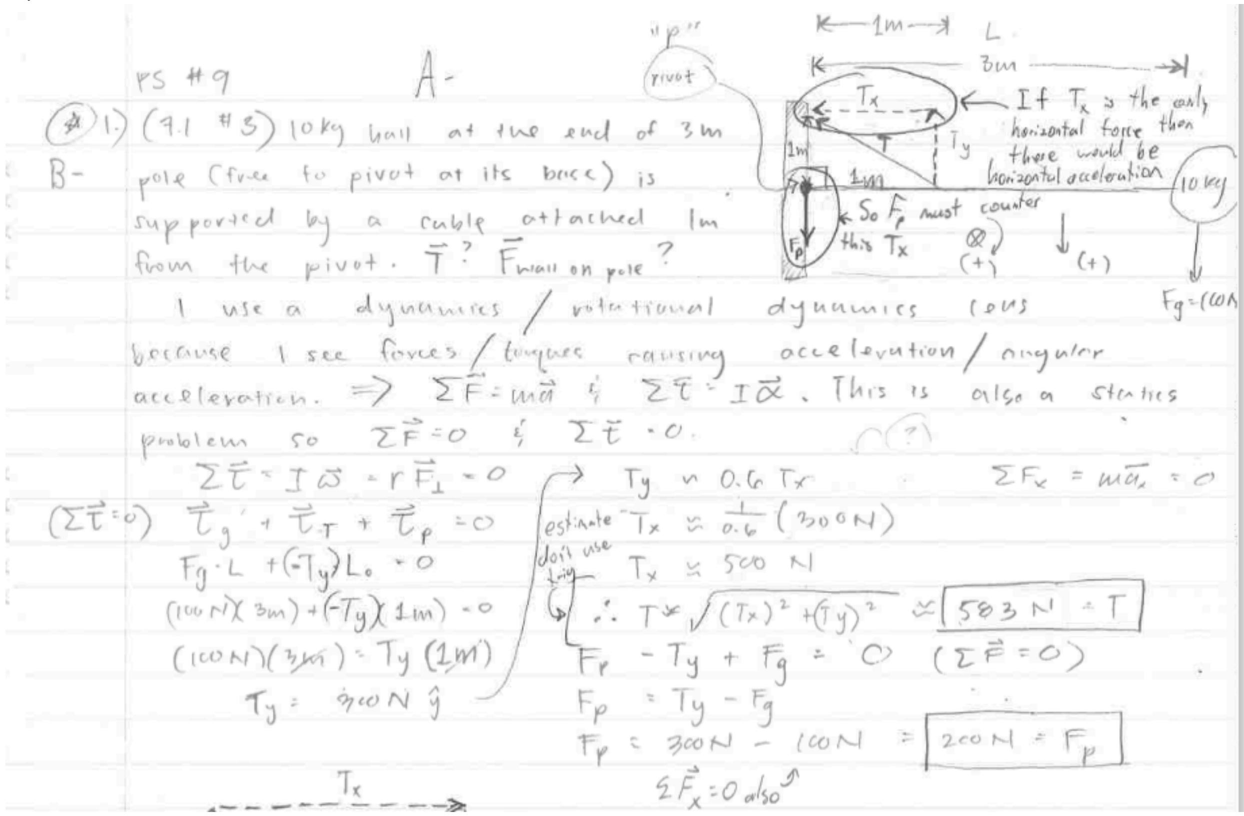
PS#9 Due in Class Monday, Nov. 13. Please pay good attention to describe the lens you are using and explain your method.

1. 7.1, Exercise 3



PS #9 A-

(#1) 1.) (7.1 #3) 10 kg ball at the end of 3 m pole (free to pivot at its base) is supported by a cable attached 1m from the pivot. $\vec{T}$? $\vec{F}_{wall\ on\ pole}$?

B- I use a dynamics / rotational dynamics lens because I see forces / torques causing acceleration / angular acceleration. $\Rightarrow \Sigma\vec{F} = m\vec{a}$ & $\Sigma\vec{\tau} = I\vec{\alpha}$. This is also a statics problem so $\Sigma\vec{F} = 0$ & $\Sigma\vec{\tau} = 0$.

