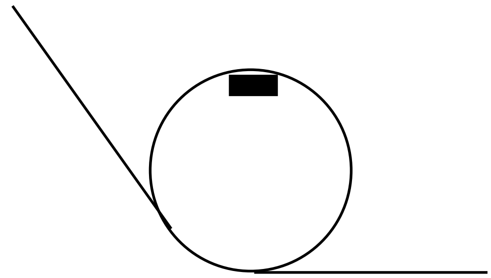
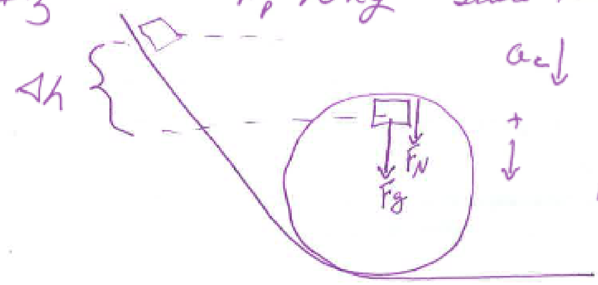
PS#9 Solutions and direction

1) There is a video about this firehose problem for week 11

2) *If you think you've already done this problem before, just skip down to part d.* I summarize a)-c) for the solutions below and focus on d)
   d) Let's say that the object is instead a hollow sphere that is rolling without slipping. How would this change the problem? Can you do part a) and c) above for this scenario? Essentially, this is what we're asking: for a) would the rolling sphere be going faster than the frictionless cart or slower – how much faster or slower – is the acceleration greater or less?; and for b) in order to make it around the loop without falling, would the hollow sphere need to start from higher or lower than the frictionless cart? How much higher or lower? The physics involved would be an energy lens… and dynamics. Using dynamics, we understand that the acceleration at the top has to be at least gravity. So this centripetal acceleration dictates the cart's minimum velocity to stay connected to the track.
   So we have to start the cart on the incline *higher* than the cart is at the top of the loop. If the cart is a hollow rotating ball, then the ball has to be rotating as well as moving, so it needs more energy, so it has to start from a greater elevation to provide that extra rotational kinetic energy. The exact equations follow below, but to me are less important. What is important is that you can do the FBD and show the energy conversions.



#3 $m_p = 70\,kg$ Scale reads F_N



$a_c \downarrow$ Dynamics Z

$\Sigma F_y = m a_c$

$F_g + F_N = m a_c$

or $a_c \geq g$

The way we solved this was

$\Delta PE \Rightarrow KE$ (energy)

$mg\Delta h = \frac{1}{2} m v^2$ and $a_c = \frac{v^2}{R} \geq g$

$2g\Delta h = v^2$

so $v^2 \geq Rg$

But Now we have to consider $KE_{Rot} = \frac{1}{2} I \omega^2$ is needed to have the sphere roll.

$I_{\text{hollow sphere}} = \frac{2}{3} MR^2$, so now

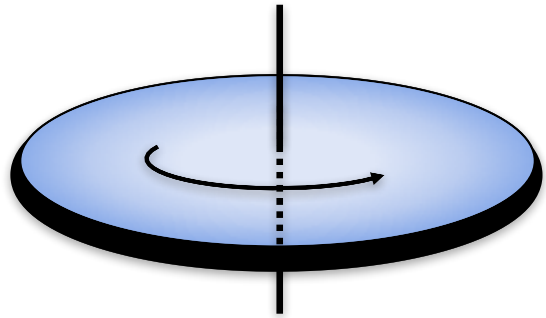
$mg\Delta h = \frac{1}{2} m v^2 + \frac{1}{\cancel{2}}\left(\frac{\cancel{2}}{3} m \underbrace{R^2)\omega^2}_{v^2}\right.$

$mg\Delta h = \frac{1}{2} m v^2 + \frac{1}{3} m v^2 = \frac{5}{6} m v^2$

$\cancel{m}\cancel{g}\Delta h = \frac{5}{6} \cancel{m} R \cancel{g}$

$\Delta h = \frac{5}{6} R$ which $> \frac{1}{2} R$ we got for a frictionless cart.

