Problem Set #4 due beginning of class, Thursday, Oct. 20.

#1 Please redo your midterm showing work, but not providing way too much information. Please find guidance for this on the main class webpage under exams.

#2 From chapter 10.1 on rotational variables, start with the unnumbered exercise about the 777's engine: The GE90-110B1. Note that this turbo fan has a diameter of 3.25 m, but read more about it yourself at: https://en.wikipedia.org/wiki/General_Electric_GE90
This is straight up kinematics… rotational kinematics. We also know because it says the blades speed up constantly, we know the instantaneous angular acceleration is the same as the average angular acceleration. First Make a good drawing and change everything into angular coordinates, radians. So answer (a) = answer (b) = $\vec{\alpha} = 12.6/s^2$, or *12.6 radians/s^2* and in the drawing, the direction of alpha is out of the paper at you.

Please answer a) and b) from the question itself. Then add to it:
c) What is the linear speed of the outer edge of the turbo fan?
Kinematics because I know how fast it's spinning and the radius. I get a speed of about *400 m/s*, slightly more than the speed of sound, and about 30% faster than the speed of the 777 itself:
How does it compare to the top speed of the 777… Oh WOW. The top speed of the 777 is Mach 0.89 – 89% the speed of sound! The speed of sound at room temperature is about 340 m/s.
d) What is the average tangential linear acceleration of the outer edge of the turbo fan blade? Again, kinematics: about 2 gravities ~ 20 m/s^2
e) What is the acceleration of the 777 at a mass of 200,000 kg, with two of these engines? I want acceleration, and I know the force, so this must be a dynamics problem… I get about *4.5 m/s^2*, so like 1/2 a gravity. The take-off speed is about *100 m/s* so we'd expect it to take about 20 s to take off… check it the next time you take off.


Check Your Understanding The fan blades on a turbofan jet engine (shown below) accelerate from rest up to a rotation rate of 40.0 rev/s in 20 s. The increase in angular velocity of the fan is constant in time. (The GE90-110B1 turbofan engine mounted on a Boeing 777, as shown, is currently the largest turbofan engine in the world, capable of thrusts of 330–510 kN.)

(a) What is the average angular acceleration?

(b) What is the instantaneous angular acceleration at any time during the first 20 s?

Figure 8. (credit: "Bubinator"/ Wikimedia Commons)

#3) I take off northward from a light on my bicycle with 700 mm wheels (diameter) at constant acceleration. After 2 seconds, I'm traveling 10 m/s. Please find: Kinematics, only motion.

1) The rotational velocity of my wheel. $\vec{\omega} = 28.6/s$ *west*
2) The angle I've rotated my wheel through. $\vec{\theta} = 28.6$ *west*
3) The rotational acceleration of the wheel. $\vec{\alpha} = 14.3/s^2$ *west*
4) The direction of the rotational velocity of the wheel. *west*

Angles should be radians

#3 so d = .700m

700mm constant acc $\rightarrow$ 2 s v = 10 m/s

1. Kinematics lens bc finding rotational velocity ω

$v = r\omega$

$(10 m/s) = (.35 m)\omega$

$\boxed{\omega = 28.57 \frac{rad}{s}}$

$l = \theta r$ $v = \frac{dl}{dt} = \frac{d\theta r}{dt} = r\omega$

$\omega = \frac{d\theta}{dt}$

2. Kinematics bc finding θ

$\omega = \frac{d\theta}{dt}$

$\omega_{ave} = \frac{\omega_f}{2} = \frac{28.57 \frac{rad}{s}}{2} = \frac{d\theta}{2 s}$

$\boxed{\theta = \frac{54 \, rad}{2}}$ = 28.57 rad

3. Kinematic bc finding α

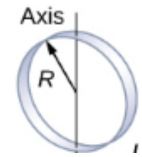
$\alpha = \frac{d\omega}{dt}$

$\alpha = \frac{28.57 \frac{rad}{s}}{2 s} = \boxed{14 \frac{rad}{s^2}}$

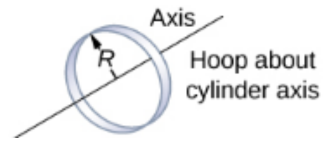
4. ω is $\xleftarrow[west]{}$ using Right Hand Rule

#4 Please do the first problem with the three rotating masses in chapter 10.4. *The solutions are in the example. If there's a lens here, you could think of rotational energy, and the moment of inertia depends crucially on the radius, because if you double the radius, you double the speed, you increase the kinetic energy by 4! So you see that leaving out the center washers has almost no effect on the moment of inertia. How about if we removed the two outer masses?*