$\Sigma\vec{\tau} = I\vec{\alpha} = rF_{\perp} = 0$

$(\Sigma\vec{\tau} = 0)$ $\vec{\tau}_g + \vec{\tau}_T + \vec{\tau}_p = 0$

$F_g \cdot L + (-T_y)L_0 = 0$

$(100\text{ N})(3\text{ m}) + (-T_y)(1\text{ m}) = 0$

$(100\text{ N})(3\text{ m}) = T_y(1\text{ m})$

$T_y = 300\text{ N}\ \hat{y}$

$T_y \sim 0.6\ T_x$

estimate, don't use trig: $T_x \simeq \frac{1}{0.6}(300\text{ N})$

$T_x \simeq 500\text{ N}$

$\therefore T = \sqrt{(T_x)^2 + (T_y)^2} \simeq \boxed{583\text{ N} = T}$

$(\Sigma\vec{F} = 0)$ $F_p - T_y + F_g = 0$

$F_p = T_y - F_g$

$F_p = 300\text{ N} - 100\text{ N} = \boxed{200\text{ N} = F_p}$

$\Sigma F_x = ma_x = 0$

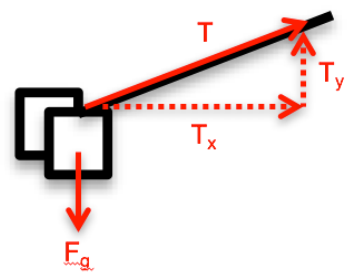
$\Sigma\vec{F}_x = 0$ also

"p" pivot; 1m; 3m; L; T_x; T_y; 1m; 1m; F_p; 10 kg; $F_g = 100\text{ N}$; (+); (+)

If T_x is the only horizontal force then there would be horizontal acceleration

So F_p must counter this T_x

2. You are watching the fuzzy dice from the rearview mirror. As you take off on level ground, it makes an angle as shown at right.



a) **** state how you will inform your choice of axis.

This is a dynamics lens because we have forces acting on the dice causing them to accelerate. We write $\sum \vec{F} = m\vec{a}$ and identify forces and direction of acceleration. Because the acceleration is horizontal, we break things up into horizontal and vertical components.

b) Estimate the acceleration of the car.

We see that $T_y = F_g$ because $a_Y = 0$. This we can see that the x component of tension (which is what's accelerating the dice in the x direction) is about twice $3*T_y = 3*F_g$ So the acceleration must be about 3 gravities or 20 m/s^2.

c) What must be the coefficient of friction of your tires for this to happen?

Because the normal force on the car = F_g between the car and road (because there's no vertical acceleration), we can see that the coefficient of friction would have to be an extraordinarily high value of 3.0 in order to attain such a high acceleration.

d) Is this realistic?

It's possible, but very unlikely, and not possible for regular tires and cars.

e) If the mass of the dice is 100 g, what is the tension in the string?

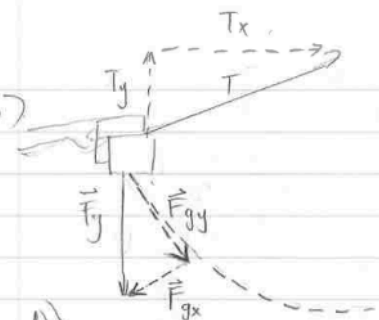
Looking back at the work we did for b) we can see that the tension should be a little more than $3*F_g$ or about 35 N.

3. Consider the fuzzy dice above. Now the car is stationary and you are sitting it in. You grab the dice and pull them to one side exactly as in the diagram above. Then you let go of them.
   a) **** Choose a good axis. Is the direction of acceleration the same as above? State how this direction will inform your choice of axis.

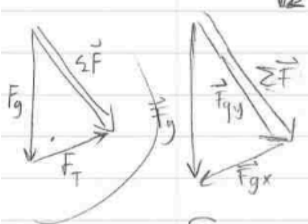
Now the acceleration is tangential, perpendicular to the radius, so our axes are radial and tangential, so we keep tension as a single force and decompose gravity into parallel (tangential) and perpendicular (radial) components.

   b) Again find the acceleration of the dice with direction.
   c) Again, if the mass of the dice is 100 g, please find the tension in the string. Is it the same as the string above? Why might this make sense?

B 3)



a) I use a dynamics lens since I see forces causing acceleration and since I see a body moving in a circular path; it has a_{cp} caused by some force; The $\vec{a}$ this time is in the direction of $\Sigma\vec{F}$ along the circle



b) I use the same lens for the same reason.