3) At right is shown a spinning solid disk on a vertical axis 80 cm long (40 cm above and 40 cm below) spinning in space in front of you, balanced on its axes. It has a mass of 2 kg and a radius of 0.5 m and is spinning around 10 times per second!



a) Show that the angular momentum of this disk is almost 15 Js, and define the direction of the angular momentum vector.

b) Then, looking at the paper, you grab the top of the axel with one hand and the bottom of the axel with the other hand. You pull the top out toward you with 20 N and push the bottom away from you with 20 N of force. You do this for a tenth of a second (0.1 s). Calculate the torque you apply to the wheel (include direction), and the angular momentum you impart onto the wheel. Remember to include direction.

c) After you do the above act for half a second, you let the wheel go again. Is the wheel tilted now? If not, why? If so, in which direction and by about how much? Draw the wheel in its new position. It tilts to the right, rotating in the direction into the paper, by about 1/10 of a radian… ~ 10 degrees.

d) What will the wheel do now if I let it balance on the bottom of the axel? Describe the motion as exactly as possible. Now that it's tipping, gravity will provide a torque into the paper, so that will make the top precess around in the upward direction (counter clockwise when viewed from above).

4) Take a bicycle wheel and spin it very fast, then support the horizontal axel only at one end (some distance “x” from the center of the wheel’s hub, letting the other side “fall”.

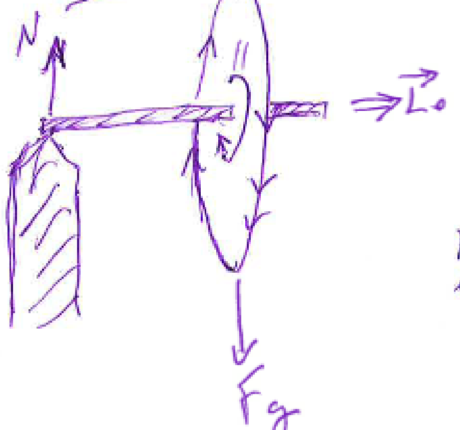
#4 see solution Provided

see video (Big Picture Rotation video at 8:59 to 9:25)

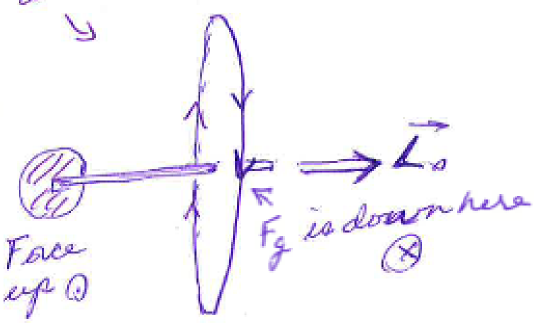
From The side

Look at it from above:

$I\vec{\omega} = \vec{L}$ in this direction →



From above



τ is ⊗ into page

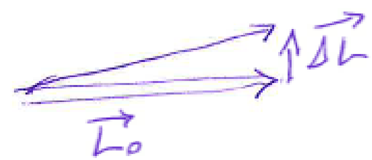
$\vec{\Delta L} = \vec{\tau} \cdot dt$ is ⊗

So wheel precesses into page or counter-clockwise from above.

τ is ↑

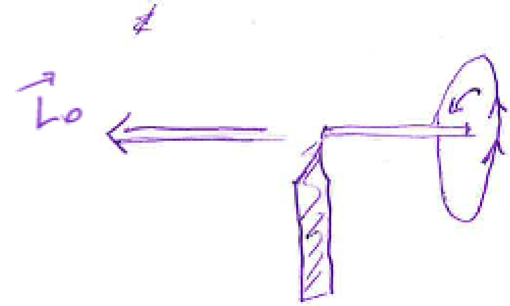
$\vec{\Delta L} = \vec{\tau} \cdot dt$ is in ⇑ direction

so $\vec{L} = \vec{L}_0 + \vec{\Delta L}$



Precession is counter clockwise when viewed from above

b) if we change the direction of $\vec{\omega}$, then $\vec{L}$ is in the other direction, but the torque from gravity is in same direction, so $\Delta\vec{L}$ is same direction



