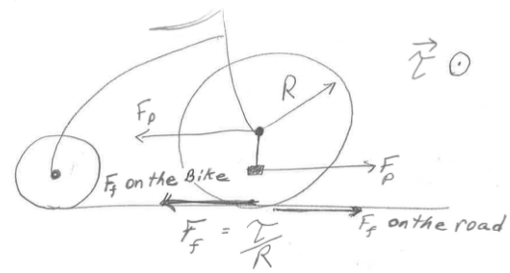
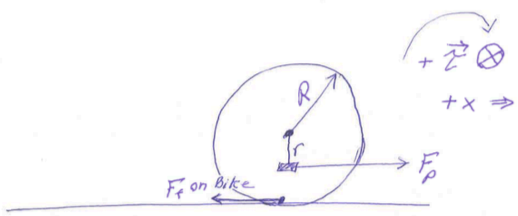
In solving why when pushing the bottom pedal forward, the tricycle goes forward when pushing it from the outside and backwards when pushing it when you're on the bike would be better explained starting from when you are on the bicycle.



When you are on the bicycle we can look at your body as in near equilibrium. It's true that your body is accelerating, but let's say for the sake of making the problem easier, that the acceleration is small, so the net force on your body is near zero and the net torque on the wheel is near zero. It's like your body is a motor – it simply provides torque to the wheel. Thus, the net force on you is zero. So, as hard as you push the pedal forward, you are pushing backwards on the handle bars, or seat or someplace else on the bike connected to the wheel at its center. The center of the wheel is the center of rotation. In this case, the only outside force on the bike is that from friction under the drive wheel. I've also put the equal and opposite frictional force on the ground, but this doesn't belong in the FBD because it's not a force on the bike. The force on the lower pedal provides a torque out of the paper at you, and thus pushes the ground forward, pushing the wheel backwards. And this is your experience, if you push the lower pedal forward while sitting on a trike, you accelerate backwards.

However, if the forward force is applied from the outside, then you have to compare the forward force on the pedal to the backwards force provided by the force of friction on the wheel. The easiest way to solve the problem is recognize that the point of contact between the wheel and the ground is the center of rotation, and pushing the pedal forward applies a torque into the paper about the point of contact. However, you can also solve the equations of motion using the center of the wheel as the center of rotation. The way I've defined the positive direction of torque and displacement, if alpha is positive, so is acceleration. If $F_p > F_f$, then alpha can be positive on both sides because R > r and the tricycle accelerates to the right. If $F_p < F_f$, then alpha on the right is negative, and alpha on the left is positive… impossible. If you want to do the explicit math, it's below.



$$\sum \tau_{center} = -\tau_p + {}^{+}\tau_f$$
$$I\alpha = -F_p r + F_f R$$
$$\alpha = \frac{-F_p r + F_f R}{I}$$

$$\sum \vec{F} = -F_f + F_p$$
$$ma = -F_f + F_p$$
$a = \alpha R$, so
$$\alpha = \frac{-F_f + F_p}{mR}$$

$$\alpha = \frac{-F_p r + F_f R}{(c)mR^2} = \frac{-F_f + F_p}{mR}$$ multiply by cmR^2

(c) = constant of rotational body

$$-F_p r + F_f R = -F_f cR + F_p cR$$
$$F_f(R + cR) = F_p(cR + r)$$
$$F_f\frac{(R + cR)}{(r + cR)} = F_p$$

because $R > r$, $F_p > F_f$

Because the forward push on the pedal is greater than the frictional force, the acceleration is in the positive direction. By multiplying by r and R, you can find the torque for each of the two forces and show that the torque from friction is greater than the torque from pulling on the pedal and the net torque is in the positive direction, again accelerating the bike forward.