$\vec{a}_{radial} = 0$

∴ $\vec{F}_{gx} = T$

∴ $\Sigma\vec{F} = F_{gy}$

$F_{gy} \sim 0.9 F_g = 0.9(mg)$

$\Sigma\vec{F} = m\vec{a} = F_{gy}$

$0.9 \cancel{m}g = \cancel{m}\vec{a}_{tang}$

$\vec{a} = 0.9g$

$\boxed{\vec{a} = 9\ m/s^2}$ good

c) I use the same lens for the same reason.

I don't understand

$T = F_{gx} \sim 0.4 F_g$

$T = 0.4(mg)$

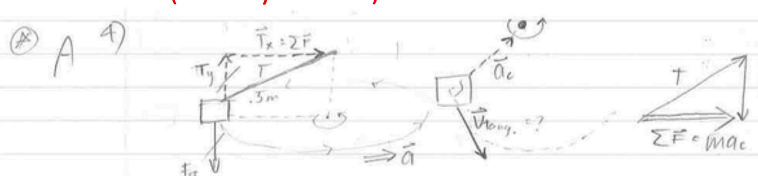
$T = 0.4(0.1\ kg)(10\ m/s^2)$

$\boxed{T = 0.4\ N}$

4. Consider the fuzzy dice in above. Now you are holding them from the end of the 50 cm string, and spinning the dice around. The path of the dice is a circle in the horizontal plane. Estimate the speed of the dice.

Now we have uniform circular motion, so there is centripetal acceleration radially inward, directly to the right. We use a dynamics lens because the forces acting on the dice are causing it to accelerate (radially inward).

A 4)



I use a dynamics lens since I see forces causing acceleration. $\Sigma\vec{F} = m\vec{a}$.

$\vec{a}_y = 0 \Rightarrow F_g = T_y$

$T_x \sim 3T_y$

∴ $T_x \sim 3F_g$

∴ $T_x = \Sigma\vec{F}$

$T_x = \Sigma\vec{F} = m\vec{a}_c = m\frac{v^2}{r}$

$3F_g = m\frac{v^2}{r}$

$3\cancel{m}g = \cancel{m}\frac{v^2}{r}$

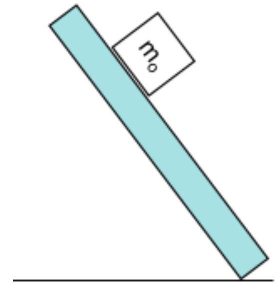
$3gr = v^2$

$\vec{v} = \sqrt{3gr}$

$\vec{v} = \sqrt{3(10\ m/s^2)(0.5\ m)}$

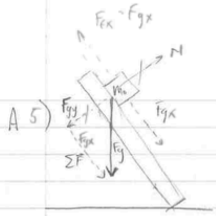
$\boxed{\vec{v} = 3.87\ m/s}$

5. Consider the mass on the inclined plane shown at right.



   a) ****, make a good choice of axis. We can't really do that yet because we don't know the direction of acceleration. However, let's assume the ramp is stationary. Then the acceleration would be *down* the ramp
   b) Estimate the acceleration if the surface is frictionless.
   c) What coefficient of friction is necessary for the block to slide at a constant speed?

A 5)



a) I use a dynamics lens since I see forces causing acceleration $\Rightarrow \Sigma\vec{F} = m\vec{a}$. I see that $\vec{a}$ is along the plane, so

y, x

b) I use the same lens for the same reason.

Since $\vec{a}_y = 0$, then $\vec{F}_{gy} = N$, $\Rightarrow \Sigma\vec{F} = \vec{F}_{gx} = m\vec{a}_x$

$\vec{F}_{gx} \sim 0.8\, F_g$

$m\vec{a}_x = 0.8\, m\vec{g} = 0.8(10\, m/s^2)$

$\boxed{\vec{a}_x = 8\, m/s^2}$

c) I use the same lens for the same reason. Constant $\vec{V}_x \Rightarrow \vec{a}_x = 0 \Rightarrow \Sigma\vec{F}_x = ma_x = 0$