$\Delta\vec{L}$ is still $\otimes$, so the $\vec{L}$ would get pushed into the paper, and the wheel would come out, precessing in opposite direction.

c) if $\vec{L}_o$ is bigger, then the same $\Delta\vec{L} = \vec{\tau}_g \Delta t$ would change the angle of $\vec{L}$ by less, so precession would decrease

d) if r_+ is less, $\tau_g \Downarrow$ so $\Delta\vec{L} = \vec{\tau} dt$ is less, so it would precess slower

e) $I \Downarrow$, so $\vec{L}_o$ decreases, but mass is the same, so τ_g is the same. So the same $\Delta\vec{L} = \vec{\tau} dt$ will rotate the ~~wh~~ wheel's axis more, so precession would increase in rate

5) In class, you saw me spin a turntable at an elevation of about 1 m, with a penny at a radius of about 20 cm. The penny slides off the turn table and hits the floor about 0.5 meters from where it was on the turn table. Please estimate the coefficient of friction between the penny and the turn table.

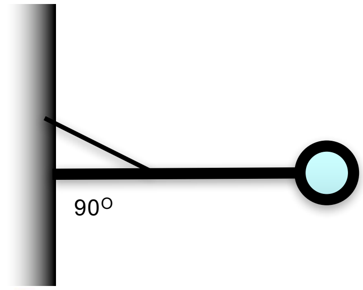
is a combination of kinematics (projectile motion) and (circular) dynamics. It requires two drawings, which I expect of you, but I am not supplying right now:

Using dynamics, make a good FBD and follow the protocol and you can see that the force of friction is the only radial force, so you set it equal to mass*centripetal acceleration. This would provide us with the coefficient of friction, except that we don't know the speed… but we know how far it moves horizontally when it falls, so we go to the kinematics lens.

Please show the parabolic trajectory as the coin falls from the edge of the spinning turntable. We want the initial horizontal velocity = dx/dt. dx = 0.5 m, but how about dt? This is revealed in the vertical component because the coin is falling from rest (vertically speaking) and hits the ground. I could use energy to find v_{final} and then use $v_{average} = ½v_{final}$ = dx/dt. Or I could just use kinematics that dx = $½at^2$. You should find that it takes about 0.45 s to fall 1 m from rest, so the initial horizontal speed is about 1.1 m/s.
Now we can go back to our circular dynamics lens and see that the centripetal acceleration at this speed at a radius of 20 cm is about 6 m/s^2, requiring a frictional force of m*6 m/s^2, or a coefficient of friction of 0.6.

Could we test this? Sure! Please show that if we slowly tipped the surface, an object with a coefficient of friction of 0.6 will slide when the surface is inclined at about 30 degrees.

6) Remember the problem #1 from PS#8? What happens to the tension as we bring the string closer to the pivot point? Please estimate the tension in the string at right for a mast supporting a 100 kg sphere and the reactive force at the pivot. There is no motion. This requires a dynamics lens, but in particular a statics lens. There is no acceleration (so the vector sum of the forces = 0) and there is no angular acceleration (so the vector sum of the torques = 0). Then I label my forces with a good FBD and begin to solve these two equations! Because there's two unkown forces at the pivot, I will make this point the center of rotation and start with the torque equation. I also know there's only two other torques – a known torque from the force of gravity on the ball, and an unknown torque from the *perpendicular* component of tension. These two torques must balance each other.