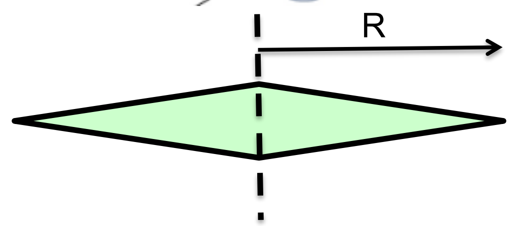
#5 The following objects have the same mass and same radius. Please put them in order of lowest moment of inertia to highest moment of inertia by thinking about the radial distribution of mass: It's all about how the mass is distributed. I think I got these right. I wrote "3" twice because I think that they are the same.



a) A solid sphere 2
b) A hollow sphere (remember these are the same size and same mass, so this one must have a very dense shell for just the shell to have the same mass as the solid sphere in a) above. 4
c) A ring spinning about a diameter as shown at right. The axis of rotation is the vertical line. 3
d) A flat disk of uniform thickness rotating about a diameter as shown above. 1
e) A ring spinning about the central cylindrical axis as shown at right (below). 2
f) A flat disk of uniform thickness rotating about the central cylindrical axis. 3



Check your answers in chapter 10.4 on moment of inertia.



#6 You invent a new kind of round discus that spins about a vertical axis (dotted line) as shown at right. The object has a thickness of t_o at the axis (at $r=0$) that tapers evenly to a sharp edge at $r=R$, or $t = t_o(1-r/R)$. The mass of the discus is M,

a) Judging from moments of inertia of other objects (above question), please guess as best you can what should be the moment of inertia about the axis in terms of the variables given, and support your estimate with reasons. For starters, you might consider if this moment of inertia is greater or less than a rim of mass M, a disk of mass M, a hollow or solid sphere of mass M.
b) Calculate exactly what the moment of inertia is by integrating over the mass. *Hint: You'll have to do two integrations for this: one to find the volume, and the next to find the moment of inertia. A similar problem was done in the moment of inertia video.*
You can find this solution here:
http://sharedcurriculum.wikispaces.com/file/view/PS7_SUS_W15_Q5%20solution.jpg/542965166/PS7_SUS_W15_Q5%20solution.jpg

#7 You have an ax to grind, and you decide to grind it on the outer rim of a round 5 kg stone grinding wheel of uniform thickness and radius 30 cm. The coefficient of friction between steel and stone is 0.3. You spin the wheel up to 1000 rpm with a 100 W motor. There are two concepts here that come to mind. I give you the power of the motor. Over time, this motor will deliver rotational kinetic energy to the wheel. We think about energy. Then I put the ax to the wheel and there is friction and the wheel slows down. I can use rotational dynamics to solve this one because I can calculate the torque from the friction and from that I can calculate the time it takes to slow down.

a) How long does it take to spin the wheel up to 1000 rpm? What lens do you use?

b) Then I push the ax against the wheel with a force of 100 N and the sparks fly! But as soon as you start, the electricity goes out and the wheel is spinning freely without power. What is the angular acceleration of the wheel as you push against it with the ax?

#7

$1000 \frac{rev}{min}$ power = 100W

$\omega = \frac{2000\pi \, rad}{min}$, $104.7 \frac{rad}{s}$

$33\frac{1}{3}\pi$

mass = 5kg

$\mu = 0.3$

a) Energy lens because power = $\frac{\Delta E}{\Delta t}$ and we can use this to find time

$100W = \frac{\frac{1}{2}I\omega^2}{\Delta t}$, $\frac{1233J}{100W} = \Delta t$ $\boxed{t = 12.33\ s}$

solving for KE = $\frac{1}{2}\left(\frac{mr^2}{2}\right)(\omega^2)$

$\frac{\frac{1}{2}(5kg)(.3m)^2(104.7\frac{rad}{s})^2}{2} = 1233J$

b) F = 100N

power = 0W

α = ?

Dynamics lens use frictional force to find the torque, then set $\tau_f = I\alpha$ to find α

$F_{f\perp} = \mu N$

$= (0.3)(100N)$ →

$F_{f\perp} = 30N$

$\tau_f = F_f \times r$

$= 30N \times 0.3m$

$\tau_f = 9N \cdot m$

Knowing τ_f, set it = $I\alpha$

$I \to \frac{mr^2}{2}$

$9N \cdot m = \frac{(5kg)(0.3m)^2}{2}\alpha$ = $0.225\ kgm^2\ \alpha$

$\boxed{\alpha = 40 \frac{rad}{s^2}}$

c) How long does it take to stop the ax?

Kinematics lens bc finding time and $\alpha = \frac{\omega_f - \omega_o}{\Delta t}$

and we know that $\alpha = 40\frac{rad}{s^2}$, $\omega_f = 0\,rad/s$ and ω_o is $104.7\frac{rad}{s}$

$40\frac{rad}{s^2} = \frac{104.7\frac{rad}{s} - 0\ rad/s}{t}$ → $\frac{104.7\frac{rad}{s}}{40\frac{rad}{s^2}} = t$

$\boxed{t = 2.6\ s}$

wow! that's really short. I shouldn't push so hard on my ax!