$\therefore \vec{F}_f = \vec{F}_x$

$\mu N = m\vec{a}_x$

$\mu F_{gy} = m\vec{a}_x$

$\mu(0.6\vec{F}_g) = m\vec{a}_x$

$\mu(0.6)\, mg = ma_x$

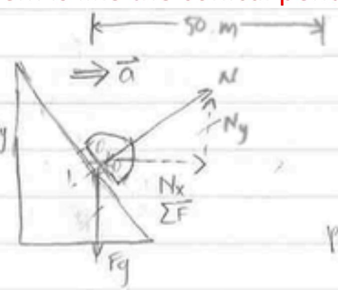
$\mu = \frac{8\, m/s^2}{6\, m/s^2}$

$\boxed{\mu = 1.3}$

6. Assume instead that the image above is for a car on a freeway off ramp as illustrated in 7.3 exercise 2. If the off ramp has a radius of 50 m, what is the ideal speed for a car not to skid if the road is very slippery?

This problem is like the conical pendulum you just solved in #4 above.

A- 6)



I use a dynamics lens since I see forces causing acceleration $\Rightarrow \Sigma\vec{F} = m\vec{a}$. I also see that the car is moving in a circular path so it has a_c caused by the net force on it.

$a_y = 0$

$\therefore \vec{F}_y = \vec{N}_y$

$\therefore N_x = \Sigma\vec{F}$

$N_y = 0.8 N_x$

$N_x = \frac{5}{4} N_y$

$\therefore N_x = \frac{5}{4}\vec{F}_g$

little small

$\Sigma\vec{F} = m\vec{a}_c$

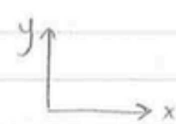
$N_x = m\frac{\vec{v}^2}{r}$

$\frac{5}{4} mg = m\frac{\vec{v}^2}{r}$

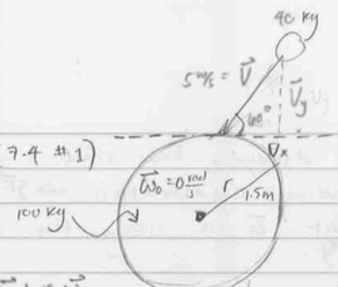
$\frac{5}{4} gr = \vec{v}^2$

$\vec{v} = \sqrt{\frac{5}{4}(10\, m/s^2)(50\, m)}$

$\boxed{\vec{v} = 25\, m/s}$



7. 7.4 Exercise 1



$\vec{V}_x = (5\,m/s)\cos 60$ $\quad \vec{V}_y = (5\,m/s)\sin 60$

$\vec{V}_x = 2.5\,m/s$ $\quad \vec{V}_y = 4.33\,m/s$

A-7) (7.4 #1)

a) I use an angular momentum $\vec{L}$ lens since I see that w/ no external torques, then $\vec{L}$ is conserved.

$\Rightarrow \Sigma\Delta\vec{L} = 0 \Rightarrow \vec{L}_i = \vec{L}_f \qquad \vec{L} = I\vec{\omega}$

$\vec{V} = r\vec{\omega}$

$\vec{\omega} = \frac{\vec{V}}{r} \qquad \vec{L} = I\vec{\omega}$

$\vec{L} = mr^2\left(\frac{\vec{V}}{r}\right) = mr\vec{v}$

$I_{\substack{point\\mass}} = mr^2 \quad I_{disk} = \frac{1}{2}mr^2$

$\vec{L}_{child} + \vec{L}_{disk} = \vec{L}_{child+disk}$

$m_c r\vec{v}_x + \left(\frac{1}{2}m_d r^2\right)\vec{\omega}_0 = \left(m_c r^2 + \frac{1}{2}m_d r^2\right)\vec{\omega}$

$\vec{\omega} = \frac{m_c r\vec{v}_x}{m_c r^2 + \frac{1}{2}m_d r^2} = \frac{(40\,kg)(1.5\,m)(2.5\,m/s)}{(40\,kg)(1.5\,m)^2 + \frac{1}{2}(100\,kg)(1.5\,m)^2}$

$\vec{\omega} = \frac{150\,kg\,m^2/s}{202.5\,kg\,m^2} = \boxed{0.74\,rad/s}$

b) I use an energy lens since I see transformations of energy as the child jumps onto the carousel. $\Sigma E_i = \Sigma E_f$.

