum of the forces = 0) and there is no angular acceleration (so the vector sum of the torques = 0). Then I label my forces with a good FBD and begin to solve these two equations!





we estimate that the string is fastened at a point on the post at a point about $\frac{1}{4}$ of the posts' length from the pivot. I pick the pivot as center of rotation because then I can ignore the 2 dimensional forces acting on the pivot.

$\boxed{\text{Tension} \perp \text{ to post, so Torque} = T \cdot \frac{L}{4}}$

$$\sum \vec{\tau} = 0 = \tau_{string} + \tau_{mass}$$

$$= \frac{L}{4} \cdot T + F_{g\perp} L$$

53° is a 3-4-5 rt Δ, so $F_{g\perp} = 600\ N$

$$T_{string} = 4 F_{g\perp} = 2400N$$

now find reaction Force:

$\sum F_{\parallel} = 0$ $\quad F_{g\parallel} + F_{React\parallel} = 0$ $\quad F_{React\parallel} = 800N$

$\sum F_{\perp} = 0$ $\quad F_{g\perp} + T + F_{R\perp} = 0$ so $\quad F_{R\perp} = 1800N$

$-600N + 2400N = -F_{R\perp}$