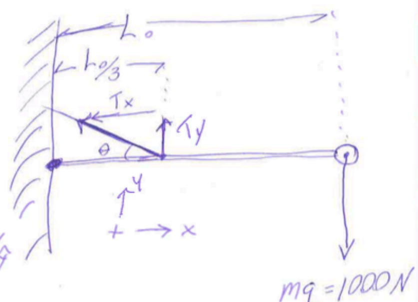
$(+\circlearrowright)\quad \otimes$

$\Sigma \vec{\tau} = 0$

$\tau_{gravity} + \tau_{tension} = 0$

$mgL + -T_y \frac{L}{3} = 0$

$T_y = 3mg = 3000\,N\,\hat{y}$



$\tan\theta = \frac{T_y}{T_x} \approx \frac{1}{2}$, so $T_x \approx 2T_y \approx -6000\,N\,\hat{x}$

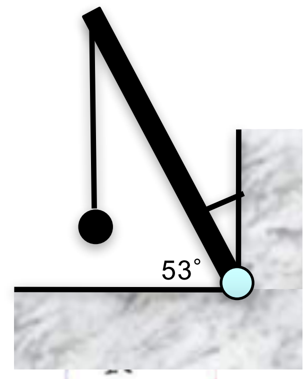
$\Sigma \vec{F} = 0$, so $\Sigma F_x = 0$ ∴ There is a reactive force from the wall on the pivot of $+6000\,N\,\hat{x}$

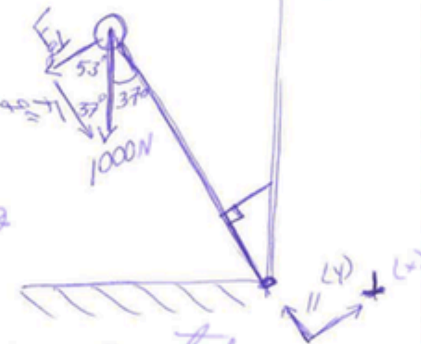
$\Sigma F_y = 0 = F_{pivot} + T_y + F_{g\,Ball}$

$0 = F_{pivot} + 3000N - 1000N$

$F_{pivot} = 2000 \Downarrow$

7) In the diagram at right, a post of some length supports a 100 kg ball. From the drawing at right (make your own better drawing), estimate the tension on the string and the reactive force at the pivot. There is no motion. This requires a dynamics lens, but in particular a statics lens. There is no acceleration (so the vector sum of the forces = 0) and there is no angular acceleration (so the vector sum of the torques = 0). Then I label my forces with a good FBD and begin to solve these two equations!





we estimate that the string is fastened at a point on the post at a point about $\frac{1}{4}$ of the posts' length from the pivot. I pick the pivot as center of rotation because then I can ignore the 2 dimensional forces acting on the pivot. $+\theta$

$\boxed{\text{Tension} \perp \text{ to post, so torque} = T \cdot \frac{L}{4}}$

$\sum \vec{\tau} = 0 = \tau_{string} + \tau_{mass}$

$= \frac{L}{4} \cdot T + F_{g\perp} L$

53° is a 3-4-5 rt Δ, so $F_{g\perp} = 600\,N$

$T_{string} = 4 F_{g\perp} = 2400\,N$

now find reaction Force:

$\sum F_{\parallel} = 0 \quad F_{g\parallel} + F_{React\parallel} = 0 \quad F_{React\parallel} = 800\,N$

$\sum F_{\perp} = 0 \quad F_{g\perp} + T + F_{R\perp} = 0 \quad F_{R\perp} = 1800\,N$

$-600\,N + 2400\,N = -F_{R\perp}$

7) You need to make a freeway off ramp of radius 20 m that allows cars at 20 m/s to execute a turn without slipping off the ramp when it is icy in winter. We think it is a good idea to tip the ramp. Why will this work? Draw this ramp with approximately the right amount of inclination. Does it have to be tipped a lot or just a little? You have to tip it a lot… a way lot… like 63 degrees from the horizontal. So what you have to do is recognize that this is a dynamics problem and write down $\sum \vec{F} = m\vec{a}$ and then look for the forces to give me the mass times acceleration. You draw a FBD of the car on the ramp and note that the acceleration is horizontal because the car is moving in a circle, so the horizontal component of the normal force of the road is causing centripetal acceleration. You see this drawing at right. The car is driving away from you as it rounds the corner, and the center of the circular track is to the left. Then you have to carefully draw in the forces on the car and add them as vectors so that they point in that horizontal direction. Note you have a right triangle and you can use trig.