$\Sigma E_i = KE_{child} = \frac{1}{2}mv^2 = \frac{1}{2}(40\,kg)(5\,m/s^2) = 500\,J$

$\Sigma E_f = KE_{child+disk} = \frac{1}{2}\left[(40\,kg)(1.5\,m)^2 + \frac{1}{2}(100\,kg)(1.5\,m)^2\right](0.74\,rad/s) = 75\,J$

oops, while this student did a great job, there is a little mistake here. The final kinetic energy is $E_{K-Rot} = \frac{1}{2}I\omega^2, where\ I = \frac{1}{2}m_{disk}R^2 + m_{girl}R^2$. This student forgot to square the rotational velocity. I get a final rotational kinetic energy of only 55 J., so close to 90% of the kinetic energy is transformed to thermal energy. Also, please note that although the kinetic energy is correctly calculated, the square for the speed needs to be outside the parenthesis.

Angular momentum lens

c) The collision would decrease the rotation rate, since the carousel is rotating in the opposite direction that the child would hit and provide angular momentum.

$\vec{L}_o = \vec{L}_f$

$mvr_\perp - I\vec{\omega} = L_f$

d) momentum lens. Momentum is always conserved since there are no outside forces. The girl's momentum is transferred to the Earth, but since the Earth's mass is so large, velocity is negligable.

e) momentum lens because no outside forces.

$P_o = P_f$

$\cancel{P_{co}}^{\,0} + P_{po} = P_f$

$mv_o = m_{c+p}V_f$

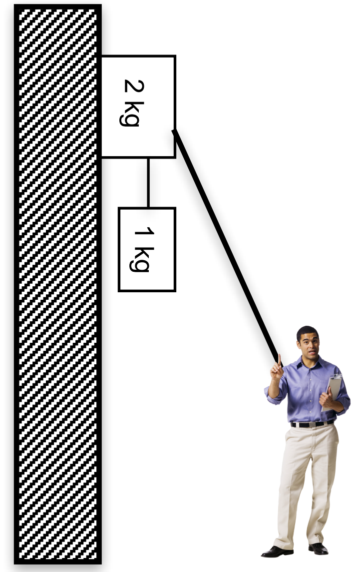
$40kg(5\,m/s) = 140kg\,V_f$

$V_f = 1.4\,m/s$ southwest

Also, again note that momentum is conserved (*all components* of momentum) independent of conserving the angular momentum (where only the tangential component contributes).

8. Please start (and possibly finish) your final project. Please let me know if there's anything I can do to support the project.

9. Please see a picture of me pushing a system of masses up a vertical wall with a coefficient of friction of 0.4. If I push the stick as shown with a force of 80 N, please find the approximate acceleration of the system of masses, and the tension in the string holding the 1 kg mass.

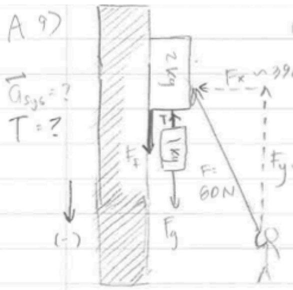


In this problem, we use a dynamics lens and recognize that the acceleration is upward (or downward), so we choose x-y coordinates. $a_x=0$, so we know the normal force = the x component of the force I put on the stick.

A 9)

$\vec{a}_{sys} = ?$

$T = ?$



a) I use a dynamics lens since I see forces causing acceleration. $\Rightarrow \Sigma F = m\vec{a}$

$$\vec{F}_y - F_g - F_f = (m_1 + m_2)\vec{a}_{sys}$$

$$\vec{a}_{sys} = \frac{F_y - (m_1 + m_2)g - \mu F_x}{m_1 + m_2} \qquad \mu = 0.4$$

$$\vec{a}_{sys} = \frac{70N - (3kg)(10 m/s^2) - (0.4)(39N)}{3kg}$$

$$\boxed{\vec{a}_{sys} \approx 8.2 \ m/s^2}$$

b) I use a dynamics lens for the same reason.

$$T - F_{g_2} = m_2\vec{a}$$

$$T = m_2\vec{a} + m_2 g$$

$$T = (1 kg)(8.2 m/s^2 + 10 m/s^2) \approx \boxed{18.2N}$$

We're glad to see that the tension in the string > F_g because the mass attached to the string is accelerating upwards.